International Conference on Progress in Science and Technology
LONDON
کنفرانس بین المللی پیشرفت در علوم و تکنولوژی
London - 20 November2016 England
۳۰ آذر ۱۳۹۵ لندن، انگلستان

Application of the central force optimization (CFO) method to the soil slope stability analysis

Mohammad Behniya
Msc. Student, Civil Engineering Department, Faculty of Engineering, Electronic(Branch) Islamic Azad University, Tehran, Iran

Amir Houshang Ayati
Civil Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

Ali Derakhshani
Civil Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran

Ali Haghighi
Civil Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran.

Corresponding author: *Amir Houshang Ayati*
Email: Saha.science@gmail.com

Abstract

This paper introduces a methodology for soil slope stability analysis based on optimization, limit equilibrium principles and method of slices. In this study, the slope stability analysis problem is transformed into a constrained nonlinear optimization problem. To solve that, a Central Force Optimization (CFO) is utilized. In this study, the slope stability safety factors are the objective functions, slip surface parameters are the decision variables and, the equilibrium equations are the problem constraints. The proposed model satisfies all conditions of the equilibrium completely. It is also applicable to problems with deferent soil layers, variable soil properties and including pore water pressure. The model is applied against a benchmark example and the results are compared with previous studies. Accordingly, it is found computationally efficient and reliable.

Keywords: CFO, Slope stability analysis, Method of slices, Equilibrium analysis, Safety factor

Introduction

Slope stability analysis is of geotechnical engineering problems that has received considerable attention from researchers worldwide. Equilibrium analyses of slope stability are widely used in design of excavation and embankment slopes. There exist many successful applications and experiences on the limit equilibrium methods, which are very popular because of their easiness and accuracy. In fact, the limit equilibrium methods have been the most widely used schemes for the slope stability analysis (Duncan 1996). These methods, in general, satisfy the force and moment equilibrium; boundary conditions and the failure criterion along the slip surface. In context of the limit equilibrium methods, the method of slices is extensively used to cope with complex slope geometries, variable soil properties and the existence of pore water pressure.

Reviewing the literature, the slope stability methods can be categorized in two major groups of the numerical methods, mostly by using the finite element method (Griffiths and Lane 1999; Griffiths and Fenton 2004; Griffiths and Marquez 2007) and analytical methods, mostly based on the methods of slices. The latter encompasses; the ordinary method (Fellenius 1936), simplified Bishop method (Bishop 1955), simplified Janbu method (Janbu and Kjaernsli 1956), Corps of Engineers method (U.S. Army Corps of Engineers 1967), Spencer method (Spencer 1967), Morgenstern-Price method (Morgenstern and Price 1965) and optimization-based methods(Samani and Meidani 2003). These are somehow different from each other with respect to the definition of the safety factors as well as the initial assumptions to derive the governing equations. Table 1. summarizes some well-known approaches of the slices method for the slope stability analysis.

Table 1. Different methods of slices for slope stability analysis

Method	*Assumptions*	*Equations used*	*Slip surface*
Ordinary method of slices (1936)	• Resultant of side forces (Ei) is parallel to the base of the slice	• Overall moment	Circular
Bishop (1955)	• Resultant of side forces is horizontal	• Overall moment • Vertical forces	Circular
Janbu (1956)	• Location of side force resultants on the sides of the slice (location can be varied) • Uses a correction factor f_o to account for the effect of the inter-slice shear forces.	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Morgenstern and price (1965)	• Inter-slice forces (Xi) related by • $V = \lambda f(x) E$ form of $f(x)$	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Spencer (1967)	• Inter-slice forces are parallel	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Samani and Meidani (2003)	• No Assumption	• Overall moment • Vertical forces • Equilibrium of forces in tangential direction to the base of slices	Circular

Note: E_i, X_i are introduced in Fig. 2.

