Assessment of observer based fault estimators for TS fuzzy models

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Fault detection of nonlinear systems become more feasible when it is conducted over Takagi-Sugeno (TS) approximated fuzzy models. Proportional plus integral observer (PIO) and robust observer (RO) have already been developed for the estimation of the system states and actuator/sensor faults. In this paper, the algorithms are implemented for the detection of valve and level sensor faults of a two-tank system. Our simulation results indicate that both algorithms run well in estimating states and sensor fault, however, there is obvious differences in how they detect actuator fault in the presence of noise. From viewpoint of estimation variance, RO renders cleaner estimate of the fault than PIO, whiles PIO has faster fault tracking speed than RO. According to the achieving result, RO algorithm is recognized to be a more attractive in estimating actuator faults in noisy environments. The results are validated through simulations.

Published in: Fuzzy and Intelligent Systems (CFIS), 2017 5th Iranian Joint Congress on

Date of Conference: 7-9 March 2017

INSPEC Accession Number: 17097708

Date Added to IEEE *Xplore*: 18 August 2017

DOI: 10.1109/CFIS.2017.8003682

ISBN Information:

Publisher: IEEE

Conference Location: Qazvin, Iran

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I. Introduction

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Keywords

IEEE Keywords
Observers, Actuators, Fault detection, Nonlinear systems, Robustness, Noise measurement

Assessment of Observer Based Fault Estimators for TS Fuzzy Models

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I. INTRODUCTION

Any unacceptable deviation of system parameters from their nominal values, that may be a sign of near future complete failure, is called fault. Naturally, all processes are vulnerable to faults, therefore, maintaining long run system performance in the form of product quality and or behavior demands certain measures that can be accomplished by automatic fault diagnosis and isolation (FDI) to give early warning and to ensure safety and quality of operations.

Among a few fault diagnosis approaches, the quantitative model based techniques and in particular the observer based method continues to gain an increasing attention in the fundamental and applied research fields [1]. The methods embark on analytical, rather than, measurement redundancies. In practice, due to model uncertainties, disturbances, perturbations and noises, accurate fault estimation, such as the fault size and shape, is a very challenging procedure [2-4].

Most nonlinear behavior systems can be appropriately approximated by Takagi-Sugeno (TS) fuzzy models [5]. Therefore, the problem of fault detection in nonlinear systems is transformed to the fault analysis of TS fuzzy system models. In this respect, plenty of researches have been conducted. A proportional integral (PI) observers has been introduced in [6-8], which detects and estimates the size of actuator faults. The algorithms performance degrades in case of measurement noise [4]. Another technique for actuator fault estimation for T–S fuzzy model has been reported in [9, 10]. Estimation of actuator and sensor faults is the subject of study in [11], where the faults are introduced as auxiliary variables to be estimated. A revised version of PI observer for the detection of sensor and actuator faults has been detailed in [12]. Design of a reduced order robust observer with unknown input for estimation of faults in continuous and discrete TS models has been investigated in [13] .

In this paper, the problem of state estimation and actuator/sensor fault detection in a noisy environment of a two-tank system is investigated. The algorithms are the PI observer (PIO) [14] and the robust observer (RO) [4]. It is exhibited that both algorithms operate well in estimating the state and sensor fault; however, contrasts' are recorded in the detection of the actuator faults. PIO quickly tracks the actuator fault as soon as it occurs but the estimate is noisy, whiles low speed fault tracking RO yields lower variance estimate, which is demanded.

In section 2, the general TS fuzzy model and PIO and RO observer based fault detection algorithms are introduced. In section 3, the simulation results in detecting actuator and sensor faults in a two tank system is investigated and lastly conclusion comes in section 4.

II. TS FUZZY MODEL FAULT DETECTION

A. TS Fuzzy model

Most nonlinear physical systems can be accurately modeled by Takagi-Sugeno (TS) fuzzy systems [5]. The TS fuzzy set approximates a nonlinear system by a combination of several linear local dynamics where each is represented by one of the fuzzy implications. Thus, the global behavior is obtained by the contribution of each of the linear sub-models. It is argued that if nonlinear system state variables are used as the fuzzy decision variables, the fuzzy model is capable of replicating a large class of nonlinear systems [15]. Thus, it is worthwhile to exploit observer based fault detection on these models which it would be applicable to many nonlinear systems [16].

