A Self-starting Control Chart for Monitoring Binary Response Profiles

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Abstract

In some processes, the quality characteristic is characterized by a functional relationship between a response variable and predictor variables which is named as a profile. The response variable in profiles usually follows a normal distribution. However, sometimes the response variables have different distributions such as Bernoulli or Binomial. On the other hand, in many situations in real applications the in-control (IC) parameters of the processes are not known a priori and also there are no sufficient samples to execute Phase I analysis to estimate the in-control parameters. In this case, we desire to use self-starting charting schemes that monitor the state of processes. In this study, we consider logistic regression profiles which have binary response variable and propose a self-starting procedure to monitor the process. The performance of the proposed method is evaluated by using average run length (ARL) criterion. The results showed a satisfactory performance of the proposed control chart.

Keywords: Control chart, Self-starting, Profile monitoring, Binary response, Logistic regression profile, Average run length (ARL)

1. Introduction

Quality control charts help us to analyze quality both mathematically and visually by charting the quantity of interest such as the mean value of a quality characteristic of the process. We estimate the parameters of a process by collecting a sufficient amount of data from the in-control process. This analysis is referred to as Phase I. We can then determine the state of the process based on the estimated parameters. If we determine the process has deviated significantly from its in-control state, the engineer should take corrective action to resolve the quality issues. In some cases, the quality of a process is characterized by a functional relationship between a response variable and one or more explanatory variables referred to as profile in the literature. [1] considered simple linear profiles (SLP) and developed two control charts for Phase II monitoring. [2] proposed three exponentially weighted moving average (EWMA) control charts to monitor SLP parameters. [3] proposed a likelihood ratio (LR) based control chart to monitor SLPs. [4] developed three methods to monitor polynomial profiles in Phase I and also developed likelihood ratio test (LRT) to identify the change point. [5] proposed a new reduction method to overcome the problems related to dimensionality of old monitoring methods for multiple linear regression profiles (MLRP) in Phase II. [6] developed two sum of squares control charts for monitoring multivariate multiple linear regression profiles (MMLRP) in Phase II.

In the above mentioned papers, the distribution of the response variable is normal. Sometimes, in industrial processes it is necessary to categorize the product or quality level into two categories. For example, the quality level of the produced batches can be classified into two groups, acceptable or non-acceptable. In this situation, the response variable is a Binary variable. The logistic regression profile is suitable for modeling the regression with binary response variables. [7] applied 5 Hotelling's $T^2$ control charts to monitor binary profiles in Phase I. [8] proposed a new control chart by integrating LRT and EWMA statistics. [9] proposed two methods to monitor residuals and parameters of logistic regression profiles in Phase II. [10] applied four different monitoring procedures in Phase II monitoring of logistic regression profiles.

Usually, the time and effort taken to get enough observations and reach a proper estimation of in-control parameters is difficult and costly. Hence, we propose the charting technique which has a self-starting property and charting can be done at the start of a production run which helps to reduce the cost and time of data gathering. In order to monitor these processes, it is desired to develop a self-starting control chart to monitor the process from the start-up stages of the process. These charts start monitoring with a few initial samples and when the process is running these charts update the parameters by each new observation.
and check the out-of-control (OC) condition simultaneously. [11] initially proposed a self-starting cumulative (CUSUM) scheme that utilizes two pairs of CUSUMs one for monitoring the location of the process and the other for monitoring the spread. [12] proposed the self-starting Q chart for monitoring both the mean and variance. [13] extended his original Q chart methodology and proposed the CUSUM of Q statistic. [14] proposed solutions to account for the bias of Shewhart Q charts when the out-of-control ARL is larger than the desired in-control ARL. [15] offered an in depth analysis and improved design techniques for the CUSUM of Q statistics. A self-starting control chart proposed by [16] also uses a EWMA procedure and combines it with likelihood ratio test to monitor the process mean and variance simultaneously when the process parameters are not known prior to start up. [17] developed what they call the adaptive cumulative score (ACUSCORE) control chart which accounts for dynamic patterns in the process mean by utilizing an adaptive EWMA for a CUSCORE chart. [18] investigated monitoring the Poisson rates with varying population size and unknown process parameters. [19] investigated the average of the in-control (IC) average run length (AARL) and standard deviation of IC average run length (SDARL) to evaluate the IC run-length performance of self-starting control charts that conditioned on the initial samples used for parameters estimation. [20] proposed a self-starting control chart based on recursive residuals referred to as SS chart to monitor SLP’s parameters. In this paper, we proposed a self-starting $T^2$ control chart to monitor generalized linear models with binary response variable. The problem is modeled with logistic regression profile which is a proper method to model this type of problem.

