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H_∞ Disturbance Attenuation of Fuzzy Large-Scale Systems

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***Abstract*—This paper is concerned with the disturbance attenuation problem of fuzzy large-scale systems which consist of N interconnected subsystems which are represented by Takagi-Sugeno fuzzy models. Using Lyapunov function and linear matrix inequalities (LMIs), a criterion is proposed to have a prescribed level of disturbance attenuation. A numerical example is given to illustrate the control design procedure and its effectiveness.**

***Index Terms*—Fuzzy large-scale systems, Stability, Disturbance attenuation, Linear matrix inequalities (LMIs).**

I. Introduction

A "large-scale system" could be considered as a system which consists of several subsystems. These subsystems serve particular functions, share resources, and are usually connected by a set of interconnections [1]-[3]. Electrical power system, nuclear reactors, aerospace systems, economic systems, and process control systems, etc., are examples of large-scale systems. Since most of the real systems are complex and huge in size, many researchers are interested to study the stability problem of large-scale systems [4]-[12].

Nowadays, most complex systems are modeled as a fuzzy system with IF-THEN rules. By using fuzzy modeling, we can model complex nonlinear systems to arbitrary degrees of accuracy [13],[14]. There are too many approaches to model a process using fuzzy rules. One of them is the so-called T-S fuzzy model which is proposed by Takagi and Sugeno [15],[16]. Suppose that the large-scale system consists of a number of subsystems with interconnections, the nonlinear model of each subsystem can be modeled as a T-S fuzzy model such that each subsystem has a number of fuzzy regions having local linear dynamic in each region. The global model of large-scale system can be then achieved by connecting these local models by the use of membership functions.

Motivated from the work in [17],[18], the main task of this paper is to investigate the disturbance attenuation problem of fuzzy large-scale systems. Instead of conventional fuzzy parallel distribute compensation (PDC) design [19], nonlinear state feedback controllers are used in the stabilization of the whole system with a good disturbance attenuation.

This paper is organized as follows. Section II introduces the fuzzy large-scale model. An H_∞ controller design method for fuzzy large-scale systems is presented in Section III. An example which illustrate the effectiveness of the proposed approach is given in Section IV. Finally, some concluding remarks are given in Section V .

II. Fuzzy Large-scale Systems Model

Consider a fuzzy large-scale system (S) which consists of N interconnected subsystems $S_i (i = 1, 2, \ldots, N)$. The ith fuzzy subsystem S_i can be described by:

$$S_i^l = \begin{cases} \dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_i^l (A_i^l x_i(t) + B_i^l u_i(t) + E_i^l v_i(t)) \\ \qquad\qquad + \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{l=1}^{r_i} \mu_i^l f_{ij}^l (x_j(t)) \\ y_i(t) = \sum_{l=1}^{r_i} \mu_i^l C_i^l x_i(t) \end{cases} \tag{1}$$

where S_i^l is the lth rule of S_i, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input of S_i at time t with appropriate dimension, $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector of the ith subsystem, $v_i(t) \in \mathbb{R}^{q_i}$ is the disturbance that belongs to $L_2[0,\infty)$, $f_{ij}^l(x_j(t))$ is the interconnection between the ith and jth subsystem in the lth rule of S_i, r_i is the number of rules in subsystem S_i, $(A_i^l, B_i^l, E_i^l, C_i^l)$ is the system matrices of rule l, in subsystem S_i, such that (A_i^l, B_i^l) is controllable, and μ_i^l is normalized membership function defined in in the following, in which $M_i^k(x_{ik}(t))$ is the grade of membership of $x_{ik}(t)$ in M_i^k.

$$\begin{aligned} \mu_i^l(x_i(t)) &= {}^{w_i^l} / {}_{\sum_{k=1}^{n_i} w_i^j} \\ w_i^j &= \prod_{k=1}^{n_i} M_i^k (x_{ik}(t)) \geq 0 \end{aligned} \tag{2}$$

As it can be seen in Fig. 1, $\sum_{l=1}^{r_i} \mu_i^l = 1, i = 1, 2, \ldots, N$.