A TS Fuzzy model subject to actuator fault, sensor faults, and unknown disturbance is expressed by,

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i (A_i x(t) + B_i u(t) + F_i f_a(t) + H_i\, w(t) \\ y(t) = Cx(t) + Ff_s(t) \end{cases} \tag{1}$$

where $x(t)\in\Re^n$ is the state, $u(t)\in\Re^p$ is the input vector, $w(t)\in\Re^d$

is the exogenous disturbance, $y(t)\in\Re^{q}$ is the measured output, $f_a(t)\in\Re^{p1}$ is the actuator and $f_s(t)\in\Re^{m}$ is the sensor faults. A_i, B_i, F_i, C, F, D and H_i are constant real matrices of appropriate dimensions. The $\mu_i(x)$ is a membership function of the state variables, x. It is defined by,

$$\begin{cases} \sum_{i=1}^{r} \mu_i(x) = 1 & , \quad \forall t \geq 0 \\ 0 \leq \mu_i(x) \leq 1 & , \quad \forall i \in \{1,...,r\} \end{cases} \tag{2}$$

Where "r" represents the local models index.

B. PI observer (PIO) design

To estimate the states and faults of (1), first, the output is filtered by a stable matrix

$$\dot{y}_f = -T(y_f - y(t)) \tag{3}$$

and (1) is reformulated by introducing a new state variable, *r(t)* as follows [14],

$$\begin{cases} \dot{r}(t) = \sum_{i=1}^{r} \mu_i (A_{1i}\, r(t) + B_{1i} u(t) + F_{1i} f(t) + H_{1i}\, w(t)) \\ q(t) = C_1 r(t) \end{cases} \tag{4}$$

$$r(t) = \begin{bmatrix} x(t) \\ y_f(t) \end{bmatrix}, f(t) = \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}, A_{1i} = \begin{bmatrix} A_i & o \\ TC & -T \end{bmatrix}$$

$$F_{1i} = \begin{bmatrix} F_i & o \\ 0 & TF \end{bmatrix}, B_{1i} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, H_{1i} = \begin{bmatrix} H_i \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & I_q \end{bmatrix}$$

Where the matrices F_{1i} are full column rank. If it is assumed that the fault is in polynomial form of *k*-1 degree and their k^{th} derivatives are bounded, i.e. it obeys the following model,

$$\begin{cases} \dot{f}(t) = f_1(t) \\ \vdots \\ \dot{f}_{k-1}(t) = f_k(t) \\ f_k(t) \leq f_0 \end{cases} \tag{5}$$

A proportional integral observer is set up to estimate the states x, y_f and the fault and its derivatives as given below,

$$\begin{cases} \dot{\hat{r}}(t) = \sum_{i=1}^{r} \mu_i(\hat{r})(A_{1i}\hat{r}(t) + B_{1i}u(t) + F_{1i}\hat{f}(t) + K_{pi}(q(t) - \hat{q}(t))) + g_r(t) \\ \hat{q}(t) = C_1\hat{r}(t) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{r} \mu_i(\hat{r}) K_{li}(q(t) - \hat{q}(t)) + \hat{f}_1(t) + g_f(t) \\ \dot{\hat{f}}_j(t) = \sum_{i=1}^{r} \mu_i(\hat{r}) K_{li}^{j}(q(t) - \hat{q}(t)) + \hat{f}_{j+1}(t) + g_{fj}(t) \qquad for\ j:1...k-1 \end{cases} \tag{6}$$

Where K_{Pi}, K_{Ii} and K^{j}_{Ii} are the proportional and integral gains, respectively. The variables g_r (t) and $g_f(t)$, $g_{fj}(t)$ are introduced in order to compensate the influence of the immeasurable decision variables. For a matter of simplicity, the system (4) and the observer (6) is restructured to the following compact form,

$$\begin{cases} \dot{\bar{r}}(t) = \sum_{i=1}^{r} \mu_i (A_{2i}\bar{r}(t) + B_{2i}u(t) + H_{2i}\, w(t)) + J.f_k(t) \\ \bar{q}(t) = C_2\bar{r}(t) \end{cases} \tag{7}$$

$$\begin{cases} \dot{\hat{\bar{r}}}(t) = \sum_{i=1}^{r} \mu_i (A_{2i}\hat{\bar{r}}(t) + B_{2i}u(t) + K_i\,(\bar{q}(t) - \hat{\bar{q}}(t))) + g(t) \\ \hat{\bar{q}}(t) = C_2\hat{\bar{r}}(t) \end{cases}$$