In the remaining of this paper the following contents are organized: model and parameters estimation of logistic regression profile are explained in Section 2. In Section 3, the proposed self-starting method is proposed. The performance assessment of the proposed method is considered in Section 4. A numerical example is presented in Section 5. Concluding remarks and future studies are given in the final section.

2. Logistic regression Profile Model

The Generalized Linear Models (GLM) is greatly used to model profiles under the situation that the responses are not continuous. When there is a binary variable in the process, the logistic regression profile (LRP) is the most frequently model used to characterize the process. Suppose that there is $n$ independent settings and $p$ explanatory variables in each setting which is denoted by $X_i = (X_{i1}, X_{i2}, ..., X_{ip})^T$, the corresponding response variable is noted by $z_{i}$, $i = 1,2,3,...,n$. Each of the response variables $(z_{i})$ is assumed to follow Bernoulli distribution with success probability $\pi_{i}$, $i = 1,2,3,...,n$. According to mean and variance of Bernoulli distribution, $E(z_{i}) = \pi_{i}$ and $\text{Var}(z_{i}) = \pi_{i}(1-\pi_{i})$. Since the probability $\pi_{i}$ is a function of $X_i$, there is a link function which relates $\pi_{i}$ and $X_i$ in LRP. The Logit link function is suitable for linking and denoted by $g(\pi_{i})$. The logistic regression profile defined as Equation (1).

$$g(\pi_{i}) = \log\left(\frac{\pi_{i}}{1-\pi_{i}}\right) = \beta_1 X_{i1} + \beta_2 X_{i2} + ... + \beta_p X_{ip},$$

(1)

where $\beta = (\beta_1, \beta_2, ..., \beta_p)^T$ is the vector of the model parameters. To show intercept in the logistic regression model, it is common to set $X_{i1} = 1$. The success probability in the logistic regression model is defined as follows:

$$\pi_{i} = \frac{\exp(X_i^T \beta)}{1+\exp(X_i^T \beta)},$$

(2)

It is also assumed that for $i$th setting of the explanatory variables, there are $m_i$ observations, $i = 1,2,3,...,n$. The total number of observations is denoted by $M = \sum_{i=1}^{n} m_i$. It is clear that $y_i = \sum_{j=1}^{m_i} z_{ij}$ is the total number of success and follows a binomial distribution with parameters $(m_i, \pi_{i})$, so the mean and variance of the response variable are calculated by Equations (3) and (4), respectively.

$$E(y_i) = m_i \pi_{i},$$

(3)

$$\text{Var}(y_i) = m_i \pi_{i}(1-\pi_{i}),$$

(4)

Besides, for estimating the parameters values we should use likelihood function which is written as Equation (5):

$$L(\pi,y) = \prod_{i=1}^{n} \left( \frac{m_i}{m_i} \right)^{\pi_{i}} \left( \frac{1}{m_i} \right)^{1-\pi_{i}},$$

(5)

where $\pi = (\pi_1, \pi_2, ..., \pi_n)^T$ and $y = (y_1, y_2, ..., y_n)^T$. Taking the logarithm of two sides of Equation (5), result is written as follows:

$$\log L(\pi,y) = \sum_{i=1}^{n} \log \left( \frac{m_i}{m_i} \right)^{\pi_{i}} + \sum_{i=1}^{n} y_i (X_i^T \beta)$$

