On the Average Achievable Rate of Block Fading Decentralized Interference Channel

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Abstract—**This paper aims to explore the average achievable rate of two-user decentralized Gaussian interference channel in a non-ergodic block fading environment, assuming the channel gains are constant throughout one transmission block and vary independently for the next block. It is assumed the channel gains are not available at the transmitters, while direct and cross channel gains are perfectly available at the corresponding receivers. In this regard, under Gaussian codebook assumption and considering the signal arising from the other co-channel communication link is treated as an interference, a multi-layer coding strategy is exploited at each transmitter. Then, under individual transmit power constraints and in a Rayleigh block fading environment, the optimum power allocation across code layers is derived, showing the average achievable rate of using multi-layer coding outperforms that of using a single layer code.**

Index Terms—**Decentralized interference channel, multi-layer coding, fading channel.**

I. Introduction

THIS paper concerns the average achievable rate of block fading decentralized interference channel in which there is not a central controller to share the existing resources, meaning time sharing and/or bandwidth partitioning is not allowed. Moreover, the interference signal is treated as an additive noise, since each receiver merely knows the codebook associated with the corresponding transmitter.

Recently, there have been some attempts to explore the achievable rate of decentralized interference channels, however, the availability of channel state information at the transmitters (CSIT) is the integral part of most of existing works (e.g., [1]). This motivated us to explore the achievable rate of such channel when the channel gains are not available at the transmitters.

This paper assumes block fading decentralized interference channel in which the channel gains associated with direct and cross links are merely available at the receiver sides. To get an insight regarding the coding strategy exploited in the current work, we simply assume a special case in which the channel gains of cross links are zero. In this case, the network subsumes two individual point-to-point communication links for which due to the lack of CSIT, each link can be cast as a degraded broadcast channel with an infinite number of virtually ordered users, corresponding to channel realizations, called the broadcast strategy [2]. Accordingly, the superposition code can be exploited to achieve the sum-rate capacity of such channel. The objective in this work is to find the best power allocation policy across code layers to maximize the average sum-rate capacity of each link.

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In what follows, Section II presents some background information for point-to-point communication channel in the absence of CSIT, where the broadcast strategy is employed to maximize the throughput. Section III aims to extend the terminology to decentralized interference channel in the presence of Gaussian noise. Finally, Section IV illustrates the results and Section V summarizes findings.

II. Background Information

We assume a point-to-point communication link, owing to transmit power constraint p_T, in a block fading environment. In this case, the received signal can be written as follows,

$$y_i = hx_i + n_i \text{ for } i = 1, \ldots, N \ , \tag{1}$$

where x_i, y_i, and n_i denote, respectively, the transmitted signal, the received signal and additive Gaussian noise of unit variance, i.e., $\mathcal{N}_c(0, 1)$. Moreover i denotes the i^{th} channel uses index of current transmission block. Also, it is assumed the channel gain (h) is constant throughout one transmission block and varies independently for the next block. In this case, when the channel gain is not available at the transmitter, it is demonstrated that the broadcast strategy is optimal [2].

According to the broadcast strategy, the system can be thought as if there are infinite number of virtual users, each corresponds to a channel realization, i.e., $s = |h|^2$, which is designated to get a fractional rate, i.e., $dR(s)$. Accordingly, as long as the channel strength exceeds s, this user is able to decode this fractional rate. Moreover, according to the broadcast approach, the users are indexed such that the j^{th} user can decode the corresponding signals of all users indexed from 1 to $j-1$. Thus, the achievable fractional rate of the j^{th} user corresponding to the channel strength s can be expressed as $dR(s) = \ln(1 + \frac{sp(s)ds}{1+sI(s)}) \simeq \frac{sp(s)ds}{1+sI(s)}$, where $p(s)ds$ is the transmit power of the j^{th} layer parameterized by s, and $I(s)$ is the interference power arising from layers indexed $j+1$ to ∞, that is $I(s) = \int_s^\infty p(u)du$. It is worth mentioning that $I(0) = p_T$, and $I(\infty) = 0$. Moreover, if the instantaneous fading power is s, the total rate, $R(s)$, is the sum of all fractional rates designated for the receivers corresponding to the channel strengths lower than s, i.e. $R(s) = \int_0^s dR(u) = \int_0^s \frac{up(u)du}{1+uI(u)}$. Finally, taking expectation with respect to the fading power, i.e., s, implies,