Where

$$\bar{r}(t) = \begin{bmatrix} r(t) \\ f(t) \\ f_1(t) \\ \cdots \\ f_{k-1}(t) \end{bmatrix}, \hat{\bar{r}}(t) = \begin{bmatrix} \hat{r}(t) \\ \hat{f}(t) \\ \hat{f}_1(t) \\ \cdots \\ \hat{f}_{k-1}(t) \end{bmatrix}, g(t) = \begin{bmatrix} g_r(t) \\ g_f(t) \\ g_{f1}(t) \\ \cdots \\ g_{fk-1}(t) \end{bmatrix}, J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{n_f} \end{bmatrix}$$

$$H_{2i} = \begin{bmatrix} H_{1i} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, A_{2i} = \begin{bmatrix} A_{1i} & F_{1i} & 0 & 0 & \cdots & 0 \\ 0 & 0 & I_{n_v} & 0 & \cdots & 0 \\ 0 & 0 & 0 & I_{n_v} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & I_{n_v} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_{2i} = \begin{bmatrix} B_{1i} \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$$

$$K_i = \begin{bmatrix} K_{pi} \\ K_{li} \\ K_{li}^{1} \\ \cdots \\ K_{li}^{k-1} \end{bmatrix}, C_2 = \begin{bmatrix} C_1 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{8}$$

Where $n_f = n_{fa} + n_{fs}$ and I_{nf} is an identity matrix.
The observer estimation error,

$$\bar{e}(t) = \bar{r}(t) - \hat{\bar{r}}(t) \quad , e_y(t) = \bar{q}(t) - \hat{\bar{q}}(t) \tag{9}$$

Is expressed by,

$$\dot{\bar{e}}(t) = \sum_{i=1}^{r} \mu_i(\hat{\bar{r}})\bar{A}_i.\bar{e}(t) + G.\varepsilon(t) + \Delta.z(t) - g(t)$$

$$\bar{A}_i = A_{2i} - K_i C_2 \ , \bar{\Delta}A = \sum_{i=1}^{r} \bar{\mu}_i A_{2i} \ , \bar{\Delta}B = \sum_{i=1}^{r} \bar{\mu}_i B_{2i} \ , \bar{\mu}_i = \mu_i(r) - \mu_i(\hat{r}) \tag{10}$$

$$G = \begin{bmatrix} J & H_{2i} \end{bmatrix}, \Delta = \begin{bmatrix} \bar{\Delta}A & \bar{\Delta}B \end{bmatrix}, \quad \sigma_1 = \frac{1}{\alpha\lambda_0} \qquad \sigma_2 = \frac{\lambda}{\lambda\lambda_0(1+\alpha) - 1}$$

$$\varepsilon(t) = \begin{bmatrix} f_k(t) \\ w(t) \end{bmatrix}, \qquad z(t) = \begin{bmatrix} \bar{r}(t) \\ u(t) \end{bmatrix}$$

$$\begin{cases} g = 0 & if \quad |e_y| \prec \varepsilon \\ g = \sigma_1\delta_1^2 \dfrac{\hat{\bar{r}}^T\hat{\bar{r}}}{2e_y^T e_y} P^{-1}C_2^T e_y + \sigma_2\delta_2^2 \dfrac{u^T u}{2e_y^T e_y} P^{-1}C_2^T e_y & if \quad |e_y| \geq \varepsilon \end{cases}$$

The estimation error $\bar{e}(t)$ asymptotically stable and L_2 stability is guaranteed [14], if there exist a matrix P>0, matrices N_i >0 and the positive scalars λ and λ_0 for each fuzzy implication that satisfies the following LMI condition under minimized μ,

$$\begin{bmatrix} \varphi_i & PJ & PH_{2i} & P & \delta_1.I & I \\ * & -\mu.I & 0 & 0 & 0 & 0 \\ * & * & -\mu.I & 0 & 0 & 0 \\ * & * & * & -\lambda.I & 0 & 0 \\ * & * & * & * & -\lambda_0 I & 0 \\ * & * & * & * & * & -\mu I \end{bmatrix} < 0 \tag{11}$$

$$\varphi_i = (PA_{2i} - N_iC_2) + (PA_{2i} - N_iC_2)^T$$

Due to ϵ (t), z (t) and g (t) some resides remain on the estimation as (10) indicates. The gain of the PI observer for each fuzzy implication is given by,

$$K_i = P^{-1}.N_i \tag{12}$$

C. *Robust state/fault observer (RO)*

By introducing a new state variable $\bar{x}$ from the series of x and the sensor fault *fs*, a new arrangement for TS fuzzy model (1) is derived which is expressed by [4],

$$\begin{cases} Q\dot{\bar{x}}(t) = \sum_{i=1}^{r} \mu_i (A_{1i}\bar{x}(t) + B_{1i}u(t) + F_{1i}f_a(t) + H_{1i}w(t) + E_1 f_s(t)) \\ y(t) = C_1\bar{x}(t) \end{cases} \tag{13}$$