(6)

$$- \sum_{i=1}^{n} m_i \log(1+\exp(X_i^T \beta))$$

After taking partial derivative with respect to $\beta$ and solving the equation $X_i^T (y - \mu) = 0$, the best estimations of process parameters are obtained.
\[
\frac{\partial \log L(\pi, y)}{\partial \beta} = \sum_{i=1}^{n} y_i X_i - \sum_{i=1}^{n} m_i \exp(X_i^T \beta) + 1 + \exp(X_i^T \beta) X_i
\]

\[
= X^T (y - \mu),
\]

where \( \mu = (\mu_1, \mu_2, ..., \mu_p)^T = E(y) = (m_1 \pi_1, m_2 \pi_2, ..., m_p \pi_p)^T \), \( X = (X_1, X_2, ..., X_p) \) is a \( n \times p \) matrix and \( \theta = (0, 0, ..., 0)^T \) is a \( p \)-dimensional zero vector. In this paper, the iterative weighted least square (IWLS) method is applied to obtain MLE of \( \beta \), shown by \( \hat{\beta} \) notation (see [7], [9] and [10] for more information). According to logistic regression profile model and the estimation of parameters, \( \hat{\beta} \) asymptotically follows \( N_p(\beta, (X^T WX)^{-1}) \) in which \( W \) value of is obtained as follows:

\[
W = \text{diag}\{m_1 \pi_1 (1 - \pi_1), m_2 \pi_2 (1 - \pi_2), \ldots\}
\]

3. Proposed Self-starting Control Chart

In this section, a self-starting \( T \) control chart is applied to monitor LRP model. In designing the proposed control chart the process parameters are updated continually during the sampling procedure sample by sample and also the out-of-control condition is checked simultaneously. Since the parameters of the logistic regression profile are unknown a priori, we should use the estimated values to construct the self-starting \( T \) control chart. The proposed self-starting control chart statistic for monitoring the regression model parameters is given as Equation (9).

\[
SST_j^2 = (\hat{\beta}_j - \hat{\beta}^{(j-1)})^T S^{-1} (\hat{\beta}_j - \hat{\beta}^{(j-1)}), \ j = 1, 2, ...
\]

where \( \hat{\beta}_j = (\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_p)^T \) represents the estimated parameters vector of the \( j \)th sample, and \( \hat{\beta}^{(j-1)} \) is the estimated vector of process parameters up to time \( j - 1 \), (the estimated parameters based on \( 1, 2, ..., j - 1 \) previous observations). Since the estimated covariance matrix \( \hat{S} = (X^T WX)^{-1} \) is a function of parameters vector, hence, we also used \( \hat{\beta}^{(j-1)} \) to estimate the covariance matrix up to time \( j - 1 \).

The designed control chart triggers an OC signal when \( SST_j^2 \) value exceeds the upper control limit (UCL), \( (SST_j^2 > UCL) \). The UCL value is set by using simulation runs to achieve a desirable in-control ARL. In the self-starting methods, it is needed to pool all \( j - 1 \) in-control historical samples and future \( j, j + 1, j + 2, ... \) observed samples in one sample and estimate the IC parameters to use as \( \hat{\beta}^{(j-1)} \) up to \( j - 1 \) in proposed methods. When the number of pooled samples gets larger, the time of calculations for estimation gets larger as well. Hence, a recursive equation as Equation (10) is used to update the estimations with each new observed sample.

\[
\hat{\beta}^{(j)} = \hat{\beta}^{(j-1)} + \frac{\hat{\beta}^{(j-1)} - \hat{\beta}^{(j-2)}}{j}
\]

4. Performance evaluation

In this section, the performance of the proposed method is evaluated by using simulation studies under different shifts in profile parameters. The IC ARL of the proposed control chart is set equal to be 200.

