Research Article

Quality and Reliability Engineering International

(wileyonlinelibrary.com) DOI: 10.1002/qre.1291

Published online 17 January 2012 in Wiley Online Library

Economic-Statistical Design of Acceptance Control Chart

Fatemeh Mohammadian[a] and Amirhossein Amiri[b]*†

Acceptance control charts are effective tools to monitor capable processes in which the fraction of the produced nonconforming items is very low. In these charts, some controlled changes in the process mean are allowed, and the production of a specified number of defectives is tolerated. Designing these acceptance control charts by considering the cost of sampling, detecting, and investigating out-of-control signals as well as the probable correction of assignable cause(s) can result in economic advantages. Moreover, the statistical properties of the control charts can be satisfied. In this article, an economic-statistical model is developed to design acceptance control charts. An illustrative example is used to compare the results of economic versus economic-statistical design of the acceptance control charts. In addition, a comprehensive sensitivity analysis is conducted on the basis of the parameters of the model. Copyright © 2012 John Wiley & Sons, Ltd.

Keywords: acceptance control chart; acceptance process level; acceptance quality level; average time to signal; economic-statistical design; nonlinear constraint model

1. Introduction

Shewhart $\bar{X}$ control charts have been widely used to stabilize and monitor the mean of different processes when the quality characteristic is a variable. Traditionally, the width of the control limits in $\bar{X}$ control charts covers six standard deviations of the quality characteristic. However, in processes with a high level of process capability, for example, $cp_k \geqslant 2$, the specification limits can be six standard deviations or more. Thus, it may be useful to relax the control limits width and allow the process mean to vary over a specific range because it does not significantly affect the nonconforming fraction of the process. Acceptance control charts (ACCs), introduced by Freund[1], is an alternative to Shewhart $\bar{X}$ control charts for monitoring the mean of high capable processes. Tool wear monitoring is an example in which the utilization of an ACC is useful in determining the intervals of tool replacement. This is because ACCs allow the process mean to vary within an interval and signal in case the change in the mean leads to the production of a considerable number of defective items.

The goal of ACC is different from that of the Shewhart control charts. The Shewhart control charts mainly aim to verify whether the process mean is stable over time, whereas ACC is concerned with maintaining the process mean at a specific range so that the nonconforming fraction does not exceed a desired value. Duncan[2] and Wadsworth *et al.*[3] compared ACC with acceptance sampling and explained their differences. Wesolowsky[4] proposed these charts to control two correlated quality characteristics simultaneously. He also obtained a cost function to find an economically optimal sample size and control limits. Wesolowsky[5] developed ACC to monitor multiple independent characteristics. Steiner *et al.*[6] extended this model to the general case of multiple correlated characteristics. To determine the parameters of the control chart, they minimized the sampling costs using a convex nonlinear optimization model. Mohammadian *et al.*[7] also proposed an economic model for ACC.

The statistical design, the economic design, and the economic-statistical design of control charts are the three main approaches used by authors to determine the parameters of different control charts, including the sample size, the sampling interval, and the control limits multiplier. In the statistical design of the control charts, the control limit multiplier is often calculated considering the desired probability of a Type I error (α). The sample size can be determined so that the power of the control chart to detect a particular shift in the parameters of quality characteristics becomes greater than a specified value. The sampling interval is usually chosen on the basis of some practical considerations, such as the production rate, the Type and frequency of assignable causes, and their possible consequences in the process outcome. Authors such as Page,[8] Prabhu *et al.*,[9] Wu *et al.*,[10] Reynolds and Stoumbos,[11] Zhao *et al.*,[12] Lee *et al.*,[13] and Gadre *et al.*[14] designed control charts on the basis of statistical properties. However, the design of control charts by considering only statistical criteria may have economic drawbacks in the costs associated with process monitoring. Thus, by considering economic criteria, another approach, known as the economic design of control charts, was proposed. A primary work on this

