Monitoring Correlation Within Simple Linear Profiles for AR(1) Processes

[1]Mehdi Koosha, [2]Amirhossein Amiri

[1, 2] *Industrial Engineering Department, Faculty of engineering, Shahed university, Teh*ran, *Iran*

[1]*Koosha@shahed.ac.ir*, [1]*Amiri@shahed.ac.ir*

ABSTRACT

In some statistical process control applications, a relationship between a response variable and an explanatory variable referred to as profile characterize the quality of a process or product and should be monitored over time. Many researches have been done in this area but in most of them, the successive observations in different levels of the explanatory variables are assumed to be independent. This assumption is violated in many real case problems for example when observations are taken in short periods of time. If one neglects the correlation between the observations in different levels of explanatory variable, it leads to misleading results on the Average Run Length (ARL) criterion. In recent years, some researchers have proposed some methods to account for the autocorrelation whitin a simple linear profile. Soleimani et al. [1] proposed a transformation technique to consider the correlation between observations in each profile. This paper specifically concentrates on the autocorrelation whitin simple linear profile in Phase II and proposes the use of real variance of autocorrelated observations to take the autocorrelation into consideration. Our simulation studies show the superiority of the proposed technique over the transformation technique in terms of average run length criterion.

Key words

Profile monitoring, Phase II, Autocorrelation, Average Run Length (ARL)

1- Introduction

Sometimes the relationship between a response variable and some explanatory variables called as profile should be monitored over time instead of the quality characteristic itself. According to the type of this relationship, profiles are classified into some categories such as simple linear profiles, multiple linear profiles, Polynomial profiles, etc. In the recent years, many researches have been done in the area of profile monitoring. Mestek et al. [2], Stover and Brill [3] and Amiri et al. [4] presented some applications of profiles. Kang and Albin [5], Kim et al. [6] and Mahmoud et al. [7] proposed some methods for monitoring simple linear profiles. Zou et al. [8] and Mahmoud [9] proposed some methods for monitoring multiple linear profiles. Monitoring multivariate profiles was considered by authors such as Noorossana et al. [10] and Eyvazian et al. [11]. Authors including Vaghefi et al. [12] and Williams et al. [13] proposed methods to monitor nonlinear profiles.

In all of the aforementioned researches there is a basic assumption that the observations within each profile are independent. There are some papers investigating the effect of autocorrelation and proposed some methods to take this effect into account. Jensen et al. [14] proposed a method based on Linear Mixed Models (LMM) to consider the effect of autocorrelation in simple linear profiles. A Non Linear Mixed Model (NLMM) based method is proposed by Jensen and Birch [15] that could be applied to monitor nonlinear autocorrelated profiles.

Soleimani et al. [1] proposed a method based on data transformation to eliminate the effect of autocorrelation when the autocorrelation structure is first order auto regressive-AR (1)- in simple linear profiles. AR (1) is a standard structure of autocorrelation in which each observation is correlated to its prior observation with a correlation coefficient (ρ). So the correlation matrix is as follows:

$$\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \tag{1}$$

Let Y_{ij} be the ith observation of the jth profile. So:

$$\begin{aligned} Y_{ij} &= a_0 + a_1 x + \varepsilon_{ij} \\ \varepsilon_{ij} &= \rho\varepsilon_{(i-1)j} + a_{ij} \end{aligned} \tag{2}$$

In Eq. (2), ρ is the correlation coefficient a_{ij}'s are independent random error variable with mean zero and variance σ^2. It can be easily seen that ε_{ij} is correlated with $\varepsilon_{(i-1)j}$.

This paper proposes a new method based on a modification in the variance estimator to take the effect of autocorrelation into account in phase II monitoring of correlation within simple linear profiles. The rest of this paper is structured as follows: Section 2 is dedicated to introducing one of the methods proposed by Soleimani et al. [1], T^2 based method. Section 3 illustrates the effect of autocorrelation on the performance of T^2 control chart when it is neglected. The proposed method is introduced in Section 4. A numerical example which shows the performance of the proposed method is illustrated in Section 5. Finally, conclusions and future research are in Section 6.