The jth rule of the isolated subsystem S_i, which is represented by a T-S fuzzy model, is of the following form:

$$\begin{aligned} &\text{Rule}\, j: \\ &\text{if } x_{i1}(t) \text{ is } M_i^l \text{ and } \ldots\ x_{in_i}(t) \text{ is } M_i^{n_i} \\ &\text{then } \dot{x}_i = A_i^l x_i(t) + B_i^l u_i(t) + E_i^l v_i(t) \\ &\qquad\qquad + \sum_{\substack{j=1 \\ j\neq i}}^{N} f_{ij}^l (x_j(t)) \\ &y_i(t) = C_i^l x_i(t) \end{aligned} \tag{3}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in_i}(t)]^T$, and M_i^ls are the fuzzy sets.

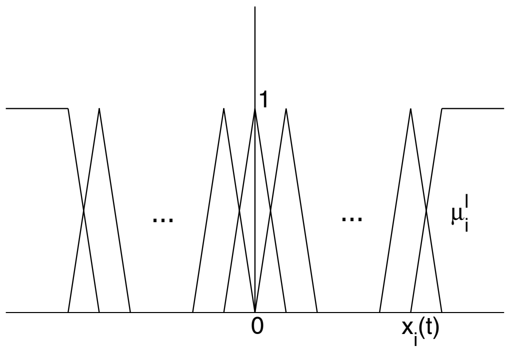

Fig. 1. Normalized membership function

III. H_∞ Controller Design with Nonlinear State Feedback Controller

In this section, based on Lyapunov functions we consider the H_∞ controller design for the large-scale fuzzy system (1). The objective is to find a fuzzy control law $u_i(t)$ such that the system (1) has a good level of disturbance attenuation, i.e., to have a good level of performance in the H_∞ sense. The H_∞ norm of a transfer function like $F(s)$, with output $y(t)$ and input $v(t)$, can be defined as follows [20]:

$$\|F(s)\|_\infty := \sup_{Re(s)>0} \bar{\sigma}[F(s)] = \sup_{\omega \varepsilon R} \bar{\sigma}[F(j\omega)] \tag{4}$$

Now, by defining the H_2 norm of $y(t)$ and $v(t)$ as

$$\begin{array}{l} \|y\|_2^2 = \int_{-\infty}^{+\infty} y^T(t)y(t)dt \\ \|v\|_2^2 = \int_{-\infty}^{+\infty} v^T(t)v(t)dt \end{array}$$

it can be concluded that

$$\frac{\|y\|_2}{\|v\|_2} \le \|F\|_\infty$$

In the large-scale fuzzy system (1), by defining $F(s)$ the transfer function from the disturbance $v(t)$ to the output as $y(t)$, having a disturbance attenuation γ is equal to have $\|F\|_\infty \le \gamma$, or equally:

$$\|y(t)\|_2 < \gamma \|v(t)\|_2 \tag{5}$$

for all non-zero $v(t) \in L_2$, where $y(t) = [y_1^T(t), y_2^T(t), \ldots, y_N^T(t)]^T$ and $v(t) = [v_1^T(t), v_2^T(t), \ldots, v_N^T(t)]^T$. Now, the closed-loop control system is said to be stable with disturbance attenuation γ. For this purpose, a nonlinear state feedback controller is considered as follows

$$u_i(t) = -\sum_{k=1}^{c_i} m_i^k(t) K_i^k(t) x_i(t) \tag{6}$$

in which

$$\sum_{k=1}^{c_i} m_i^k(t) = 1 \quad , \quad 0 \le m_i^k(t) \le 1$$

$$(i = 1, 2, \ldots, N)(k = 1, 2, \ldots, c_i) \tag{7}$$

and K_i^k is the state feedback gain with appropriate dimension. $m_i^k(t)$ is a nonlinear function defined by (9) and (10), c_i is an optimal number of controller in each subsystem. $M_i^{l,\acute{l},h}$s and $|M_i^{l,\acute{l},h}|$s are given in (11) and (12), in which, P_is are symmetric positive definite matrices, ε_{i1} and ε_{i2} are positive scalars, $f_{ij}^l(x_j(t)) = G_{ij}^l x_j(t)$, and n_{ij} is either 0 or 1. Now, by proposing the nonlinear state feedback controller, the closed-loop fuzzy subsystem becomes