There are two kinds of solutions for the problem. The first is a simplified solution where, the conditions of static equilibrium are not rigorously satisfied. In this solution, some assumptions are made to obtain the solution in a simple form. The second is a rigorous solution where, the equilibrium conditions are completely satisfied with no simplification (Sarma 1979). In general, the main features of limit equilibrium methods can be summarized as the following (Zhu et al. 2003):

1. The sliding body above an assumed slip surface is divided into a number of vertical (or inclined) slices.
2. The strength of the slip surface is mobilized by the same factor of safety, where the cohesion component and the friction component of the strength are reduced equally.
3. Assumptions regarding inter-slice forces are employed to render the problem determinate.
4. The factor of safety is derived from the force or/and moment equilibrium equations.

The number of equations and unknowns associated with the limit equilibrium methods are presented in Table 2. It shows that the number of available equilibrium equations is less than the number of unknowns in slope stability problems. As a result, the problem is inherently indeterminate. An indeterminate system of equations has an infinite number of solutions. Using engineering judgment and experiences, one may confine the unknown values between a lower and upper bound in order to manage the solution process. In this context, the problem's complexity can be more systematically handled using the optimization techniques. On this basis, the present study applies the central force optimization (CFO) method to solve the system of equations of slope stability analysis. The applied procedure satisfies all conditions of equilibrium with a high degree of precision. For this purpose, a slope stability analyzer model is developed and coupled to the CFO. The proposed model is applied against a benchmark example and the results are compared with the other conventional methods.

Table 2. Summary of equations and unknowns associated with limit equilibrium methods

Number of equations	*Type of equations*
N	Horizontal force equilibrium
N	Vertical force equilibrium
N	Moment equilibrium
N	Mohr-Coulomb failure criterion at the base of slice
$4N$	Total number of equations
Number of unknowns	*Type of unknowns*
N	Total normal force at the base of slice, P_iShear force at the base of slice, S_i
N	
$N-1$	Inter-slice total normal force, E_i
$N-1$	Inter-slice shear force, X_i
$N-1$	Point of application of the Inter-slice total normal force
N	Point of application of the total normal force at the base of a slice
1	Factor of safety
$6N-2$	Total number of unknowns

Note: N is the number of slices, P_i, S_i, E_i, X_i are introduced in Fig. 2.

Governing Equations

The method of slices is the most commonly used technique to solve the slope stability problems. The method is popular because for the sake of its easiness in concept and implementation as well as ability to accommodate complex geometrics and variable soil and water pressure conditions (Terzaghi and Peck 1967).

Fig. 1 shows a potential sliding mass along a trial slip surface through a homogenous slope. The sliding mass is subdivided into a number of vertical slices. The free body diagram of a slice is illustrated in Fig. 2. The forces acting on the slice are consisting of its own weight W_i , slide forces, shear component X_i, normal component E_i, shear resistance S_i and normal force P_i acting on the base of slice. Equating the moment of weight of the sliding mass with the moment of external forces acting on the slip surface about the center O of the slip circular surface yields:

$$\sum W_i \,.x_i = \sum S_i \,.r \tag{1}$$

where, x_i and r are the perpendicular distances shown in Fig. 1. The relation between the shear strength of failure and equilibrium shear stress along the slide surface can be expressed as follows.

$$\tau = \frac{\tau_f}{F} \tag{2}$$

in which, F is the factor of safety and τ_f is the soil shear strength of failure calculated based on the Mohr-Clumb equation as the following.

$$\tau_f = C' + \left(\frac{P_i}{l_i} - u_i\right).\tan\phi' \tag{3}$$

where C' is the drained cohesion of the soil, $\phi'$is drained internal friction angle, l_i is the slice's base length and u_i is the pore water pressure.