$$\bar{x}(t) = [x^T(t), f_s^T(t)]^T,\ Q = \begin{bmatrix} I_n & 0 \\ 0 & 0_{q\times m} \end{bmatrix}, A_{1i} = \begin{bmatrix} A_i & 0 \\ 0 & -F \end{bmatrix},$$

$$F_{1i} = \begin{bmatrix} F_i \\ 0_{q\times p1} \end{bmatrix},\quad B_{1i} = \begin{bmatrix} B_i \\ 0_{q\times p} \end{bmatrix},\ H_{1i} = \begin{bmatrix} H_i \\ 0_{q\times d} \end{bmatrix},\ E_1 = \begin{bmatrix} 0_{n\times m} \\ F \end{bmatrix},\ C_1 = [C \quad F]$$

Adding $K_{1i}\dot{y}$ to both sides of (13) yields,

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^{r} \mu_i (A_{2i}\bar{x}(t) + B_{2i}u(t) + F_{2i}f_a(t) + H_{2i}w(t) + E_{2i}f_s(t) + G_{1i}^{-^*} K_{1i}\dot{y}(t)) \\ y(t) = C_1\bar{x}(t) = C_0\bar{x}(t) + Ff_s(t) \end{cases} \tag{14}$$

$$A_{2i} = Q_{1i}^{-^*} A_{1i},\ B_{2i} = Q_{1i}^{-^*} B_{1i},\ E_{2i} = Q_{1i}^{-^*} E_1,\ H_{2i} = Q_{1i}^{-^*} H_{1i},\ F_{2i} = Q_{1i}^{-^*} F_{1i}$$

$$Q_{1i}^{-^*} = \begin{bmatrix} I & 0 \\ -F^- C & F^- K_{12i}^{-1} \end{bmatrix},\ K_{12i} = diag(\lambda_{i1},\dots,\lambda_{iq}),\ Q_{1i}^{-^*} K_{1i} = \begin{bmatrix} 0 \\ F^- \end{bmatrix}$$

It is shown [4] that under the defined conditions, there is a transformation matrix, *T* that transforms (14) to the following equation,

$$T = \begin{bmatrix} C_1^{\perp T} \\ C_1 \end{bmatrix}^{-1},\quad \bar{x}(t) = Tr(t)$$

$$\begin{cases} \dot{r}(t) = \sum_{i=1}^{r} \mu_i (A_{3i}r(t) + B_{3i}u(t) + F_{3i}f_a(t) + H_{3i}w(t) + E_{3i}f_s(t) + N\dot{y}(t)) \\ y(t) = C_1 Tr(t) = [0_{q\times(n+m-q)} \quad I_q] r(t) \end{cases} \tag{15}$$

$$A_{3i} = T^{-1}A_{2i}T,\quad B_{3i} = T^{-1}B_{2i},\quad E_{3i} = T^{-1}E_{2i},\ H_{3i} = T^{-1}H_{2i},$$

$$N = T^{-1}Q_{1i}^{-^*} K_{1i},\quad F_{3i} = T^{-1}F_{2i},\quad N = T^{-1}\begin{bmatrix} 0 \\ F^- \end{bmatrix}$$

Now, system (15) is decomposed into two set of variables, variables dependent and independent of the measured y as follows,

$$\begin{cases} \dot{r}_1(t) = \sum_{i=1}^{r} \mu_i (A_{11i}r_1(t) + A_{12i}r_2(t) + B_{31i}u(t) + F_{31i}f_a(t) + H_{31i}w(t) + E_{31i}f_s(t) + N_1\dot{y}(t)) \\ \dot{r}_2(t) = \sum_{i=1}^{r} \mu_i (A_{21i}r_1(t) + A_{22i}r_2(t) + B_{32i}u(t) + F_{32i}f_a(t) + H_{32i}w(t) + E_{32i}f_s(t) + N_2\dot{y}(t)) \\ y(t) = [0_{q\times(n+m-q)} \quad I_q]\begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = r_2(t) \end{cases} \tag{16}$$

$$r_1(t) \in \Re^{n+m-q},\ r_2(t) \in \Re^q,\ r(t) = [r_1(t)^T \ r_2(t)^T]^T$$

$$\begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix} = A_{3i},\ \begin{bmatrix} B_{31i} \\ B_{32i} \end{bmatrix} = B_{3i},\ \begin{bmatrix} E_{31i} \\ E_{32i} \end{bmatrix} = E_{3i},\ \begin{bmatrix} H_{31i} \\ H_{32i} \end{bmatrix} = H_{3i},\ \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = N,\ \begin{bmatrix} F_{31i} \\ F_{32i} \end{bmatrix} = F_{3i}$$