$$\begin{array}{l} \dot{x}_i(t) = \sum_{l=1}^{r_i} \sum_{k=1}^{c_i} \mu_i^l(t) m_i^k(t) \left\{ (A_i^l - B_i^l K_i^k) x_i(t) \right. \\ \left. \quad + E_i^l v_i(t) \right\} + \sum_{l=1}^{r_i} \sum_{\substack{j=1 \\ j \ne i}}^{N} \mu_i^l(t) f_{ij}^l(x_j(t)) \end{array}$$

$$y_i(t) = \sum_{l=1}^{r_i} \mu_i^l C_i^l x_i(t) \tag{8}$$

in which

$$m_i^1(t) = \begin{cases} 1 - \dfrac{\sum_{k=2}^{c_i} M_i^{l,\acute{l},k}}{\sum_{h=2}^{c_i} abs(M_i^{l,\acute{l},h})} & , if \ \sum_{h=2}^{c_i} |M_i^{l,\acute{l},h}| \ne 0 \\ \dfrac{1}{c_i} & , if \ \sum_{h=2}^{c_i} |M_i^{l,\acute{l},h}| = 0 \end{cases} \tag{9}$$

and

$$m_i^k(t) = \begin{cases} \dfrac{\sum_{k=2}^{c_i} M_i^{l,\acute{l},k}}{\sum_{h=2}^{c_i} |M_i^{l,\acute{l},h}|} & , if \ \sum_{h=2}^{c_i} |M_i^{l,\acute{l},h}| \ne 0 \\ \dfrac{1}{c_i} & , if \ \sum_{h=2}^{c_i} |M_i^{l,\acute{l},h}| = 0 \end{cases} \tag{10}$$

where

$$\begin{array}{l} M_i^{l,\acute{l},k} = \sum_{l=1}^{r_i} \mu_i^l \mu_i^{\acute{l}} x_i^T(t) \Big(\left(A_i^l - B_i^l K_i^k\right)^T P_i \\ + P_i \left(A_i^l - B_i^l K_i^k\right) + {C_i^l}^T C_i^{\acute{l}} + \varepsilon_{i1}^{-1} P_i^T P_i + \\ \sum_{j=1}^{N} \varepsilon_{i2}^{-1} n_{ij} P_i^T P_i + \sum_{j=1}^{N} \varepsilon_{i2} n_{ji} {G_{ji}^l}^T G_{ji}^l \Big) x_i(t) \end{array} \tag{11}$$

and

$$\begin{array}{l} |M_i^{l,\acute{l},k}| = \sum_{l=1}^{r_i} |\mu_i^l \mu_i^{\acute{l}} x_i^T(t) \Big(\left(A_i^l - B_i^l K_i^k\right)^T P_i \\ + P_i \left(A_i^l - B_i^l K_i^k\right) + {C_i^l}^T C_i^{\acute{l}} + \varepsilon_{i1}^{-1} P_i^T P_i + \\ \sum_{j=1}^{N} \varepsilon_{i2}^{-1} n_{ij} P_i^T P_i + \sum_{j=1}^{N} \varepsilon_{i2} n_{ji} {G_{ji}^l}^T G_{ji}^l \Big) x_i(t)| \end{array} \tag{12}$$

Now, our task is to design K_i^k in such a way that the whole fuzzy large-scale system satisfies the condition (5). This condition is summarized by the following theorem.

Theorem 3.1: The fuzzy large-scale system (1) has a guaranteed performance in H_∞ sense, if there exist symmetric positive definite matrices P_i, positive scaler ε_{i1} and ε_{i2}, and state gain K_i^1, such that the following condition is satisfied:

$$\begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{bmatrix} < 0 \tag{13}$$

where

$$\begin{aligned}\Omega_1 &= m_i^1\left(A_i^l - B_i^l K_i^1\right)^T P_i + P_i\left(A_i^l - B_i^l K_i^1\right)\\ &\quad + C_i^{l^T} C_i^{\acute{l}} + \varepsilon_{i1}^{-1} P_i^T P_i + \textstyle\sum_{j=1}^N \varepsilon_{i2} n_{ji} G_{ji}^{l^T} G_{ji}^l \\ \Omega_2 &= \varepsilon_{i1} E_i^{l^T} E_i^l - \gamma^2 I.\end{aligned} \tag{14}$$