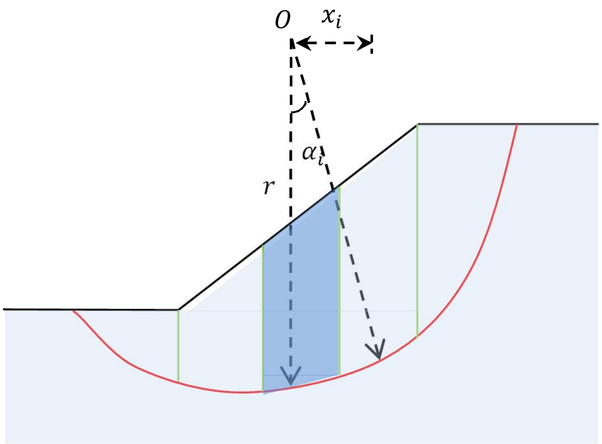

Figure 1. Sliding circular surface subdivided into vertical slices

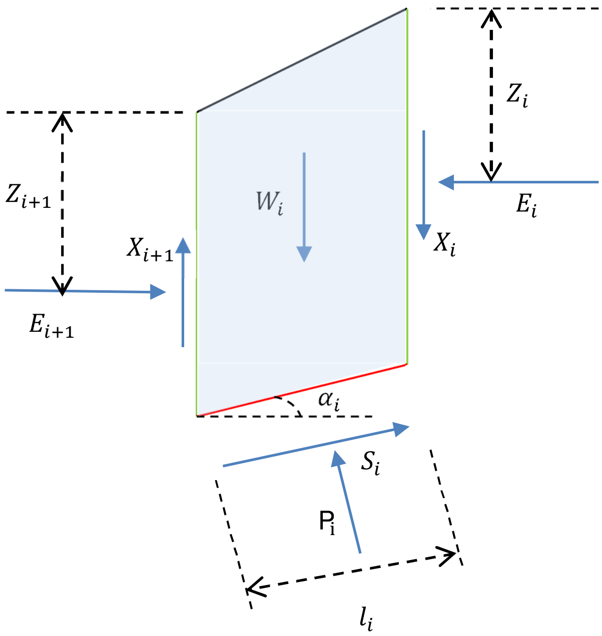

Figure 2. Free body diagram of a slice

Combining equations (2) and (3) results in the following equation.

$$\tau = \frac{1}{F}[C' + \left(\frac{P_i}{l_i} - u_i\right).\tan\phi'] \quad (4)$$

The vertical equilibrium for the slice i gives:

$$W_i + X_i - X_{i+1} = P_i\,.\cos\alpha_i + S_i\,.\sin\alpha_i \quad (5)$$

Rearranging the equation for P_i yields:

$$P_i = (W_i + X_i - X_{i+1}).sec\alpha_i - S_i.tan\alpha_i \tag{6}$$

Substituting the last expression in equation (4) and simplifying the result gives:

$$S_i = \frac{1}{F+tan\alpha_i.tan\phi'}\{C'.l_i + [(W_i + X_i - X_{i+1}).sec\alpha_i - u_i.l_i].tan\,\phi'\} \tag{7}$$

Hence, by substituting the last expression for S_i in equation (1) the following equation is obtained.

$$r.\sum\frac{\{C'.l_i+[(W_i+X_i-X_{i+1}).sec\alpha_i-u_i.l_i].tan\,\phi'\}}{F+tan\alpha_i.tan\phi'} = \sum r.W_i.\,sin\alpha_i \tag{8}$$

The summation of the normal inter-slice forces should also be zero. Accordingly,

$$\sum(E_i - E_{i+1}) = 0 \tag{9}$$

Resolving the force acting on the slice in a tangential direction to the base of the slice results in the following.

$$S_i = (E_i - E_{i+1}).cos\,\alpha_i + (W_i + X_i - X_{i+1}).sec\alpha_i \tag{10}$$

Therefor:

$$\sum(E_i - E_{i+1}) = \sum[S_i.sec\,\alpha_i + (W_i + X_i - X_{i+1}).tan\alpha_i] \tag{11}$$

Insertion of the value of S_i from equation (7) into equation (11) yields:

$$\sum\{\frac{C'.l_i + [(W_i + X_i - X_{i+1}).sec\alpha_i - u_i.l_i].tan\,\phi'}{F + tan\alpha_i.tan\phi'}.sec\alpha_i - (W_i + X_i - X_{i+1}).tan\alpha_i\} = 0 \tag{12}$$

Equations (8) and (12) are respectively the moment and force equilibrium equations. These two should be solved to determine the slope's unknowns X_i for every slice as well as the safety factor F.