2. The method by Soleimani et al. [1]

This section analyzes the method proposed by Soleimani et al. [1]. They first proposed transformations in Eq. (3) to make the observations independent.

$$\begin{aligned} y'_{ij} &= y_{ij} - \rho y_{(i-1)j} \\ \beta_0 &= a_0(1-\rho) \\ \beta_1 &= a_1 \\ x'_i &= x_i - \rho x_{i-1} \end{aligned} \tag{3}$$

By applying these transformations in Eq. (2), the following equation is obtained in which y'_{ij}'s are independent.

$$y'_{ij} = \beta_0 + \beta_1 x'_i + a_{ij} \tag{4}$$

Then the T^2 Hoteling control chart with the following statistic was used to monitor the regression parameters estimator of the transformed model in Eq. (4)

$$T_j^2 = (\hat{\beta}_j - \bar{\beta})^T \Sigma^{-1} (\hat{\beta}_j - \bar{\beta}), \tag{5}$$

where Σ is computed by Eq. (4)

$$\Sigma = \begin{bmatrix} \sigma^2(\frac{1}{n-1} + \frac{\bar{X}^2}{S_{XX}}) & -\sigma^2 \frac{\bar{X}}{S_{XX}} \\ -\sigma^2 \frac{\bar{X}}{S_{XX}} & \frac{\sigma^2}{S_{XX}} \end{bmatrix} \tag{6}$$

In the above equations, β_j is vector of estimated coefficients of jth profile. $\bar{\beta}$ is the mean vector of the coefficients of all profiles and S_{XX} is computed by $\sum(X_i - \bar{X})^2$. Upper control limit (UCL) for this control chart is calculated as Eq. (5):

$$UCL = \chi^2_{p,\alpha}, \tag{7}$$

where p is the number of regression parameters estimators. Note that n-1 in the first component of the covariance matrix is due to the transformation and reduction of one of the observations.

3- The effect of autocorrelation

In this section we first illustrate the effect of autocorrelation under different values of ρ on the ARL criterion. β_0 and β_1 are set to 3 and 2, respectively. ε_{ij}'s are normally distributed with mean zero and variance 1. The x-values are equal to 2,4,6, and 8.Table 1 shows the effect of autocorrelation with AR(1) structure on the performance of the traditional T^2 control chart under different values of ρ and when the shifts of $\lambda\sigma$ are applied in the intercept (β_0)**.** Note that UCL under **ρ** equals to zero is obtained by Eq. (5) and the other upper control limits are obtained by simulation studies to obtain in-control ARL of roughly 200 for all correlation coefficients considered.

Table 1 ARL comparison when β_0 shifts to $\beta_0 + \lambda\sigma$ and under different correlation coefficients (***ρ***)

UCL	λ / ρ	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
10.62	0	200	136.03	64.75	27.41	13.1	6.94	3.99	2.65	1.86	1.45	1.23
13.35	0.2	200.37	147.83	72	34.48	17.43	9.38	5.52	3.58	2.48	1.86	1.50
17.85	0.4	199.55	153.25	84.3	43.69	23.93	13.63	8.41	5.45	3.75	3.68	2.72
25.96	0.6	200.67	166.33	105.63	61.24	36.18	21.86	13.86	9.19	6.42	4.61	3.44
40.5	0.8	200.41	177.79	130.73	85.92	56.23	37.35	25.29	17.29	12.46	8.91	6.71

Table 2 shows the effect of autocorrelation on the performance of T^2 control chart when shifts of $\nu\sigma$ are applied on the slope (β_1) under different values of correlation coefficients.

