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# ROTA THEOREM FOR FINITE DIMENSIONAL BANACH SPACES

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ABSTRACT. Rota theorem states that for a Hilbert space $H$ and $T \in B(H)$, the equality

$$\rho(T) = \inf\{\|P^{-1}TP\| : P \text{ is invertible in } B(H)\},$$

holds, where $\rho$ denotes the spectral radius. In this paper, using an elementary method, prove this result for finite dimensional Banach spaces endowed by an absolute norm. Also we show that for every arbitrary induced norm $\|.\|$ on $M_n$, the algebra of $n \times n$ matrices with complex entries, the following equality holds

$$\rho(T) = \inf\{r(P^{-1}TP) : P \text{ is invertible in } M_n\},$$

where $r$ denotes the numerical radius related to $\|.\|$.

Keywords: Spectral radius; Numerical radius; Absolute norm.

## 1. INTRODUCTION

A direct result of Rota theorem [3] is that for every bounded linear operator $T$ on a Hilbert space, the spectral radius of $T$ is equal to the infimum of the norms of all operators which are similar to $T$. In this paper, using an elementary method, we refine this result for finite dimensional Banach spaces that which are endowed with absolute norms. Before it, we need some terminologies.

Let $M_n$ be the algebra of all $n$ by $n$ matrices with complex entries. We denote a diagonal matrix with the entries $\lambda_1, \cdots, \lambda_n$ on its diagonal by $diag(\lambda_1, \cdots, \lambda_1)$. The spectral radius of a matrix in $A \in M_n$ is displayed by $\rho(A)$. For every two vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \cdots, y_n)$ in $\mathbb{C}^n$, the product partial order is defined on $\mathbb{C}^n$ by $(x_1, \ldots, x_n) \leq (y_1, \cdots, y_n)$, if $|x_i| \leq |y_i|$, for all $i = 1, \cdots, n$. A norm $\|.\|$ on $\mathbb{C}^n$ is said to be *absolute* if

$$\|(x_1, \ldots, x_n)\| = \|(|x_1|, \ldots, |x_n|)\|, \quad x = (x_1, \ldots, x_n) \in \mathbb{C}^n,$$

and it is *monotone* if

$$\|(x_1, \ldots, x_n)\| \leq \|(y_1, \ldots, y_n)\|,$$

for every $x, y \in \mathbb{C}^n$ with $x \leq y$. It is well known that a norm on $\mathbb{C}^n$ is absolute if and only if it is monotone [2, Theorem 5.5.10] and [1, Proposition IV.1.1]. The numerical range and numerical radius related to a norm $\|.\|$ on $\mathbb{C}^n$ is defined as follows respectively

$$W(A) = \{y^*Ax : \|y\|^D = \|x\| = y^*x = 1\},$$

$$r(A) = \sup\{|\lambda| : \lambda \in W(A)\}.$$

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