$$R_{ave} = \int_0^\infty f(s)R(s)ds = \int_0^\infty (1 - F(s))dR(s), \tag{2}$$

where in (2), $f(.)$ and $F(.)$ denote, respectively, the probability density function (pdf) and cumulative distribution function (cdf) associated with the fading power s. Noting $p(s) = -\frac{dI(s)}{ds} = -I'(s)$, the problem is to find the optimum power allocation strategy across layers, such that R_{ave} is maximized, i.e,

$$\begin{aligned} R_{ave} &= \max_{p(.)} \int_0^\infty (1-F(s)) \frac{sp(s)}{1+sI(s)} ds \\ &\text{s.t.} \int_0^\infty p(s)ds = p_T \;\text{ and }\; p(s) \geq 0 \;. \end{aligned} \tag{3}$$

Noting $p(s) = -I'(s)$, using the method of Lagrange multipliers and after some straight manipulations, we have,

$$\begin{aligned} L = & \int_0^\infty \big((1-F(s))\frac{-sI'(s)}{1+sI(s)} + I'(s)(\lambda - v(s))\big)ds \\ & + \lambda p_T, \end{aligned} \tag{4}$$

where $v(s)$ and λ are assumed to be, respectively, as an arbitrary non-negative function and a non-negative coefficient associated with the constraints of (3). The integrand of (4) can be thought as a function $G(s, I, I') = (1-F(s))\frac{-sI'(s)}{1+sI(s)} + I'(s)(\lambda - v(s))$. As a result, the optimization problem can be solved using the Euler equation [3] as $G_I - \frac{dG_{I'}}{ds} = 0$, where $G_I = \frac{\partial G}{\partial I}$ and $G_{I'} = \frac{\partial G}{\partial I'}$. Thus, we have,

$$G_I - \frac{dG_{I'}}{ds} = \frac{1-F(s)-sf(s)-s^2f(s)I(s)}{(1+sI(s))^2} + v'(s) = 0 \;. \tag{5}$$

Moreover, from complementary slackness conditions, we have

$$\lambda(p_T + \int_0^\infty I'(s)ds) = 0, \;\text{ and }\; I'(s)v(s) = 0 \;. \tag{6}$$

Referring to (6), $v(s) = 0$ when $I'(s) = -p(s) < 0$. In this case, noting (5), we have [2],

$$I(s) = \frac{1-F(s)-sf(s)}{s^2f(s)}. \tag{7}$$

Thus, $I'(s) = \frac{(F(s)-1)(2sf(s)+s^2f'(s))}{s^4f(s)^2} = \frac{F(s)-1}{s^4f(s)^2}\frac{d(s^2f(s))}{ds}$. As a result, the following condition is required to have $I'(s) = -p(s) < 0$, i.e., to have non-zero power at point s ,

$$\frac{d(s^2f(s))}{ds} > 0. \tag{8}$$

In the sequel, we are going to show that if there are totaly K disjoint intervals, i.e., $[s_{k_l}, s_{k_u}]$ for $k = 1, \ldots, K$, in which (8) holds, then there is at most one subinterval with positive power allocation within each disjoint interval. Suppose otherwise, assume $s_{k_l} < c_1 < c_2 < d_1 < d_2 < s_{k_u}$ such that (8) holds for $s \in [s_{k_l}, s_{k_u}]$ and there are two positive subintervals, i.e., $[c_1, c_2]$ and $[d_1, d_2]$, with zero power interval (c_2, d_1) between them. In [3], it is demonstrated that for a piecewise continuous extremal solution, the following Weierstrass-Erdmann (corner) conditions hold at any corner point, i.e., s_c,

$$\begin{aligned} G_{I'}|_{s=s_c^+} &= G_{I'}|_{s=s_c^-} \\ G - I'G_{I'}|_{s=s_c^+} &= G - I'G_{I'}|_{s=s_c^-} \end{aligned} \tag{9}$$