The Optimization Problem

The slope stability analysis using the limit equilibrium methods is performed in two steps: First, the safety factor for a given slip surface is calculated and, second, the critical slip surface with the minimum factor of safety of the slope is sought. As earlier discussed, the number of equations is less than the number of unknowns and consequently, the problem's system of equations is indeterminate. The critical slip surface is found when the slope's safety factor is minimized. This issue introduces a nonlinear optimization problem in which, equation (13) is the objective function while, the slope's unknowns, X_i, xc, yc, and r are the optimization decision variables. Also, the acceptable bound of variation for each variable imposes a constraint on the objective function. . By minimizing the objective function subjected to the following inequality constraints, optimum values of the aforementioned decision variables are obtained.

$$\text{Minimize } F \text{ (safety factor) Subject to:} \tag{13}$$

$$X_{i\_l} \le X_i \le X_{i\_u} \quad i = 1,2,\dots,N$$
$$x_{c\_l} \le x_c \le x_{c\_u}$$
$$y_{c\_l} \le y_c \le y_{c\_u}$$
$$r_l \le r \le r_u$$
$$(Eq.8)^2 + (Eq.12)^2 \le \varepsilon \tag{14}$$

where, subscriptions "l" and "u" indicate the lower and upper bounds of decision variables respectively and, ε is an acceptable tolerance to satisfy the compatibility of equations (8) and (12).

Central Force Optimization (CFO) method

Small objects in our universe can become trapped in close orbits around highly gravitating masses (Formato 2009). This is the main concept behind the central force optimization method which searches in a decision space for the biggest object i.e., the global optimum of the problem. CFO is a new nature-inspired metaheuristic originally developed by Formato (2007). This method works based on the metaphor of gravitational kinematics, a branch of physics that deals with the motion of masses moving under the influence of gravity. Because of this metaphor utilized by CFO, all movements in the problem's decision space are governed by deterministic rules taken from the vector mechanics. As a consequence, CFO is a deterministic method which means that every initial starting point, named as probe herein, travels on a certain trajectory to reach the answer. In other words, CFO is not influenced by random-based operators which play a major role in most stochastic metaheurestics like GA, PSO and SA. In this view, CFO looks like mathematical methods which are deterministic too. Besides, CFO seems to be capable of escaping from local solutions as shown through several test functions by Formato (2007 and 2009). CFO is basically a maximization algorithm that starts to optimize an objective function; Eq., 13 herein, by flying a limited number of probes through the decision space. Every probe, p, is a feasible solution to the problem that has Nd coordinates in a Nd –dimensional problem. The position vector is therefore $\vec{R}_j^p = \sum_{m=1}^{N_d} x_j^{p,m} \hat{e}_m$ where $x_j^{p,m}$ is mth coordinate (decision variable) of probe p at time step j and $\hat{e}_m$ is the unit vector along the xm axis. As a comparison, probes in CFO are equivalent to chromosomes in GA. As each probe position represents a solution to the problem, it also takes a fitness value, mass M herein, from the objective function. In the problem's decision space, smaller probes are dragged by bigger ones and experience new accelerations making them flying through the space over time. As time progresses, probe position vectors are changed by some rules taken from the equations of motions until all probes settle around the largest mass found in the decision space.

Based on the Newton's universal law of gravitation, each probe p with position vector $\vec{R}_{j-1}^p \in R^{N_d}$ at time step j-1 experiences an acceleration vector $\vec{a}_{j-1}^p$ under the influence of gravitational central forces caused by other probes,

$$\vec{a}_{j-1}^p = G \sum_{\substack{k=1 \\ k \neq p}}^{N_p} U\left(M_{j-1}^k - M_{j-1}^p\right)\left(M_{j-1}^k - M_{j-1}^p\right)^{\alpha} \frac{\left(\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\right)}{\left|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\right|^{\beta}} \tag{15}$$

where Np = the number of probes, p = 1, 2, 3, …, Np the probe number name, j = time step of calculations which is in fact the optimization iteration number, α, β and G = the CFO constants, $M_{j-1}^p = C\left(\vec{R}_{j-1}^p\right)$, the objective function value against probe p at time step j-1 and U is the unit step function that gives U(x) = 1 if x ≥ 0 and = 0 otherwise. In fact, U keeps the CFO's gravity always attractive that means the attractive forces from bigger probes are only adopted for modifying probe position vectors. The position distance $\left|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\right|$ between two probes k and p, is obtained from the following relationship,

$$\left|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\right| = \sqrt{\sum_{m=1}^{N_d} \left(R_{j-1}^{k,m} - R_{j-1}^{p,m}\right)^2} \tag{16}$$