r_2 has already been measured, thus the unmeasured r_1 is required to be estimated. By considering the following equations for *fs*,

$$\bar{x} = \begin{bmatrix} x \\ f_s \end{bmatrix} = Tr = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \Rightarrow f_s = T_{21}r_1 + T_{22}r_2 \tag{17}$$

a reduced order estimator is designed for r_1 and fa as follows [4],

$$\begin{cases} \hat{r}_1(t) = z_1(t) + \sum_k [(N_1 + K_{2k} - K_{2k}N_2)y(t)] \\ \hat{f}_a(t) = z_2(t) + \sum_k [(K_{3k} - K_{3k}N_2)y(t)] \\ \dot{z}_1(t) = \sum_{i,j,k}^{r} [(\bar{A}_{11i} - K_{2j}\bar{A}_{21i})z_1(t) + (B_{31i} - K_{2j}B_{32i})u(t) + (F_{31i} - K_{2j}F_{32i})\hat{f}_a(t) \\ \quad + ((\bar{A}_{11i} - K_{2j}\bar{A}_{21i})(N_1 + K_{2k} - K_{2k}N_2) + (\bar{A}_{12i} - K_{2j}\bar{A}_{22i}))y(t)] \\ \dot{z}_2(t) = \sum_{i,j,k}^{r} [-K_{3j}F_{32i}z_2(t) - K_{3j}\bar{A}_{21i}\hat{r}_1(t) - K_{3j}B_{32i}u(t) \\ \quad + (-K_{3j}F_{32i}(K_{3k} - K_{3k}N_2) - K_{3j}\bar{A}_{22i})y(t)] \end{cases} \tag{18}$$

$$\bar{A}_{11i} = A_{11i} + F_{31i}T_{21},\ \bar{A}_{12i} = A_{12i} + F_{31i}T_{22}$$

$$\bar{A}_{21i} = A_{21i} + F_{31i}T_{21},\ \bar{A}_{22i} = A_{22i} + F_{31i}T_{22}$$

$$\sum_i, \sum_{i,j}, \sum_{i,j,k}, \quad denote \quad \sum_{i=1}^{g}\mu_i,\ \sum_{i=1}^{g}\sum_{j=1}^{g}\mu_i\mu_j,\ \sum_{i=1}^{g}\sum_{j=1}^{g}\sum_{k=1}^{g}\mu_i\mu_j\mu_k$$

The observer estimation error is denoted by,

$$e(t) = [(r_1(t) - \hat{r}_1(t))^T,\ (f_a(t) - \hat{f}_a(t))^T]^T \tag{19}$$

It is shown [4] that If there exist a symmetric positive-definite matrix P, and matrices $M_i > 0$ (for each fuzzy implication) and j=1, 2… g that satisfies the following linear matrix inequalities,

$$G_{ii} < 0, \qquad \frac{2}{g-1}G_{ii} + G_{ij} + G_{ji} < 0,\quad 1 \le i \ne j \le g \tag{20}$$

Where

$$G_{ij} = \begin{bmatrix} A_{4i}^T P + PA_{4i} - J_{1j}^T\bar{M}_i^T - \bar{M}_i J_{1j} + M_1 & P\bar{H}_i - \bar{M}_i J_{2j} \\ * & -\rho_1 M_2 \end{bmatrix}$$

$$A_{4i} = \begin{bmatrix} \bar{A}_{11i} & F_{31i} \\ 0 & 0 \end{bmatrix},\ J_{1j} = [\bar{A}_{21j}\ B_{32j}],\quad \bar{H}_i = \begin{bmatrix} H_{31i} & 0 \\ 0 & I \end{bmatrix} \tag{21}$$

$$J_{2j} = [H_{32j}\ 0],\ \rho_1 = \gamma_1^2$$

Then, the estimation error has H_∞ performance level γ_1 as expressed below,

$$\begin{cases} \lim_{t\to\infty} e(t) = 0,\quad \delta(t) = [(w(t))^T, \dot{f}_a^T(t)]^T = 0 \\ \int_0^t e^T(s)M_1 e(s)ds < \gamma_1^2 \int_0^t \delta^T(s)M_2\delta(s)ds,\quad if\ \delta(t) \ne 0 \end{cases} \tag{22}$$

With the following observer gain:

$$K_{4i} = \begin{bmatrix} K_{2i} \\ K_{3i} \end{bmatrix} = P^{-1}\bar{M}_i \tag{23}$$

III. SIMULATION TESTS

In this section, fault estimation in a two tank hydraulic unit using the PIO and RO algorithms are examined. There are two interconnected tanks, two level sensors, two electric valves and a fluid supplying pump. The system state variables and control inputs are,

$$x = [h_1, s_1, h_2, s_2]^T,\qquad u = [v_1, v_2]^T$$

Where hi, si and vi are the fluid level in a tank, valve opening and the valve controlling voltage of component i.