Proof: Consider a Lyapunov function $V(t)$ for the fuzzy large-scale system (1), given by

$$V(t) = \sum_{i=1}^N V_i(t) = \sum_{i=1}^N x_i^T(t) P_i x_i(t) \tag{15}$$

Now, by taking the derivative of $V_i(t)$ along the trajectories of ith subsystem, we get

$$\dot{V}_i(t) = \dot{x}_i^T P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t) \tag{16}$$

where, under zero conditions

$$\begin{aligned}&\textstyle\int_0^{+\infty}\left\{y^T(t)y(t) - \gamma^2 v^T(t)v(t)\right\}dt\\ &= \textstyle\int_0^{+\infty}\left\{y^T(t)y(t) - \gamma^2 v^T(t)v(t) + \frac{d}{dt}V(t)\right\}dt\\ &\quad - V(+\infty)\\ &\le \textstyle\int_0^{+\infty}\left\{y^T(t)y(t) - \gamma^2 v^T(t)v(t)\right.\\ &\quad \left.+ \textstyle\sum_{i=1}^N\left[\dot{x}_i^T P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t)\right]\right\}dt\\ &\le \textstyle\int_0^{+\infty}\sum_{i=1}^N\left\{\sum_{l=1,\,\acute{l}=1}^{r_i}\mu_i^l\mu_i^{\acute{l}} x_i^T(t) C_i^{l^T} C_i^{\acute{l}} x_i(t)\right.\\ &\quad - \gamma^2 v_i^T(t) v_i(t)\\ &\quad + \textstyle\sum_{l=1}^{r_i}\sum_{k=1}^{c_i}\mu_i^l m_i^k x_i^T(t)\left(\left(A_i^l - B_i^l K_i^k\right)^T P_i\right.\\ &\quad \left. + P_i\left(A_i^l - B_i^l K_i^k\right)\right) x_i(t)\\ &\quad + \textstyle\sum_{l=1}^{r_i}\mu_i^l\left(v_i^T(t) E_i^{l^T} P_i x_i(t) x_i^T(t) P_i E_i^l v_i(t)\right)\\ &\quad \left. + 2\textstyle\sum_{l=1}^{r_i}\sum_{j=1}^N \mu_i^l f_{ij}^{l^T} P_i x_i(t)\right\}dt\end{aligned} \tag{17}$$

Using the following matrix inequalities for all $a,b \in \mathbb{R}^n$, and any positive constant $\varepsilon > 0$

$$2a^T b \le \varepsilon a^T a + \varepsilon^{-1} b^T b \tag{18}$$

Now, let $f_{ij}^l(x_j(t)) = G_{ij}^l x_j(t)$, then we get

$$\begin{aligned}&\le \textstyle\int_0^{+\infty}\sum_{i=1}^N\left\{\sum_{l=1,\,\acute{l}=1}^{r_i}\sum_{k=1}^{c_i}\mu_i^l\mu_i^{\acute{l}} m_i^k x_i^T(t)\left(\left(A_i^l\right.\right.\right.\\ &\quad \left. - B_i^l K_i^k\right)^T P_i + P_i\left(A_i^l - B_i^l K_i^k\right)\\ &\quad \left. + C_i^{l^T} C_i^{\acute{l}}\right) x_i(t) - \gamma^2 v^T(t) v(t)\\ &\quad + \textstyle\sum_{l=1}^{r_i}\mu_i^l \varepsilon_{i1} v_i^T(t) E_i^{l^T} E_i^l v_i(t)\\ &\quad + \textstyle\sum_{l=1}^{r_i}\mu_i^l \varepsilon_{i1}^{-1} x_i^T(t) P_i^T P_i x_i(t)\\ &\quad + \textstyle\sum_{l=1}^{r_i}\sum_{j=1}^N \mu_i^l \varepsilon_{i2} x_j^T(t) G_{ij}^{l^T} G_{ij}^l x_j(t)\\ &\quad \left. + \textstyle\sum_{l=1}^{r_i}\sum_{j=1}^N \mu_i^l \varepsilon_{i2}^{-1} x_i^T(t) P_i^T P_i x_i(t)\right\}dt\end{aligned} \tag{19}$$