For more clarification, Fig. 3 demonstrates the utilized gravitational metaphor for a 3-dimensional space with 4 probes. Probe position vectors at time step j are then updated by applying the accelerations computed in previous time step as follows,

$$\vec{R}_j^p = \vec{R}_{j-1}^p + \frac{1}{2}\vec{a}_{j-1}^p \Delta t^2 \tag{17}$$

in which Δt = time step increment which is considered to be unity here. Using Eq. (17), the probes move to new locations that may be outside the feasible decision space. The method used for returning the errant probes can be very important to a CFO's performance. Formato (2007) suggested the following equation to repair infeasible components of errant probes,

$$\text{if } \vec{R}^{p}_{j,i} < x_i^{\min} \text{ then } \vec{R}^{p}_{j,i} = x_i^{\min} + F_{rep}\left(\vec{R}^{p}_{j-1,i} - x_i^{\min}\right) \text{ and,}$$
$$\text{if } \vec{R}^{p}_{j,i} > x_i^{\max} \text{ then } \vec{R}^{p}_{j,i} = x_i^{\max} - F_{rep}\left(x_i^{\max} - \vec{R}^{p}_{j-1,i}\right) \tag{18}$$

where $x_i^{\min}$ and $x_i^{\max}$ = respectively, the lower and upper bounds of decision variables and F_{rep} the reposition factor which is a user-specified parameter in range of 0-0.9.

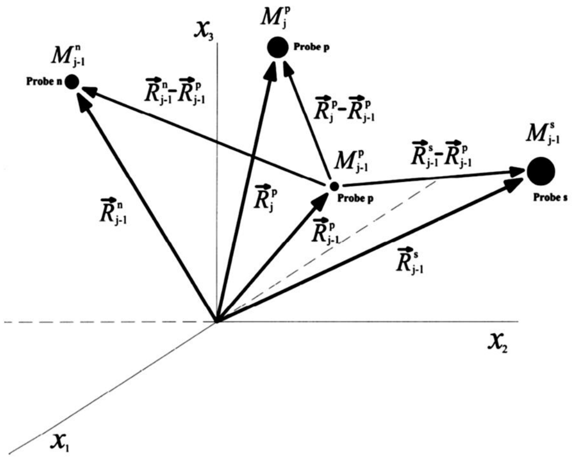

Figure 3. Typical 3-D CFO decision space (Formato 2007)

Since CFO is deterministic and there is no randomness in it, finding an optimum set for its constants is easier than stochastic metaheuristics. Formato (2009) comprehensively investigated the CFO's parameters by running it against several complex multi-dimensional test functions. It was concluded that setting α, β and G to be 2 and Frep to be 0.5 provides the best performance in most cases. However, Np and Frep require receiving more attention to be assigned. Another important factor in CFO is the initial spatial distribution of probes that in accordance with NP determines how much CFO knows about the decision space topology at the beginning of a run. For this purpose, several approaches may be applied, for example; probes may be uniformly distributed on each coordinate axis or on a pre-defined grid or they may be randomly generated. Another strategy can also be a combination of the aforementioned probe distribution methods. In fact, for a successful and efficient parameter setting it is necessary to be mathematically aquatinted with the problem at hand as well as with the CFO's behavior.

Similar to other heuristic methods, convergence of a CFO run must be systematically checked through its iterations. This is essential to prevent ineffectual computations. In general, a large number of time steps Nt is initially considered which determines the end of computations. However, the optimization may be sooner terminated for the sake of "fairly static probe distribution over several time steps" or "saturation of the best fitness over many time steps". Adding to these, in ITA problems, the global optimum value of objective function is already known which is theoretically equal to zero. This fact helps one introduce another stopping criteria to the CFO algorithm applied to ITA problems.