A sample logistic profile model which is considered by [21] is used for simulation studies. In this model, the strength of an alloy fastener which is used in manufacturing of aircrafts is considered under pressure. Ten levels of pressure in range of 2500-4300 with pounds per square inch (psi) measure are applied to the fasteners as explanatory variable. A number of fasteners are tested at each pressure level. The number of failed fasteners at each level is considered as response variable. Researchers are usually replaced the real values of explanatory variable \( x \) with \( \log(x) \), so the corresponding design matrix is as follows:

\[
X = \begin{pmatrix}
1 & 1 & \ldots & 1 \\
\log(2500) & \log(2700) & \ldots & \log(4300)
\end{pmatrix}
\]

The in-control value of the regression model parameters and the covariance matrix of the estimated regression parameters are \( \beta = (\beta_1, \beta_2) = (-42.1110, 5.1772)^T \) and \( \sum = \begin{bmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2 \\
\end{bmatrix} = \begin{bmatrix}
18.568 & -2.283 \\
-2.283 & 0.2809
\end{bmatrix} \). Respectively. To apply shifts in the parameters of the model, we change \( \beta_0 \) to \( \beta_0 + \delta_0 \sigma_1 \) and \( \beta_1 \) to \( \beta_1 + \delta_1 \sigma_1 \). Both \( \delta_0 \) and \( \delta_1 \) are constant values in the range of 0.01 to 0.1. \( \sigma_1 \) and \( \sigma_2 \) are standard deviations of the estimated parameters. \( \beta_0 \) and \( \beta_1 \), respectively.

We consider \( m_1 = 30, 50 \) and 100, and also the upper control limit (UCL) for the proposed control chart is reported in Table 1. The UCL values are calculated by 10000 simulation runs to achieve IC ARL of approximately equal to 200.

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>UCL values</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>11.872</td>
</tr>
<tr>
<td>50</td>
<td>11.389</td>
</tr>
<tr>
<td>100</td>
<td>10.906</td>
</tr>
</tbody>
</table>
Tables 2 and 3 show the ability of the proposed self-starting control chart in detecting the shift values in logistic regression profile parameters ($\beta_0$ and $\beta_1$) by assessing OC ARL criterion. According to the results of the Tables 2 and 3 by increasing the IC samples which observed before change point $\tau$, the estimation of parameters gets more accurate and as a result the performance of the chart improves (see [20] and [16]). Hence, in this study 3 historical samples is used for initial parameter estimation and the performance of the control chart is investigated under two different values of change point $\tau$ equal to 5, and 20. The results showed that the proposed control schemes are robust with respect to different values of $\tau$ and this property of robustness is useful in real applications, because the performance of the proposed charts with $\tau = 5$ is equal to $\tau = 20$. Also the results show that by increasing $m_i$ value, the performance of the proposed self-starting control chart is improved and the chart detects the OC condition faster. It is clear that the proposed self-starting $T^2$ chart performs satisfactory in moderate and large shifts with $m_i = 30$, in both values of $\tau$.

Table 2: OC ARLs of the considered proposed $SST^2$ control chart in detecting various shifts in the regression parameter $\beta_0$ with two initial samples

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>Reference Value</th>
<th>$\delta_0$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$\tau = 5$</td>
<td>174.74</td>
<td>98.03</td>
<td>51.66</td>
<td>27.18</td>
<td>15.32</td>
<td>8.68</td>
<td>5.40</td>
<td>3.56</td>
<td>2.47</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>176.64</td>
<td>97.23</td>
<td>53.40</td>
<td>27.91</td>
<td>15.12</td>
<td>8.43</td>
<td>5.22</td>
<td>3.47</td>
<td>2.43</td>
<td>1.85</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>$\tau = 5$</td>
<td>145.89</td>
<td>65.71</td>
<td>27.31</td>
<td>12.19</td>
<td>6.16</td>
<td>3.49</td>
<td>2.27</td>
<td>1.64</td>
<td>1.31</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>145.89</td>
<td>65.70</td>
<td>27.28</td>
<td>12.21</td>
<td>6.17</td>
<td>3.45</td>
<td>2.26</td>
<td>1.63</td>
<td>1.33</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>$\tau = 5$</td>
<td>95.63</td>
<td>28.04</td>
<td>9.10</td>
<td>3.75</td>
<td>2.02</td>
<td>1.39</td>
<td>1.13</td>
<td>1.04</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>95.55</td>
<td>27.99</td>
<td>9.10</td>
<td>3.69</td>
<td>2.03</td>
<td>1.39</td>
<td>1.13</td>
<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: OC ARLs of the considered proposed $SST^2$ control chart in detecting various shifts in the regression parameter $\beta_1$ with two initial samples