By defining n_{ij} as follows

$$n_{ij} = \begin{cases} 1 & \text{if } G_{ij}^l \neq 0 \\ 0 & \text{if } G_{ij}^l = 0 \end{cases} \tag{20}$$

we get

$$\begin{aligned}&= \textstyle\int_0^{+\infty}\sum_{i=1}^N\left\{\sum_{l=1,\,\acute{l}=1}^{r_i}\sum_{k=1}^{c_i}\mu_i^l\mu_i^{\acute{l}} m_i^k x_i^T(t)\left(\left(A_i^l\right.\right.\right.\\ &\quad \left. - B_i^l K_i^k\right)^T P_i + P_i\left(A_i^l - B_i^l K_i^k\right)\\ &\quad + C_i^{l^T} C_i^{\acute{l}} + \varepsilon_{i1}^{-1} P_i^T P_i\\ &\quad + \textstyle\sum_{j=1}^N \varepsilon_{i2}^{-1} n_{ij} P_i^T P_i\\ &\quad \left. + \textstyle\sum_{j=1}^N \varepsilon_{i2} n_{ji} G_{ij}^{l^T} G_{ij}^l\right) x_i(t)\\ &\quad \left. + \textstyle\sum_{l=1}^{r_i}\mu_i^l v_i^T(t)\left(\varepsilon_{i1} E_i^{l^T} E_i^l - \gamma^2 I\right) v_i(t)\right\}dt\end{aligned} \tag{21}$$

Now, by using the definition of m_i^k, we have

$$\begin{aligned}&\textstyle\int_0^{+\infty}\left\{y^T(t)y(t) - \gamma^2 v^T(t)v(t)\right\}dt\\ &\le \textstyle\int_0^{+\infty}\sum_{i=1}^N\sum_{l=1,\,\acute{l}=1}^{r_i}\mu_i^l\mu_i^{\acute{l}} \times \left\{\begin{bmatrix} x_i(t) \\ v_i(t)\end{bmatrix}^T\right.\\ &\quad \left. \times \begin{bmatrix}\Omega_1 & 0 \\ 0 & \Omega_2\end{bmatrix} \times \begin{bmatrix} x_i(t) \\ v_i(t)\end{bmatrix}\right\}dt\end{aligned} \tag{22}$$

where

$$\begin{aligned}\Omega_1 &= m_i^1\left(A_i^l - B_i^l K_i^1\right)^T P_i + P_i\left(A_i^l - B_i^l K_i^1\right)\\ &\quad + C_i^{l^T} C_i^{\acute{l}} + \varepsilon_{i1}^{-1} P_i^T P_i + \textstyle\sum_{j=1}^N \varepsilon_{i2} n_{ji} G_{ji}^{l^T} G_{ji}^l \\ \Omega_2 &= \varepsilon_{i1} E_i^{l^T} E_i^l - \gamma^2 I\end{aligned} \tag{23}$$

now if $\begin{bmatrix}\Omega_1 & 0 \\ 0 & \Omega_2\end{bmatrix} < 0$, we get

$$\int_0^{+\infty}\left\{y^T(t)y(t) - \gamma^2 v^T(t)v(t)\right\}dt < 0 \tag{24}$$

which implies that when $x_i(0) = 0; i = 1,2,\ldots,N$, $\|y(t)\|_2 < \gamma\|v(t)\|_2$. ∎

Now, it can be easily seen that if inequality (13) is satisfied, then the closed-loop fuzzy large-scale system is asymptotically stable, with disturbance attenuation γ.

According to the above theorem, the following algorithm can be developed for designing the controller.

Algorithm

1) Set a value for $\varepsilon_{ij}; j = 1,2$.
2) Solve the matrix inequality (13). This can be solved by using the MATLAB LMI toolbox.
3) If the solution is feasible, the controller parameter K_i^1 will be obtained and the procedure can be stopped. In this case $K_i^k; k \neq 1$ can be chosen according to the type of the response. That is, $K_i^k; k \neq 1$ affects the response features, like: settling time, overshoot , etc. If the solution is not feasible, set $\varepsilon_{ij} = \frac{\varepsilon_{ij}}{2}$ [18], and go to step 2.