Thus, noting the first equation in (9), we have,

$$\begin{aligned} &\big(1-F(s_c)\big)\frac{s_c}{1+s_cI(s_c^-)} + v(s_c^-) - \lambda = \\ &\big(1-F(s_c)\big)\frac{s_c}{1+s_cI(s_c^+)} + v(s_c^+) - \lambda \end{aligned} \tag{10}$$

Again, referring to slackness conditions in (6) and noting $c_2^- \in [c_1, c_2]$, we have $v(c_2^-) = 0$. Thus, noting (10) and using the fact that I(s) is a continues smooth function, i.e., $I(s^-) = I(s^+)$, thus it follows $v(c_2^+) = 0$. By the same token, we have $v(d_1^-) = 0$. On the other hand, since $p(s) = -I'(s) = 0$ in the interval $s \in (c_2, d_1)$, thus using (5), we have, $v'(s) = \frac{s^2f(s)(I(c_2) - \frac{1-F(s)-sf(s)}{s^2f(s)})}{(1+sI(c_2))^2}$. Since $s^2f(s)$ has positive derivation in $s \in (c_2, d_1)$, implying $\frac{1-F(s)-sf(s)}{s^2f(s)}$ is a decreasing function, thus $v'(s)$ is strictly positive. As a result, $v(s)$ would be an increasing function in the interval $s \in (c_2, d_1)$. However, this contradicts the earlier finding that $v(c_2^+) = v(d_1^-) = 0$. Thus, there is at most one positive power allocation interval in $s \in [s_1, s_2]$.

Although, it is proved that there is at most one positive subinterval within each interval $[s_{k_l}, s_{k_u}]$, it is still possible to have totally more than one positive subinterval, each belongs to one of disjoint intervals $[s_{k_l}, s_{k_u}]$ for $k = 1, \ldots, K$.

III. Multi-Layer Coding in Block Fading Gaussian Interference Channel

This paper concerns two-user decentralized interference channel in a block Rayleigh fading environment, as follows,

$$Y_k = h_{k1}X_1 + h_{k2}X_2 + Z_k \quad k = 1, 2 \;, \tag{11}$$

where X_k, Y_k and Z_k denote, respectively, the transmitted signal from the k^{th} transmitter, the received signal at the k^{th} receiver and additive white Gaussian noise at the k^{th} receiver which is of unit power, i.e., $\mathcal{N}_c(0, 1)$. Also it is assumed channel coefficients $h_{ki} \sim \mathcal{N}_c(0, 1)$ for $k, i \in \{1, 2\}$ are constant throughout one transmission block and vary independently for the next block. Moreover, the fading power is defined as $s_{ki} = |h_{ki}|^2$, thus having exponential distribution with probability density function $f_{s_{i,j}}(x) = e^{-x}$. As is mentioned in the preceding section, this work investigates the case in which the transmitters are unaware of channel gains, while s_{ik} for $k \in \{1, 2\}$ are perfectly available at the i^{th} receiver. Moreover, the inputs are subject to the average power constraint $E[|X_k|^2] \leq p_k$ and transmitters employ Gaussian codebooks. Thus, the interference signal is treated as an additive Gaussian noise, assuming no cooperation is allowed at the transmitter and receiver sides. Moreover, as transmissions take place in a decentralized manner, time sharing and/or bandwidth partitioning is not allowed.