Based on what described about the central force optimization method, Fig.4 as a flowchart is also presented here to better show how the method works.

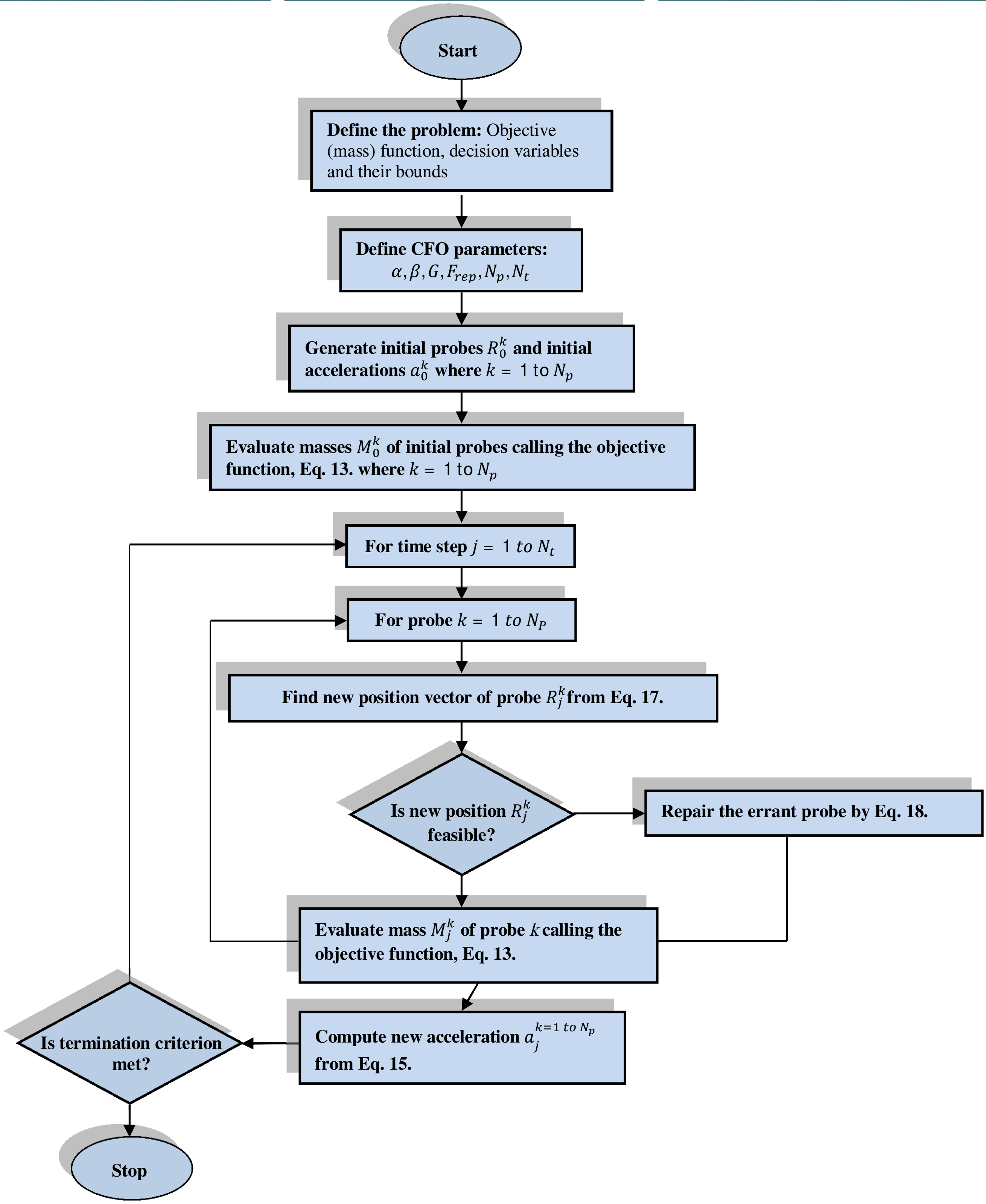

Figure 4. CFO algorithm

Illustrative Example

In this section, an illustrative example from the literature is adopted and analyzed using the proposed method. The geometry and soil parameters are presented in Fig 5. To analyze the slope stability it is supposed that the center of the coordinate system is located at point A. The safety factor and slip surface geometric parameters are considered to be unknown.