IV. Numerical Example

Here, an example is presented to verify the results of the proposed stabilization procedure of fuzzy large-scale system composed of two subsystems described by

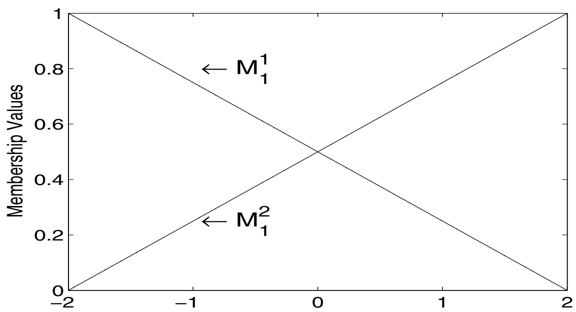

Fig. 2. Membership functions of subsystem S_1

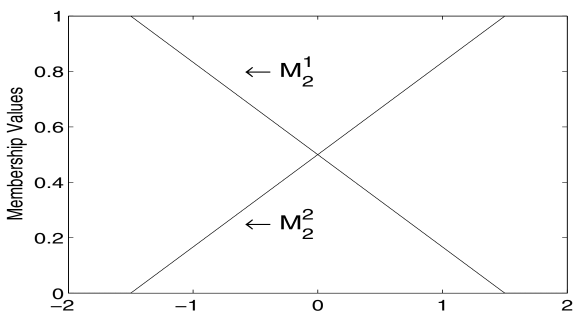

Fig. 3. Membership functions of subsystem S_2

Subsystem S_1

$$\begin{aligned} \text{Rule1}: &\text{ if } x_{11}(t) \text{ is } M_1^2 \text{ and } x_{12}(t) \text{ is } M_1^1 \\ &\text{then } \dot{x_1}(t) = A_1^1 x_1(t) + B_1^1 u_1(t) \\ &\quad + E_1^1 v_1(t) + \textstyle\sum_{j=1}^2 G_{1j}^1 x_j(t) \\ &y(t) = C_1^1 x_1(t) \\ \text{Rule2}: &\text{ if } x_{11}(t) \text{ is } M_1^2 \text{ and } x_{12}(t) \text{ is } M_1^2 \\ &\text{then } \dot{x_1}(t) = A_1^2 x_1(t) + B_1^2 u_1(t) \\ &\quad + E_1^2 v_1(t) + \textstyle\sum_{j=1}^2 G_{1j}^2 x_j(t) \\ &y(t) = C_1^2 x_1(t) \end{aligned}$$

in which $A_1^1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -1 \end{bmatrix}$, $A_1^2 = \begin{bmatrix} 0 & 1 \\ -1.2 & -1 \end{bmatrix}$, $B_1^1 = \begin{bmatrix} 2.2 \\ 0 \end{bmatrix}$, $B_1^2 = \begin{bmatrix} 1.2 \\ -0.6 \end{bmatrix}$, $D_1^1 = D_1^2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $C_1^1 = C_1^2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Moreover, the interconnection matrices among the two subsystems are given by $G_{12}^1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}$, $G_{12}^2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$. Membership functions of subsystem S_1 are shown in Fig. 2.