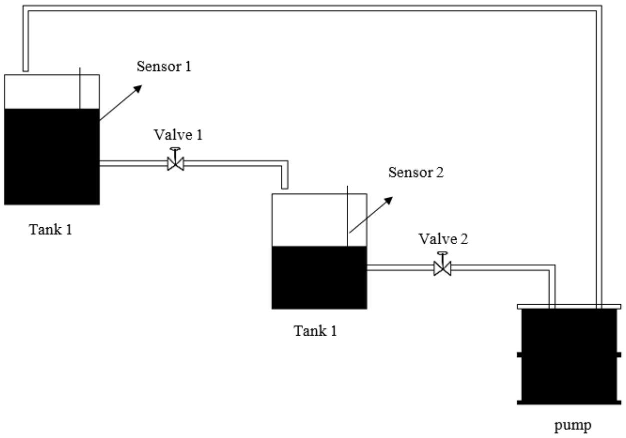

Fig. 1. The two–tank hydraulic system [17].

The approximated TS fuzzy model of the fault infected system is given by [18],

$$A_1 = \begin{bmatrix} -1.29m & -3.125 & 0 & 0 \\ 0 & -0.377 & 0 & 0 \\ 1.29m & 3.125 & -1.29m & -3.125 \\ 0 & 0 & 0 & -0.377 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 13.3\mu & 0 \\ 0 & 0 \\ 0 & 14.7\mu \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.92m & -3.69754 & 0 & 0 \\ 0 & -0.377 & 0 & 0 \\ 0.92m & 3.69754 & -0.92m & -3.69754 \\ 0 & 0 & 0 & -0.377 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 11.6\mu & 0 \\ 0 & 0 \\ 0 & 10.7\mu \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$F_1 = F_2 = 10^{-3}\begin{bmatrix} 0 & 3 & 0 & 5 \end{bmatrix}^T \qquad , H_i = 10^{-3}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$$

$$F = \begin{bmatrix} 0 & 10 \end{bmatrix}^T, \quad x_0 = \begin{bmatrix} 0.25 & 0 & 0.25 & 0 \end{bmatrix}^T$$

Where the valve no 1 and no 2 and the sensor no 2 are affected by faults. The process is also corrupted by process noise.
The gain matrices of the robust observer are obtained as follow:

$$K41 = \begin{bmatrix} -1.53 & 1.11*10^{12} \\ -1.17 & -5.9*10^{10} \\ 0.69 & -6.73*10^{10} \\ -154.03 & -1.25*10^{13} \end{bmatrix} \quad K42 = \begin{bmatrix} -1.47 & 5.78*10^{11} \\ -1.02 & 1.95*10^{11} \\ 0.62 & -1.64*10^{11} \\ -133.26 & 3.59*10^{13} \end{bmatrix}$$

Table I illustrates amount of the obtained PI observer gains.

Table I. PI observer gains

$\mu = 7.5132$	$\lambda = 3.9647*10^{11}$	$\lambda_0 = 3.6013*10^{11}$
i	1	2
K_{pi}	$\begin{bmatrix} 4.71*10^3 & -2.05*10^3 \\ -1.26*10^4 & 6.92*10^3 \\ 1.09*10^3 & -4.49*10^3 \\ -2.12*10^4 & 1.12*10^4 \\ 38.31 & -141.58 \\ 1.27 & 62.44 \end{bmatrix}$	$\begin{bmatrix} 4.65*10^3 & -2.05*10^3 \\ -1.23*10^4 & 6.93*10^3 \\ 1.18*10^3 & -4.49*10^3 \\ -2.07*10^4 & 1.12*10^4 \\ 37.78 & -141.60 \\ 1.26 & 62.44 \end{bmatrix}$
K_{Ii}	$\begin{bmatrix} -1.21*10^7 & 6.16*10^6 \\ -137.64 & 6.35*10^4 \end{bmatrix}$	$\begin{bmatrix} -1.18*10^7 & 6.17*10^6 \\ -142.38 & 6.35*10^4 \end{bmatrix}$
K_{Ii}^1	$\begin{bmatrix} -1.65*10^7 & 8.24*10^6 \\ -1.04*10^3 & 1.91*10^6 \end{bmatrix}$	$\begin{bmatrix} -1.61*10^7 & 8.25*10^6 \\ -886.79 & 1.91*10^6 \end{bmatrix}$
K_{Ii}^2	$\begin{bmatrix} -1.07*10^7 & 5.23*10^6 \\ 4.65*10^4 & 3.89*10^7 \end{bmatrix}$	$\begin{bmatrix} -1.05*10^7 & 5.24*10^6 \\ 4.88*10^4 & 3.89*10^7 \end{bmatrix}$