[a]*Faculty of Mechanical and Industrial Engineering, Islamic Azad University, Qazvin Branch, Qazvin, Iran*
[b]*Faculty of Engineering, Department of Industrial Engineering, Shahed University, Tehran, Iran*
**Correspondence to: Amirhossein Amiri, Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran.*
[†]*E-mail: amiri@shahed.ac.ir*

subject was published by Girshick *et al.*,[15] who considered cost parameters in designing control charts. Duncan[16] extended the model of Girshick *et al.*[15] and developed an optimization model to find the parameters of $\bar{X}$ control charts. A comprehensive literature review on the economic design of control charts can be found in reviews by Montgomery,[17] Ho *et al.*,[18] Keats *et al.*,[19] and Molnau *et al.*[20] For recent work on the economic design of control charts, see Baud-Lavigne *et al.*,[21] Xue *et al.*,[22] Mao *et al.*,[23] and Sun *et al.*,[24] for example. The economic design of control charts accounts for economic criteria; however, it may result in poor statistical properties.

Regarding the drawbacks of both statistical and economic design approaches, Saniga[25] suggested the economic-statistical design of control charts to consider statistical criteria as well as the costs associated with the monitoring procedure. He introduced a constrained optimization model in which an economic objective function is minimized. The constraints of the model also correspond to statistical criteria, such as the probability of Type I and Type II errors and the average time to signal (ATS). Montgomery *et al.*[26] applied the economic-statistical design approach to an exponentially weighted moving average (EWMA) control chart. Zhang *et al.*[27] provided an economic-statistical design model for a process under the Weibull failure distribution. McWilliams *et al.*[28] extended the model by Saniga[25] to $\bar{X}-R$ and $\bar{X}-S$ control charts. Rahim *et al.*[29] developed the economic-statistical design model for the $\bar{X}$ chart when in-control times follow a Gamma (λ,2) distribution. Some recent works an economic-statistical design of control charts are as follows: Kim *et al.*,[30] Torng *et al.*,[31] Chen *et al.*,[32] Celano[33], Faraz,[34] and Yin *et al.*[35]

Because there has been little work performed on the economic design of ACC, we provide a model for designing ACC that considers both economic and statistical criteria. In this article, we build an optimization model with an economic objective function and statistical constraints. The advantages of economic-statistical design compared with a purely economic design are also shown through a numerical example. We assume that the process variability does not change over time statistically, so no control chart is used for monitoring the standard deviation.

The remainder of the article is organized as follows. In the next section, ACC is briefly reviewed. In Section 3, economic and economic-statistical models for ACCs are presented. In Section 3, a numerical example is presented and subsequently solved by both models, and the results are compared. Then, in Section 5, a sensitivity analysis on the parameters of the economic-statistical model is performed. Additionally, the effect of each statistical constraint on the cost of the model is investigated. A comparison among the statistical design, the economic design, and the economic-statistical design of ACC is accomplished in Section 6. Concluding remarks are presented in the final section.

2. Acceptance control chart

Freund[1] proposed ACCs for monitoring processes with a high capability index. In this section, the principles of ACC are described. As mentioned earlier, the process mean is allowed to vary over a range specified by the limits in ACC; these limits are known as acceptance process levels (APL). APL limits are determined on the basis of the specification limits, the standard deviation, and the acceptance quality level (AQL). The AQL is defined as the maximum of the rejected product proportions, whereas the process mean lies within the APL. With the assumption that the quality characteristic is normally distributed and with the known standard deviation σ, APL limits are calculated as follows:

$$\begin{aligned} \mathrm{APL_U} &= \mathrm{USL} - Z_{\mathrm{AQL}}\sigma \\ \mathrm{APL_L} &= \mathrm{LSL} + Z_{\mathrm{AQL}}\sigma \end{aligned} \tag{1}$$

where Z_{AQL} is a standard normal value associated with the probability AQL. USL and LSL are upper and lower specification limits, respectively.

To protect the proportion of rejected products against possible assignable causes, we defined a protective range known as the rejectable process levels (RPLs). RPL is determined on the basis of the specification limits, the standard deviation, and the RPLs, where RPL is defined as the maximum of rejected product proportions that can be tolerated. Thus, the RPL limits can be obtained with the formulas

$$\begin{aligned} \mathrm{RPL_U} &= \mathrm{USL} - Z_{\mathrm{RQL}}\sigma \\ \mathrm{RPL_L} &= \mathrm{LSL} + Z_{\mathrm{RQL}}\sigma \end{aligned} \tag{2}$$

where Z_{RQL} is a standard normal value associated with the probability RQL.