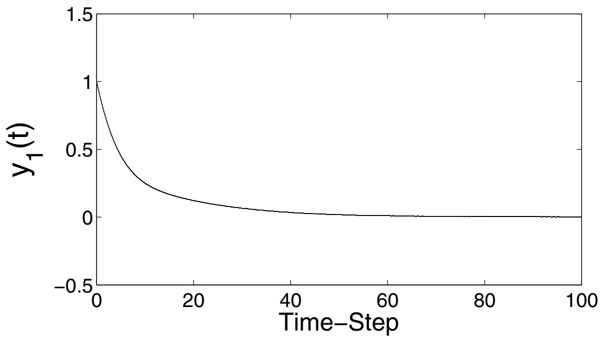

Fig. 4. $y_1(t)$, the output of the subsystem S_1

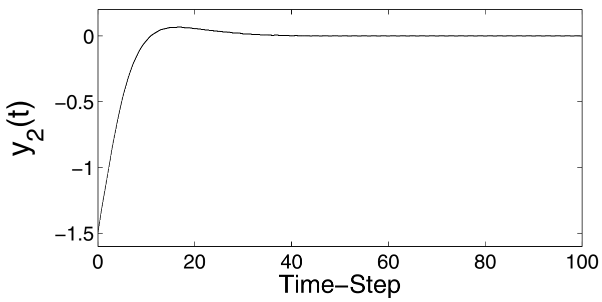

Fig. 5. $y_2(t)$, the output of the subsystem S_2

Subsystem S_2

$$\begin{aligned} \text{Rule1}: &\text{ if } x_{21}(t) \text{ is } M_2^1 \text{ and } x_{22}(t) \text{ is } M_2^1 \\ &\text{then } \dot{x_2}(t) = A_2^1 x_2(t) + B_2^1 u_2(t) \\ &\quad + E_2^1 v_2(t) + \textstyle\sum_{j=1}^2 G_{2j}^1 x_j(t) \\ &y(t) = C_2^1 x_2(t) \\ \text{Rule2}: &\text{ if } x_{21}(t) \text{ is } M_2^2 \text{ and } x_{22}(t) \text{ is } M_2^1 \\ &\text{then } \dot{x_2}(t) = A_2^2 x_2(t) + B_2^2 u_2(t) \\ &\quad + E_2^2 v_2(t) + \textstyle\sum_{j=1}^2 G_{2j}^2 x_j(t) \\ &y(t) = C_2^2 x_2(t) \end{aligned}$$

in which $A_2^1 = \begin{bmatrix} -1 & 0 \\ -0.8 & -2 \end{bmatrix}$, $A_2^2 = \begin{bmatrix} 0.08 & 1 \\ -2.5 & -0.6 \end{bmatrix}$, $B_2^1 = \begin{bmatrix} 1.2 \\ 1.2 \end{bmatrix}$, $B_2^2 = \begin{bmatrix} 2.2 \\ -0.8 \end{bmatrix}$, $D_2^1 = D_2^2 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$, $C_2^1 = C_2^2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Moreover, the interconnection matrices among the two subsystems are given by $G_{21}^1 = \begin{bmatrix} 0 & 0.1 \\ 1 & 0 \end{bmatrix}$, $G_{21}^2 = \begin{bmatrix} 0.3 & 0 \\ -0.9 & 0 \end{bmatrix}$. The normalized membership functions of subsystem S_2 are shown in Fig. 3. To design, first, we design a nonlinear state feedback controller for each subsystem so to make the fuzzy large-scale have a good performance in H_∞ sense. By choosing $\varepsilon_{11} = \varepsilon_{21} = 2$ and $\varepsilon_{12} = \varepsilon_{22} = \frac{100}{0.15}$ and using the above procedure, we can have

$$Q_1 = P_1^{-1} = \begin{bmatrix} 0.162 & 0.0066 \\ 0.0066 & 0.1626 \end{bmatrix},$$
$$Q_2 = P_2^{-1} = \begin{bmatrix} 0.1137 & 0.0157 \\ 0.0157 & 0.1851 \end{bmatrix},$$
$$K_1^1 = K_2^1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Since K_1^2 and K_2^2 don't affect the stability, by choosing $K_1^2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $K_2^2 = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}$, the procedure is completed. The initial conditions are $x_1(0) = \begin{bmatrix} 1 & -1.5 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} -1.5 & 1 \end{bmatrix}^T$.

The simulation results using $v_1(t) = 5\sin(2\pi t)$, $v_2(t) = 5\sin(\pi t)$, 100 steps of $\Delta t = 0.01$, and $\gamma = 0.0036$ are shown in Figs. 4 and 5. It can be observed that the controller proposed in this paper based on the fuzzy dynamic model not only stabilizes the original nonlinear system but also effectively attenuates the disturbances as expected.

V. Conclusion

This paper investigates the disturbance attenuation problem of fuzzy large-scale systems. Under some sufficient conditions, the nonlinear state feedback controller is designed to have a desired disturbance attenuation. LMI tools play a key role to find the symmetric positive matrices P_i. It is shown that using the method presented in this paper, the stability condition can be checked by K_i^1. That is, K_i^k (for $k \neq 1$) does not affect the stability. An example is also presented to illustrate the effectiveness of the proposed method.

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