We simply concentrate on the first link, as any findings can be readily extended to the second link by using the same token. Referring to (11), the received signal at the first receiver can be modeled as $Y_1 = h_{11}X_1 + h_{12}X_2 + Z_1$.

Note that the intended signal at this receiver is X_1, thus interfering signal X_2 is treated as an additive noise. Noting $E[|X_k|^2] \leq p_k$ ($k \in \{1, 2\}$) are drawn from Gaussian distributions and are independent of channel gains, so the interfering signal $h_{12}X_2$ follows Gaussian distribution with power $s_{12}p_2$. This implies $Y_1 = h_{11}X_1 + \hat{Z}$, where $\hat{Z}$ is the equivalent Gaussian noise with distribution $\mathcal{N}_c(0, s_{12}p_2 + 1)$. Multiplying the received signal by $\frac{1}{\sqrt{s_{12}p_2+1}}$, one arrives at $\check{Y}_1 = \frac{h_{11}}{\sqrt{s_{12}p_2+1}}X_1 + \check{Z}$, where $\check{Z} \sim \mathcal{N}_c(0, 1)$. As a result, the equivalent fading coefficient of the intended signal ($\frac{h_{11}}{\sqrt{s_{12}p_2+1}}$)

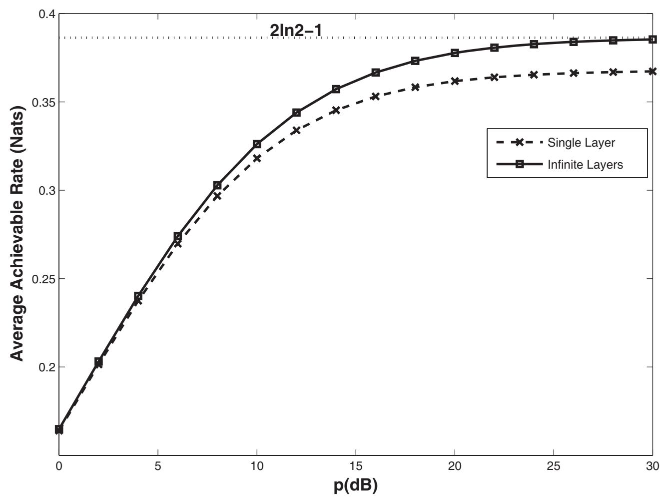

Fig. 1. The average achievable rate of single-layer and multi-layer code versus different transmit powers.

has power $s = \frac{s_{11}}{s_{12}p_2+1}$. Thus, the cdf of equivalent fading power s can be computed as,

$$F_S(s) = Pr(S \leq s) = \iint_R f_{S_{11}}(s_{11}) f_{S_{12}}(s_{12}) ds_{11} ds_{12} \ , \tag{12}$$

where $R = \{s_{11}, s_{12} \in [0,\infty) | \frac{s_{11}}{s_{12}p_2+1} \leq s\}$. For instance, in Rayleigh fading environment, it follows,

$$F_S(s) = \int_0^\infty \int_0^{ss_{12}p_2+s} e^{-s_{11}} e^{-s_{12}} ds_{11} ds_{12} = 1 - \frac{e^{-s}}{sp_2+1} \ . \tag{13}$$

Thus, the corresponding pdf of s ($s \geq 0$) becomes,

$$f_S(s) = \frac{dF_S(s)}{ds} = \frac{e^{-s}}{(sp_2+1)^2}(sp_2+p_2+1) \ . \tag{14}$$