The example is solved with 10 slices. The upper and lower bounds of the decision variables are shown in Table 3. The CFO's constants, α, β and G, are set to be 2 as proposed by Formato (2007 and 2009). Then, several pre-runs were carried out to set F_{rep} by changing it over range of 0-0.9. It was concluded that $F_{rep} = 0.26$ is the best choice in this example. Now, CFO is ready to start optimization by flying its probes through the decision space. Using the objective function in Eq. 13, the probe masses are evaluated and their positions are updated over time until the biggest probe i.e., the global optimum is achieved.

After about 110 iterations the best results of the optimization were obtained as the following; $F = 2.32$; $x_c = 4.13$ m; $y_c = 11.68$ m and $r = 9.37$ m. For more investigations, the safety factors evaluated here and in the previous studies have been reported in Table 4. Also, in Table 5 the inter-slice shear forces obtained here are compared to the previous works. Accordingly, it is concluded that the model has a good agreement with the previous well-known methods. Furthermore, the maximum constraint violation of $(\text{Eq.}8)^2 + (\text{Eq.}12)^2$ is obtained 6E-06 which means that, both moment and force equilibrium equations have been precisely fulfilled through the applied model. This is an important achievement since, only by using optimization techniques, compatibility of equations (8) and (12) both can be simultaneously met.

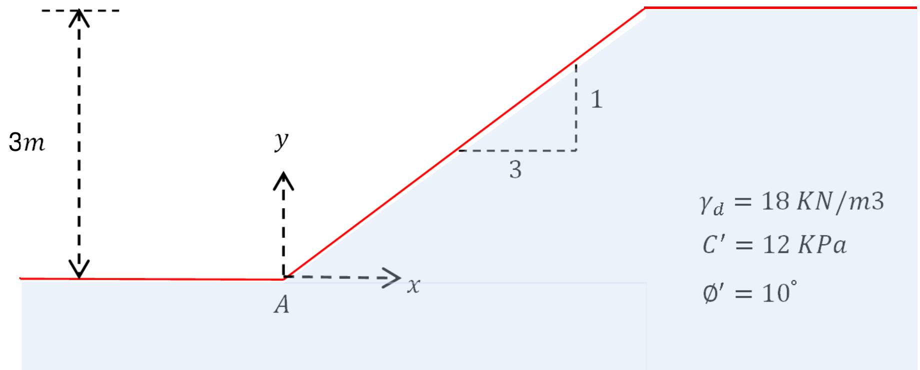

Figure 5 Geometry and soil parameters of the example

Table 3 Upper and lower bounds of unknown values

Unknowns	*Lower limit*	*Upper limit*
xc	0	15
yc	0	15
r	3	15
$X1$	1	10
$X2$	1	20
$X3$	1	22
$X4$	1	22
$X5$	1	25
$X6$	1	22
$X7$	1	20
$X8$	1	20
$X9$	1	10

Table 4. Safety Factor calculated by various methods

Bishop (1955)	*Janbu (1956)*	*Morgenstern-Price (1965)*	*Samani-Meidani (2003)*	*CFO (Present study)*
2.394	2.125	2.391	2.437	2.326

Table 5. Inter-slice shear force calculated by different methods

Shear force	*Constant-Interslice*	*Half-Sine*	*Corps of Eng.*	*Samani and Meidani (2003)*	*CFO(Present Study)*
$X1$	3.38	1.39	3.4	3	3.24
$X2$	7.27	5.49	7.3	7.4	7.38
$X3$	10.95	11.29	11	12.3	12.21
$X4$	13.54	16.28	13.6	19.7	19.39
$X5$	14.44	18.1	14.5	21.8	21.07
$X6$	14	17.04	14.06	22.5	21.52
$X7$	10.7	10.92	10.74	22.1	20.55
$X8$	7.35	6.12	7.4	21.5	19.46
$X9$	2.44	1.24	2.45	12	11.48