Table 2 ARL comparison when β_1 shifts to $\beta_1 + \nu\sigma$ and under different correlation coefficients (***ρ***)

UCL	ν / ρ	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
10.62	0	200	167.57	107.43	60.95	34.56	19.92	12.31	7.77	5.30	3.75	2.76
13.35	0.2	200.37	175.37	114.23	70.04	42.26	25.90	16.36	10.77	7.19	5.13	3.82
17.85	0.4	199.55	177.72	129.67	83.67	53.71	34.16	22.46	15.61	10.82	7.88	5.89
25.96	0.6	200.67	181.58	143.64	101.85	75.51	49.38	34.69	24.69	18.18	13.07	9.99
40.5	0.8	200.41	190.42	160.81	123.74	96.59	71.22	53.25	39.97	30.52	23.11	18.04

Table 3 shows the effect of autocorrelation on the performance of T^2 control chart when shifts of $\gamma\sigma$ are applied on the variance.

Table 3 ARL comparison when σ shifts to $\gamma\sigma$ and under different correlation coefficients ($\boldsymbol{\rho}$)

UCL	ρ \ γ	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
10.62	0	200	39.68	15.36	8.01	5.11	3.76	2.99	2.52	2.17	1.95	1.83
13.35	0.2	200.37	41.41	15.48	8.20	5.26	3.77	3.05	2.56	2.22	1.97	1.81
17.85	0.4	199.55	43.95	16.92	8.90	5.70	4.17	3.29	2.73	2.34	2.07	1.91
25.96	0.6	200.67	48.28	19.29	10.28	6.59	4.78	3.72	3.07	2.60	2.30	2.11
40.5	0.8	200.41	49.61	21	11.36	7.59	5.49	4.29	3.51	2.98	2.62	2.34

As shown in Tables 1 to 3, as the correlation coefficient increases, the control chart performs weaker in terms of ARL criterion.

4- Proposed Method

As shown in Neter [16], when data have AR(1) autocorrelation structure with correlation coefficient of ρ, the real variance of the process should be computed by Eq. (8).

$$\sigma_{real}^2 = \frac{\sigma^2}{1-\rho^2} \tag{8}$$

Hence, the variance covariance matrix of the regression parameters estimators is obtained as follows:

$$\Sigma = \begin{bmatrix} \sigma_{real}^2 (\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}) & -\sigma_{real}^2 \frac{\bar{X}}{S_{XX}} \\ -\sigma_{real}^2 \frac{\bar{X}}{S_{XX}} & \frac{\sigma_{real}^2}{S_{XX}} \end{bmatrix} \tag{9}$$

The real variance of autocorrelated data can account for the autocorrelation structure in the dataset. Then, we use the modified variance covariance matrix in T^2 Hoteling statistic for monitoring the autocorrelated simple linear profiles in Phase II.

5- Simulation study

In this section, we evaluate the performance of the proposed method in comparison with the method by Soleimani et al. [1]. For this purpose, the numerical example used in Section 3 is revisited. Figure 2 shows the results under both weak and strong autocorrelation, when shifts are applied on the intercept (β_0), the slope (β_1) and the variance, respectively.