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>Reference Value</th>
<th>$\delta_1$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$\tau = 5$</td>
<td>169.76</td>
<td>97.70</td>
<td>51.17</td>
<td>27.29</td>
<td>15.47</td>
<td>8.81</td>
<td>5.38</td>
<td>3.56</td>
<td>2.51</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>170.87</td>
<td>97.12</td>
<td>53.67</td>
<td>27.84</td>
<td>15.21</td>
<td>8.46</td>
<td>5.38</td>
<td>3.50</td>
<td>2.50</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>$\tau = 5$</td>
<td>144.82</td>
<td>64.08</td>
<td>27.27</td>
<td>12.52</td>
<td>6.16</td>
<td>3.60</td>
<td>2.32</td>
<td>1.66</td>
<td>1.33</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>143.72</td>
<td>64.02</td>
<td>27.30</td>
<td>12.51</td>
<td>6.14</td>
<td>3.58</td>
<td>2.31</td>
<td>1.64</td>
<td>1.30</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>$\tau = 5$</td>
<td>95.12</td>
<td>28.24</td>
<td>9.09</td>
<td>3.83</td>
<td>2.05</td>
<td>1.41</td>
<td>1.13</td>
<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau = 20$</td>
<td>94.97</td>
<td>28.26</td>
<td>9.08</td>
<td>3.82</td>
<td>2.06</td>
<td>1.42</td>
<td>1.12</td>
<td>1.02</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

5. An illustrative Example

In this section, a numerical example is considered to show the applicability of the proposed control chart. The profile model is the one which was used in simulation studies ([22]). In this example, first we generate three in-control initial samples and estimate the parameters of the model, and then we generate 17 samples with sustained shift in the slope $\beta_1$ of the logistic regression profile from $\beta_1 = 5.1772$ to $\beta_1 = 5.2143$ in sample number 4. The statistic values are calculated for each sample over the time and the results are given in Table 4. The $SST^2$ control chart as shown in Figure 1 detects the shift in sample 10, in fact in the 7th sample after the shift. Hence, the proposed control chart shows a satisfactory performance in numerical example.
### Table 4 - Data for numerical example with a step shift in the slope of LRP from 4th sample with 3 historical samples (IC ARL=200)