The results of the state estimation using the PIO and RO algorithms have been depicted in Fig. 2 and 3. As the figures indicate, both algorithms have successfully estimated the states despite of fault and noise. Distinctions between their outcomes are unnoticeable.

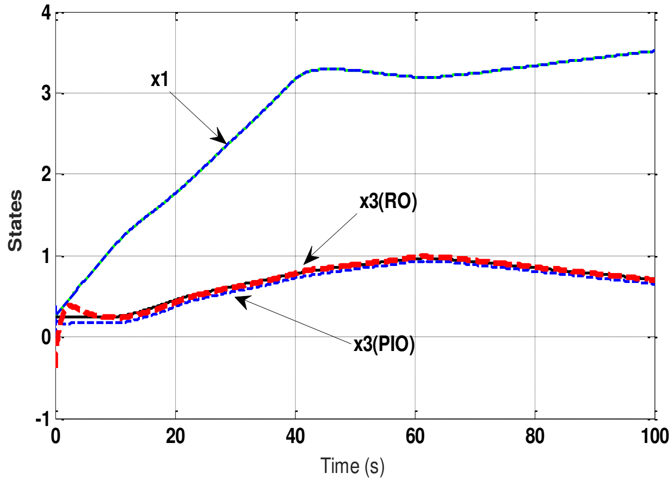

Fig. 2. The states 1 and 3 and their estimates by PIO and RO.

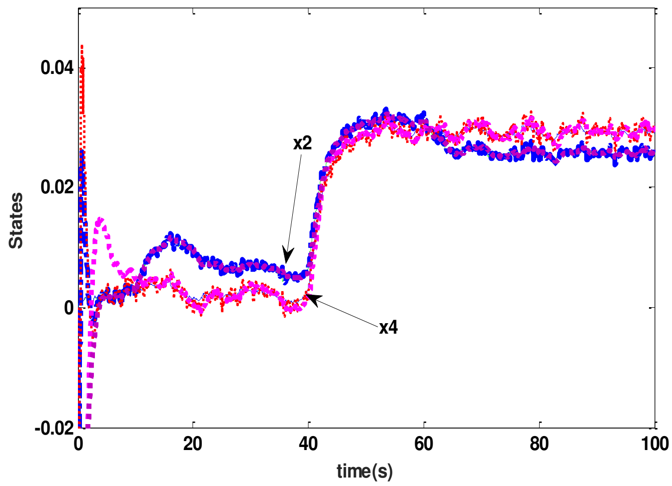

Fig. 3. The Sates 2 and 4 and their estimates by PIO and RO

Similarly, the algorithms give identical results concerning the sensor fault as it has been shown in Fig. 4. The fault consists of a step and a mounted sinusoidal disturbance, which may be infected by external interferences.

Fig. 5 gives information about error between sensor fault and fault estimation. Both algorithms RO and PIO perform as proper estimators, while the lower error has been obtained by RO algorithm in comparison to PIO algorithm.