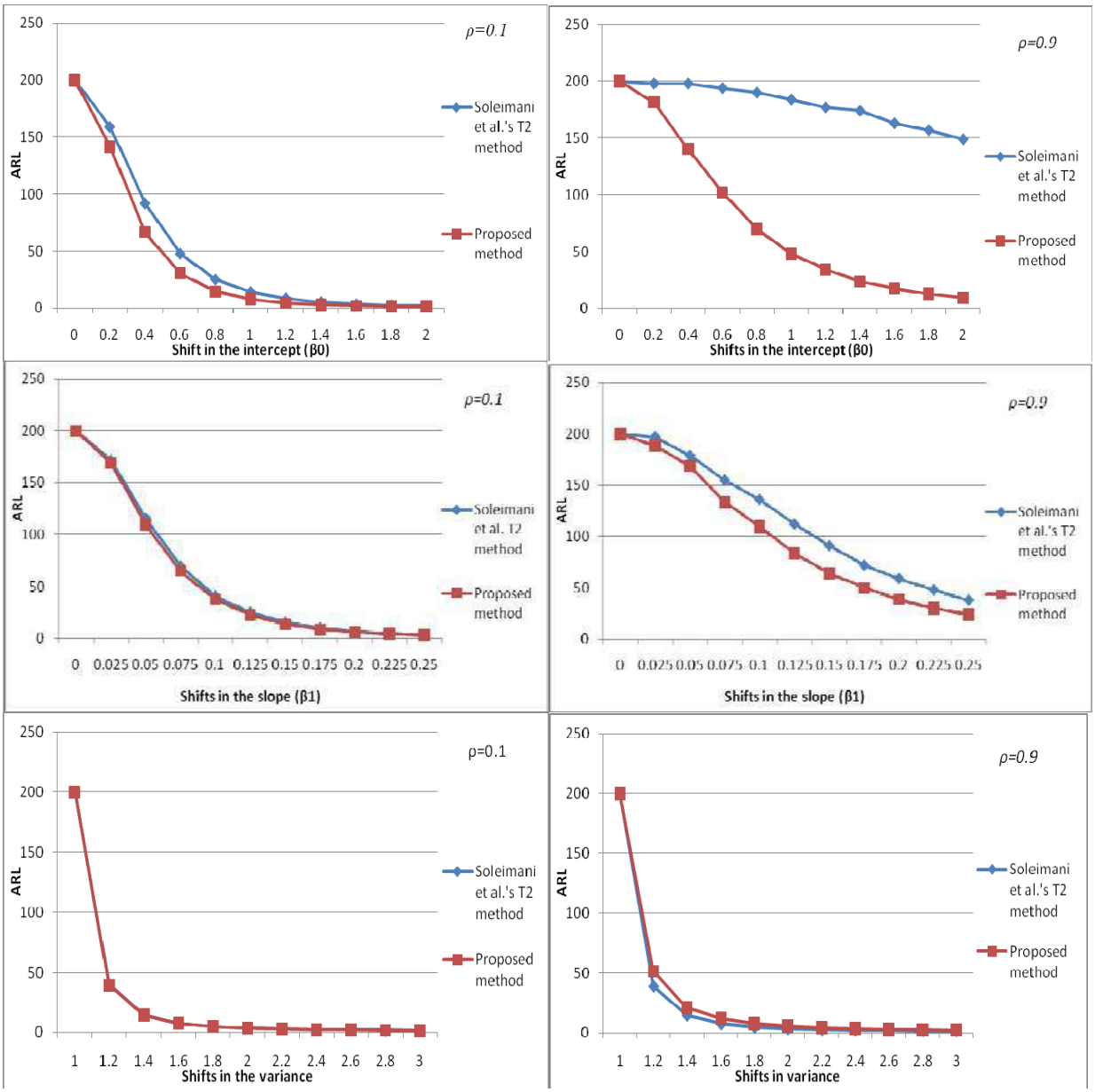

Fig 2- Comparison between the performance of the proposed method and Soleimani et al. [1] method

Figure 2 shows that the proposed method performs uniformly better than the method by Soleimani et al. [1] in various shifts in the intercept and the slope under both weak and strong correlation coefficients. The better performance of the proposed method respect to the method by Soleimani et al. [1] improves as the correlation coefficient ρ increases. On the other hand when shifts are applied in the variance, the proposed method performs roughly the same as the method proposed by Soleimani et al. [1] in weak correlation coefficients. However, it is worse than soleimani et al. [1] 's method in strong correlation coefficients.

6- Conclusions

In this paper we proposed a method based on real variance of autocorrelated dataset to monitor autocorrelated profiles with AR(1) structure. The results showed that the proposed method has better performance than the method by Soleimani et al. [1] under all shifts to the slope and

intercept considered in the paper. In addition, the simulation results showed that as the correlation coefficient increases, the performance of the proposed method gets better respect to the method by Soleimani et al. [1]. Extending this research to other types of correlation structures could be a suggestion for future research.

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