AQL, APL, RQL, and RPL are graphically shown in Figure 1. The region between APL and RPL is called the indifferent zone because the process can be neither rejected nor accepted. However, in this case, a search for possible root causes and the required corrective actions are undertaken.

On the basis of the desired Type I error probability $\bar{X}$control chart and AQL, the control limits of ACC can be written as follows (see Figure 2):

$$\begin{aligned} \mathrm{UCL} &= \mathrm{USL} - Z_{\mathrm{AQL}}\sigma + Z_{\alpha}\frac{\sigma}{n} \\ \mathrm{LCL} &= \mathrm{LSL} + Z_{\mathrm{AQL}}\sigma - Z_{\alpha}\frac{\sigma}{n} \end{aligned} \tag{3}$$

Because only upper or lower specification limits are active in ACC, the lower specification limit has been considered in this article, and all calculations are within this limit. All calculations and results could be extended to the upper limit as well.

We assume that σ is known in this article; otherwise, it could be estimated on the basis of a historical data set. Moreover, σ could be monitored with an R or an S control chart. However, in this article, we assume that the process variability does not change over time statistically; thus, no control chart is used for monitoring σ.

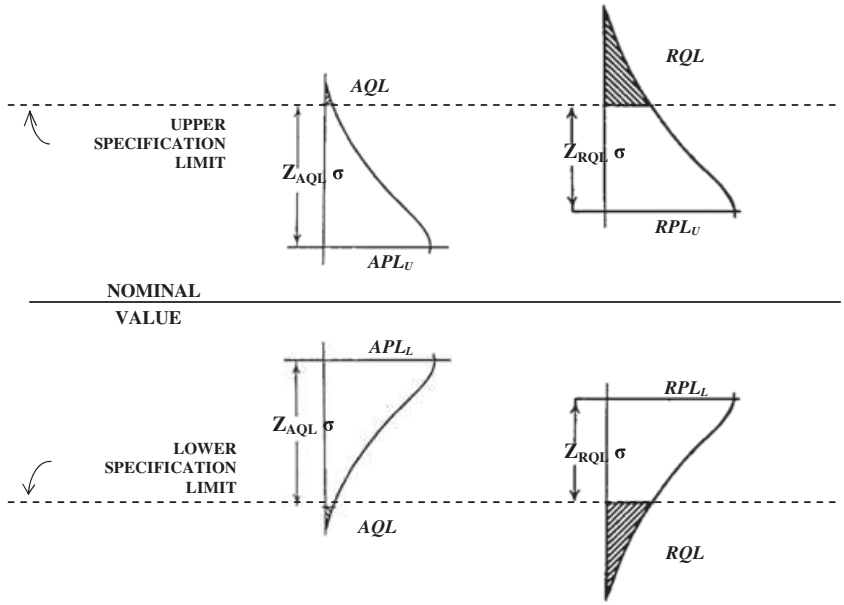

Figure 1. AQL, APL, RQL, and RPL values in an ACC

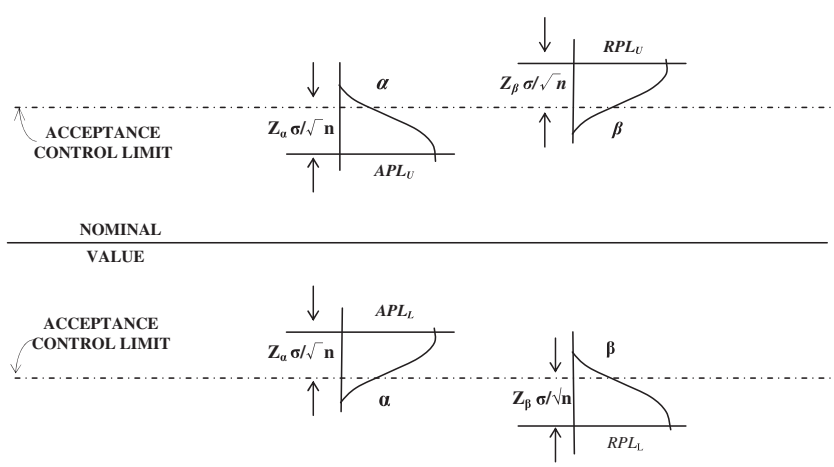

Figure 2. Control limits and probability of Type I and Type II errors of ACCs

α and $1-\beta$ corresponding to the lower control limit (shown in Figure 2) can be computed by Equations (4) and (5), respectively.