Conclusion

Design or evaluation of any embankment and slope to resist the destructive effects safely, requires to solve a complicated problem in the field of geotechnical engineering. The limit equilibrium methods are the most common technique for the slope stability analysis. However, these methods need some simplifications to the problem governing equations to solve the problem by simple algebra. In general, the problem of slope stability analysis introduces an indeterminate problem in which the number of unknowns is more than the number of available equations. To solve the problem completely without any simplification optimization methods are useful. Accordingly, the present study introduced a nonlinear constrained optimization framework to solve the slope stability analysis problem. For this purpose, a nature-inspired metaheuristic method of CFO was developed and coupled to the limit equilibrium and method of slices. Through the proposed scheme, the CFO can freely search into the problem decision space and gradually approach to the feasible regions where, the moment and force equilibrium equations are completely satisfied. The method was then applied to an illustrative example slope and, the minimum safety factor, inter-slice shear forces, coordinates of the slip circle center and radius were calculated. The results showed that the model is in a good agreement with the previous studies and the CFO can be taken account as a powerful approach to handle the problem’s complexity and constraints. The proposed procedure would be also applicable for dealing with more complicated slope stability analysis problems including deferent soil layers, variable soil properties and having pore water pressure.

References

Bishop, A. W. 1955, "The use of the slip circle in the stability analysis of slopes", Geotechnique,vol. 5, no. 1, pp. 7-17.

Duncan, J. M. 1996, "State of the art: Limit equilibrium and finite-element analysis of slopes", Journal of Geotechnical Engineering, vol. 122, no. 7, pp. 577-596.

Fellenius, W. (1936), "Calculation of the stability of earth dams", Transactions, 2nd Congress on Large Dams, Washington, pp. 445-462.

Formato RA (2007) Central force optimization: a new metaheuristic with applications in applied electromagnetics. J. Prog. in Electromagnetics Res. 77(1):425–491.

Formato RA (2009) Central force optimization: a new gradient-like metaheuristic for multidimensional search and optimization. Int. J. Bio-Inspired Computation. 1(4):217–238.

Griffiths, D.V. and Fenton, G.A., (2004). Probabilistic slope stability analysis by finite elements. Journal of Geotechnical and Geoenvironmental Engineering, 130(5), pp.507-518.

Griffiths, D.V. and Lane, P.A., (1999). Slope stability analysis by finite elements. Geotechnique, 49(3), pp.387-403.

Griffiths, D.V. and Marquez, R.M., (2007). Three-dimensional slope stability analysis by elasto-plastic finite elements. Geotechnique, 57(6), pp.537-546

IEEE International Conference on Neural Networks, IEEE Service Center, Piscataway,NJ, (1995).

Janbu, N., Bjerrum, L., & Kjaernsli, B. (1956), "Soil mechanics applied to some engineering problems", Norwegian Geotechnical Institute Publication, vol. 16, pp. 1-93.

Morgenstern, N. R. & Price, V. E. (1965), "Analysis of stability of general slip surfaces",Geotechnique, vol. 15, no. 1, pp. 79-93.

Samani, Hossein M. V., and Meidani, M. (2003). "Slope Stability Analysis Using A Non-linear Optimization Technique." International Journal of Engineering, Vol. 16, No. 2. pp. 147-156.

Sarma, S. K. 1979, "Stability analysis of embankments and slopes", American Society of Civil Engineers, Journal of the Geotechnical Engineering Division, vol. 105, no. 12, pp. 1511-1524.

Spencer, E. (1967), "Method of analysis of stability of embankments assuming parallel inter-slice forces", Geotechnique, vol. 17, no. 1, pp. 11-26.

Terzaghi. K. and Peck. R. B. (1967).Soil mechanics i n engineering practice. (2nd ed.), John Wiley and Sons lnc. New York. N.Y.Deb,

U.S.Army Corps of Engineers (1967), Stability of slopes and foundations, Engineering manual,Vicksburg, Miss. Whitman, R.V., and Bailey, W.A.

Zhu, D. Y., Lee, C. F., & Jiang, H. D. (2003), "Generalized framework of limit equilibrium methods for slope stability analysis", Geotechnique, vol. 53, no. 4, p