One can readily verify that $\frac{d(s^2 f_S(s))}{ds} = \frac{e^{-s}s}{(sp_2+1)^3}k(s)$, where $k(s) = -p_2^2 s^3 - 2p_2 s^2 + (2p_2-1)s + 2p_2 + 2$. Assuming, the derivative of $s^2 f_S(s)$ is strictly positive for the interval $s \in [s_0, s_1]$, i.e., $f_S(s)$ meets the condition of (8), thus plugging $F_S(.)$ and $f_S(.)$ into (7), it follows,

$$I_{\text{opt}}(s) = \begin{cases} p_1 & 0 \leq s \leq s_0 \\ \frac{1-s^2p_2-s}{s^3p_2+s^2+s^2p_2} & s_0 \leq s \leq s_1 \\ 0 & s \geq s_1 \end{cases} \ , \tag{15}$$

where s_0 and s_1 are unknown parameters to be determined. To this end, we note that $I_{\text{opt}}(s_0) = p_1$ and $I_{\text{opt}}(s_1) = 0$. Thus, after some manipulations, the feasible value for s_1 becomes $s_1 = \frac{-1+\sqrt{1+4p_2}}{2p_2}$ and similarly, it turns out s_0 is the solution of $p_2p_1s^3 + (p_1 + p_1p_2 + p_2)s^2 + s - 1 = 0$, which has just one positive root[1]. For instance, assuming $p_1 = p_2 = p$, we have $s_0 = \frac{1}{2p}(-p-1+\sqrt{p^2+6p+1})$.

Again, since the polynomial function $k(s) = -p_2^2s^3 - 2p_2s^2 + (2p_2-1)s + 2p_2 + 2$ associated with $\frac{d(s^2 f_S(s))}{ds}$ which is derived earlier in this section has just one variation in sign of consecutive non-zero coefficients, hence it has just one positive root [4]. On the other hand, one can readily verify that $k(s_1)$ is positive. Noting $\lim_{s\longrightarrow+\infty} k(s) \to -\infty$, thus the positive root of $k(s)$ resides in the interval $(s_1, +\infty)$, hence $k(s)$ is strictly positive for $s \leq s_1$, confirming $f_S(s)$ regardless of the power constraints p_1 and p_2 meets the condition of (8) in the interval $s \in [s_0, s_1]$. Finally, the average achievable rate associated with the first communication link (R_{ave}) is determined by substituting (15) in (2). Moreover, the limiting behavior of R_{ave} is found to be $R_{ave} = 2\ln 2 - 1$ as $p_1 = p_2 \to +\infty$, implying when the transmit powers go to infinity, we are working in interference-limited regime.

[1]The number of positive roots of a polynomial with real coefficients ordered in terms of ascending powers of the variable is either equal to the number of variations in sign of consecutive non-zero coefficients or less than this by a multiple of 2 [4].

IV. Numerical Results

This section aims at investigating the average achievable rate of multi-layer code as compared to that of single-layer code. For multi-layer code, the optimum power allocation policy is derived through the use of (15), noting $p_{opt}(s) = -I'_{opt}(s)$. On the other hand, for single layer code, the rate is set to $\ln(1+p_1s_{th})$, which is achievable as long as the channel strength s exceeds s_{th}. Thus, the average achievable rate of single layer code with parameter s_{th} becomes $R = Pr(s \geq s_{th})\ln(1+p_1s_{th}) = \frac{e^{-s_{th}}}{s_{th}p_2+1}\ln(1+p_1s_{th})$. As a result, taking derivation of R with respect to s_{th} and equating to zero, the optimum value of s_{th} for which R is maximized is computed as $(s_{th}^{opt}p_2 + p_2 + 1)\ln(1 + p_1s_{th}^{opt}) = \frac{p_1(s_{th}^{opt}p_2+1)}{s_{th}^{opt}p_1+1}$. Fig. (1) depicts the resulting average achievable rate of each link through using a single layer code for various equal transmit powers ($p_1 = p_2$) ranging 0 to 30dB and compares it to that of using multi-layer code with infinite layers.

V. Conclusion

In this paper, the broadcast strategy approach is applied to Gaussian interference channel, in which a multi-layer code is employed at each transmitter. Accordingly, in a Rayleigh block fading environment, the optimum power allocation across layers is derived, showing the average achievable rate outperforms that of using single layer code.

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