$$\begin{aligned}\alpha &= \mathrm{P}(\bar{X} \leqslant \mathrm{LCL} | \mu = \mathrm{APL_L}) \\ &= \mathrm{P}(\bar{X} \leqslant \mathrm{APL_L} - Z_\alpha {}^{\sigma}/_{\sqrt{n}} | \mu = \mathrm{APL_L}) \\ &= \mathrm{P}(Z \leqslant -Z_\alpha) = \int_{-\infty}^{-Z_\alpha} f_z(z)dz,\end{aligned} \tag{4}$$

$$\begin{aligned}1-\beta &= \mathrm{P}(\bar{X} \leqslant \mathrm{LCL} | \mu = \mathrm{RPL_L}) \\ &= \mathrm{P}(\bar{X} \leqslant \mathrm{APL_L} - Z_\alpha {}^{\sigma}/_{\sqrt{n}} | \mu = \mathrm{RPL_L}) \\ &= \mathrm{P}(Z \leqslant Z_{\mathrm{AQL}}\sqrt{n} - Z_\alpha - Z_{\mathrm{RQL}}\sqrt{n}) = \int_{-\infty}^{Z_{\mathrm{AQL}}\sqrt{n} - Z_\alpha - Z_{\mathrm{RQL}}\sqrt{n}} f_z(z)dz\end{aligned} \tag{5}$$

where $f_z(z)$ is the standard normal probability destiny function. Similarly, $\alpha$and $1-\beta$ values corresponding to UCL can be easily obtained. It should be mentioned that when the process level decreases to $\mathrm{APL_L}$ or $\mathrm{RPL_L}$, the α and $1-\beta$ corresponding to UCL is close to zero, and therefore, it can be disregarded.

3. Economic-statistical design of ACC

In this section, a constrained nonlinear model is proposed to design an ACC with economic-statistical considerations. To develop an economic-statistical design for ACC, we used the traditional objective function introduced by Duncan.[16] Hence, using Duncan's model and applying the probability of Type I and Type II errors, the objective function can be defined as follows:

$$E(L) = \frac{a_1 + a_2 n}{h} + \frac{a_4\left[h/(1-\beta) - \left(1-(1+\lambda h)e^{-\lambda h}\right)/\left(\lambda - \lambda e^{-\lambda h}\right) + gn + D\right] + a_3 + a'_3 \alpha e^{-\lambda h}/\left(1-e^{-\lambda h}\right)}{1/\lambda + h/(1-\beta) - (1-(1+\lambda h)e^{-\lambda h})/(\lambda - \lambda e^{-\lambda h}) + gn + D} \tag{6}$$

Notations of the model are as follows:

n : sample size
h : sampling interval
α : probability of Type I error
β : probability of Type II error
a_1 : fixed cost of sampling.
a_2 : variable cost of sampling.
λ : Poisson distribution parameter (in this model, it is assumed that assignable causes occur based on a Poisson distribution with the rate of λ causes per hour).
D : the time required to detect an assignable cause after a warning signal.
g : the time required to take a sample and interpret the results.
a_3 : the expected cost of detecting an assignable cause.
a'_3 : the expected cost imposed by a false alarm.
k : the hourly penalty cost of operating in the out-of-control state.
$E(L)$: the expected loss per hour that was incurred by the process and is a function of the design parameters.

In the economic-statistical design of ACC, some statistical constraints are added to the economic model to achieve proper statistical properties. We add three statistical constraints similar to those that Saniga[24] suggested. One of them is the minimum value of the power of the control chart; the other two are the maximum values of Type I error probability and ATS. These extreme values may be proposed by quality engineers.

Therefore, the economic-statistical model used to design the ACC is as follows:

$$\begin{array}{ll} \text{Minimize} & E(L) \\ \text{Subject to}: & \alpha \leqslant \alpha_i, \quad p \geqslant p_i, \quad \text{ATS} \leqslant \text{ATS}_i \end{array} \tag{7}$$

where p is the power value equivalent to $1-\beta$ and ATS is the average time to signal, which can be computed as $h/1-\beta$. To calculate the probability of Type I error, power, and *ATS* values for the *ACC*, Equations (4) and (5), should be used.