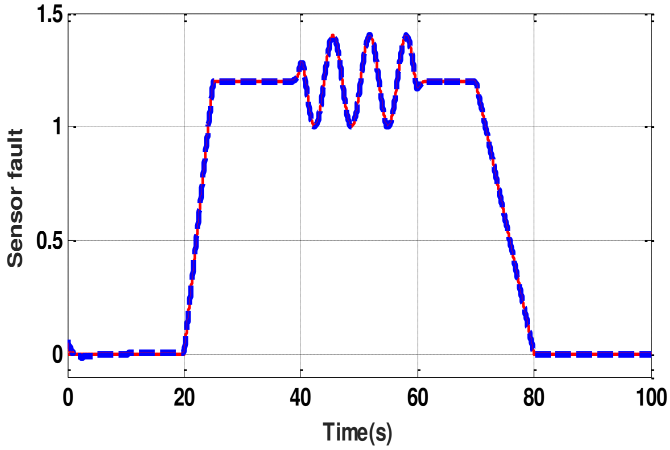

Fig. 4. The Sensor fault and its estimates by PIO and RO

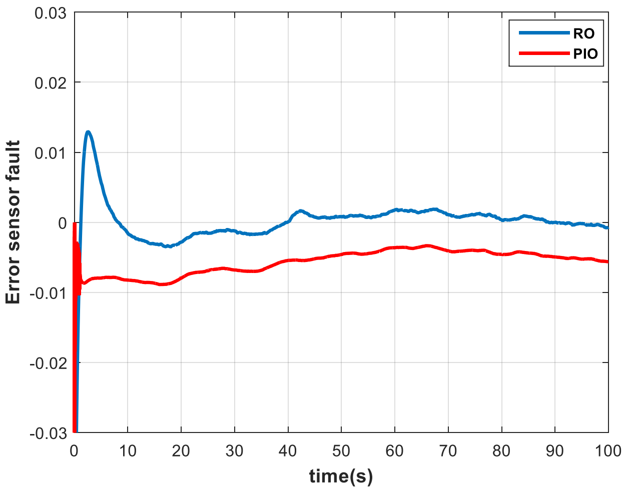

Fig. 5. Error between sensor fault and its estimate by PIO and RO

However, the difference between the two algorithms emerges in the estimation of the actuator fault. Both algorithms with diverse quality detect the step like fault, which can be seen in Fig. 6. The PIO estimation is noisy, whereas RO delivers low variance estimation. From view of point of fault tracking speed, PIO looks more sensitive than RO, particularly at the moment of occurring fault. As less variance is a more important quality factor for any estimation algorithms, one will prefer RO to PIO in the procedure of an actuator fault detection. In addition to, Fig. 7 gives more information about the resulting error between actuator fault and its estimation.

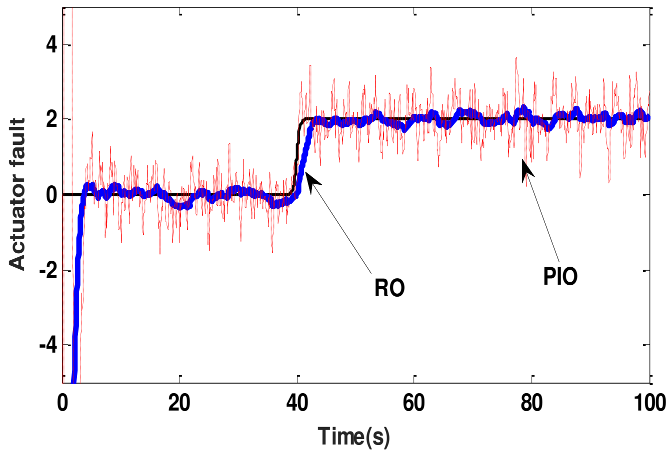

Fig. 6. The Actuator fault and its estimates by PIO and RO

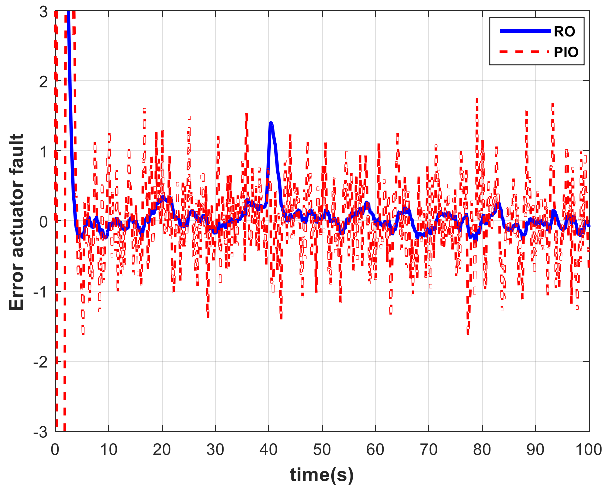

Fig. 7. Error between actuator fault and its estimate by PIO and RO

IV. Conclusion

In this work, estimation of sensor and actuator fault in a two-tank system are investigated by using the PIO and RO algorithms. Despite of the existence of the process noise, both algorithms would be able to the accurate estimation of the states and sensor faults. However, the quality of outcome differs in estimating the actuator fault. RO returns a smoother estimates of fault whiles PIO outcome is noisy. On the other hand, the fault tracking speed of PIO advances RO. In general, RO looks more appropriate for using as an actuator fault estimator in a noisy environment than more agile PIO.

Acknowledgement

This work has been partially supported by the research department of Shahed University, Tehran, IRAN.

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