<table>
<thead>
<tr>
<th>$j$</th>
<th>$y_j$</th>
<th>$(\hat{\beta}_0^{(j)}, \hat{\beta}_1^{(j)})$</th>
<th>$(\hat{\beta}<em>{0(j-1)}^{(j)}, \hat{\beta}</em>{1(j-1)}^{(j)})$</th>
<th>$SS T^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 11 7 13 10 13 20 22 18 28</td>
<td>(-42.981,5.288)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8 2 7 13 10 20 20 18 22 18</td>
<td>(-39.006,4.781)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8 7 10 12 14 17 17 20 26 22</td>
<td>(-39.025,4.813)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3 9 14 13 15 16 21 23 24 26</td>
<td>(-48.103,5.953)</td>
<td>(-42.279,5.209)</td>
<td>7.332</td>
</tr>
<tr>
<td>5</td>
<td>4 8 13 15 17 16 15 23 25 25</td>
<td>(-36.590,4.511)</td>
<td>(-41.141,5.069)</td>
<td>0.801</td>
</tr>
<tr>
<td>6</td>
<td>7 9 12 12 13 18 20 23 23 26</td>
<td>(-42.834,5.301)</td>
<td>(-41.423,5.108)</td>
<td>2.453</td>
</tr>
<tr>
<td>7</td>
<td>6 7 11 13 18 18 20 23 18 25</td>
<td>(-40.078,4.954)</td>
<td>(-41.231,5.086)</td>
<td>0.600</td>
</tr>
<tr>
<td>8</td>
<td>8 10 9 12 14 21 23 25 24 26</td>
<td>(-47.717,5.922)</td>
<td>(-42.042,5.190)</td>
<td>6.997</td>
</tr>
<tr>
<td>9</td>
<td>8 9 7 15 14 15 23 19 22 25</td>
<td>(-39.270,4.850)</td>
<td>(-41.734,5.153)</td>
<td>0.195</td>
</tr>
<tr>
<td>10</td>
<td>7 5 15 13 16 24 19 22 28 28</td>
<td>(-53.936,6.701)</td>
<td>(-42.954,5.308)</td>
<td>12.320</td>
</tr>
<tr>
<td>11</td>
<td>4 8 14 12 14 16 21 18 25 25</td>
<td>(-43.826,5.412)</td>
<td>(-43.033,5.317)</td>
<td>0.059</td>
</tr>
<tr>
<td>12</td>
<td>12 7 7 9 15 24 20 24 27 24</td>
<td>(-46.273,5.738)</td>
<td>(-43.303,5.352)</td>
<td>2.184</td>
</tr>
<tr>
<td>13</td>
<td>4 13 8 15 19 18 25 20 23 23</td>
<td>(-40.770,5.057)</td>
<td>(-43.108,5.329)</td>
<td>1.309</td>
</tr>
<tr>
<td>14</td>
<td>7 7 11 6 22 21 21 18 26 26</td>
<td>(-48.101,5.955)</td>
<td>(-43.465,5.374)</td>
<td>1.054</td>
</tr>
<tr>
<td>15</td>
<td>6 15 11 13 17 14 16 20 21 23</td>
<td>(-28.126,3.475)</td>
<td>(-42.442,5.248)</td>
<td>5.928</td>
</tr>
<tr>
<td>16</td>
<td>6 8 7 7 15 18 19 24 27 24</td>
<td>(-53.653,6.618)</td>
<td>(-43.143,5.333)</td>
<td>3.505</td>
</tr>
<tr>
<td>17</td>
<td>6 9 11 9 18 19 18 20 25 24</td>
<td>(-42.234,5.220)</td>
<td>(-43.090,5.326)</td>
<td>0.030</td>
</tr>
<tr>
<td>18</td>
<td>6 5 11 19 23 16 21 24 22 24</td>
<td>(-43.498,5.399)</td>
<td>(-43.112,5.331)</td>
<td>2.101</td>
</tr>
<tr>
<td>19</td>
<td>3 8 9 14 13 16 21 20 21 23</td>
<td>(-43.470,5.350)</td>
<td>(-43.131,5.332)</td>
<td>2.451</td>
</tr>
<tr>
<td>20</td>
<td>6 11 8 17 17 14 25 19 26 24</td>
<td>(-41.983,5.204)</td>
<td>(-43.074,5.325)</td>
<td>0.874</td>
</tr>
</tbody>
</table>

UCL = 11.872

Figure 1 - $SS T^2$ statistic values over time

### 6. Conclusion and future research

In this paper, we considered a logistic regression profile. A self-starting $T^2$ control chart was proposed to monitor logistic regression profile over time. In the proposed control chart, the self-starting property was considered, i.e. the control chart is able to monitor the process at the start-up stages. While the process is running the parameters were updated and also the out-of-control condition was checked simultaneously. In the updating procedure, by observing each sample over the time the parameters were updated. The parameters estimation was improved over time, and the performance of the control chart got better. The simulation studies were done to evaluate the performance of the
proposed control chart. Results showed that the proposed control chart has an acceptable performance under moderate and large shifts. Developing self-starting control charts for the other GLM profiles such as Poisson or Gamma regression profiles is a fruitful area for future research.

References