4. An illustrative example

In this section, an illustrative example is presented to compare the economic-statistical design versus the economic design of ACC explained in Section 2. It should be noted that the model coefficients and parameters in this example have been taken from Montgomery,[36] Freund,[1] and Saniga.[25]

Assume a process for filling bottles of a drink. The desired volume of a filled bottle is 10 ± 0.5^{cc}, so the specification limits are LSL = 9.5^{cc} and USL = 10.5^{cc}. The process is accepted when 0.1% of the filled bottles have the drink volume outside of the 10 ± 0.5 limits, and the process is rejected when 2.5% of the filled bottles have the drink volume outside of the 10 ± 0.5 limits. Thus, AQL and RQL are equal to 0.001 and 0.025, respectively. In this example, it is assumed that the quality characteristic follows a normal distribution, and the inherent variability of the process is estimated as $\hat{\sigma} = 0.01$. Because the standard deviation of this process is small compared with the specification limits width, the manufacturer has decided to use ACC to control this process. However, to reduce the costs and improve the power of the control chart, the manufacturer wishes to use an economic-statistically optimized acceptance chart. To estimate the related costs, we assumed the parameters noted in the next paragraph.

The fixed (a_1) and variable cost (a_1) of sampling is estimated as \$0.5 and \$0.1 per bottle, respectively. It takes approximately (g) 0.05 h to measure and record the volume of liquid in each bottle. Process shifts occur randomly with the exponential distribution and with parameter $\lambda = 0.01$, which is a reasonable model of the in-control run length. The average time required to investigate an out-of-control signal (D) is 2 h, and the cost of detecting and taking an action to eliminate an assignable cause (a_3) is \$25, whereas the cost of investigating a false alarm is (a'_3) \$50. The penalty cost of operating in the out-of-control state for 1 h (a_4) is \$100. The right-hand-side values of constraints on the probability of a Type I error, power, and ATS are defined as $a_i = 0.004$, $p_i = 0.98$ and $\text{ATS}_i = 4$, respectively.

The model is optimized in two ways: the pure economic model without statistical constraints and the economic-statistical model. To optimize these nonlinear models with the mentioned coefficients, we used a medium-scale algorithm called the sequential quadratic programming method in Matlab 7. This method is based on the research presented by Powell[37] and Han.[38]

The output results from optimizing the unconstrained (economic) model indicate that when taking nine samples every 1.61 h and $k = 2.46$, then the expected loss is minimized at \$4.79 per h. Optimizing the constrained (economic-statistical) model results with 17 as the sample size, 2.24 h as the sampling interval, and 2.65 as the control limit coefficient. In this case, the expected loss is 5.20, which is the optimal value of the objective function. All power, ATS, and probability of Type I error constraints are also satisfied. Obviously, the loss associated with the economic-statistical design of ACC is slightly more than the loss of the economic design. In fact, this additional cost is the amount paid for improving the statistical performance of the ACC.

5. Sensitivity analysis

In this section, the effects of the changes in the parameters of the economic-statistical model on the optimal solutions obtained from the previous example are studied. The results of the sensitivity analysis are shown in Table I. The last four columns in Table I are associated with the optimal settings obtained from the input parameters in previous columns. Null places in the table are duplications of the first row.

As considered in Table I, increasing AQL and RQL has no effect on the control limits but leads to the reduction of the sample size and the sampling interval. Consequently, the expected cost decreases. Increasing λ increases the sampling interval but has no effect on sample size and control limits. As a result, the expected cost increases. Increasing the parameters g and D one at a time increases h. However, it has no effect on sample size and control limits. Note that increasing g, D, a_1, and a_2 leads to an increase in the expected cost. If a_4 decreases, then h is the only parameter that increases, and the expected cost decreases.

Finally, increasing a_4 and a'_3 simultaneously increases all three parameters of ACC, especially sample size, and causes an increase in the expected cost.

It would be useful for practitioners to know the effects of the right-hand-side values of constraints on the objective function. Therefore, the effects of each right-hand-side value on the expected cost are considered. Figures 3–5 depict the effect of a particular constraint on the result when the others are assumed to be fixed at their initial values.

It is clear from Figures 3–5 that if one of the constraints is relaxed, such as increasing the desired probability of a Type I error or ATS or decreasing the power, the expected cost decreases. It is logical because the feasible region increases when a constraint is relaxed. It can also be understood from Figures 3–5 that the objective function does not improve for right-hand-side values greater than the threshold values of each constraint. Hence, selecting right-hand-side values greater than these thresholds does not have any benefit, and this decreases the sensitivity of the control chart. These thresholds are 0.011, 0.8, and 2.28 for α, p, and ATS, respectively.

6. A comparison among statistical, economic, and economic-statistical designs of ACC

In this section, we first design the ACC statistically and then compare it with economic and economic-statistical designs, which are presented in Section 5 and by Mohammadian *et al.*,[7] respectively. For an accurate study of these three designs, we considered the

Table I. Results of sensitivity analysis for the economic-statistical model

Experiment no.	RQL	AQL	λ	a_4	g	D	a'_3	a_3	a_1	a_2	n	h	k	Expected cost
1	0.025	0.001	0.01	100	0.05	2	50	25	0.5	0.1	17.33	2.24	2.65	5.20
2	0.05	0.002									14.56	2.11	2.65	4.95
3	0.075	0.003									12.94	2.02	2.65	4.79
4	0.1	0.004									11.79	1.96	2.65	4.68
5			0.02								17.33	1.63	2.65	8.87
6			0.03								17.33	1.37	2.65	12.15
7				1							17.33	3.92	2.65	0.90
8				10							17.33	3.92	2.65	1.32
9					0.5						17.33	2.40	2.65	11.88
10						20					17.33	2.62	2.65	19.31
11							5	2.5			17.33	2.16	2.65	4.91
12							500	250			20.51	2.56	3.07	7.77
13							5000	2500			26.06	3.17	3.71	29.67
14									5		17.33	3.81	2.65	6.69
15										1	17.33	3.92	2.65	9.52
16										10	17.33	3.92	2.65	49.32
17			0.02	50							17.33	2.33	2.65	5.28
18				1						1	17.23	3.9	2.65	4.88
19	0.05	0.002		12.87							14.56	3.92	2.65	1.37
20	0.05	0.002		128.7							14.56	1.85	2.56	5.99
21	0.05	0.002		12.87			500	250			18.78	3.92	2.65	3.65
22	0.05	0.002		12.87					5		14.56	3.92	2.65	2.52
23	0.05	0.002		12.87						1	14.56	3.92	2.65	4.71
24	0.075	0.003		2.25							12.94	3.92	2.65	0.84
25	0.075	0.003		225							12.94	1.345	2.65	9.01
26	0.075	0.003		2.25					5		12.94	3.92	2.65	1.99
27	0.075	0.003		2.25						1	12.94	3.92	2.65	3.82

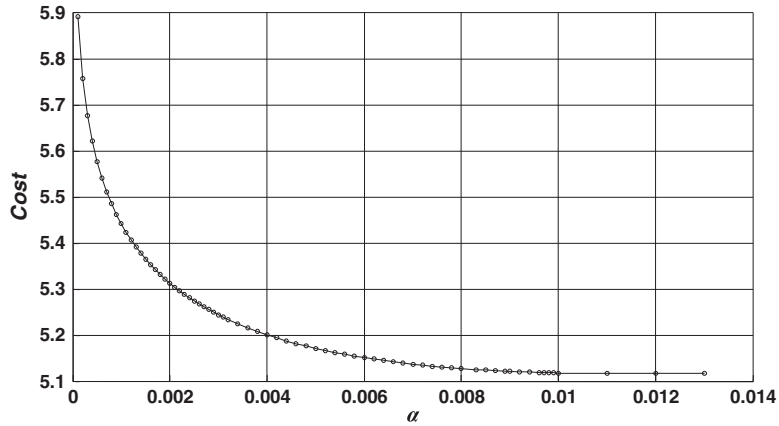

Figure 3. The effect of changes in right-hand-side value of α constraint on the expected cost per hour in economic-statistical design of ACC

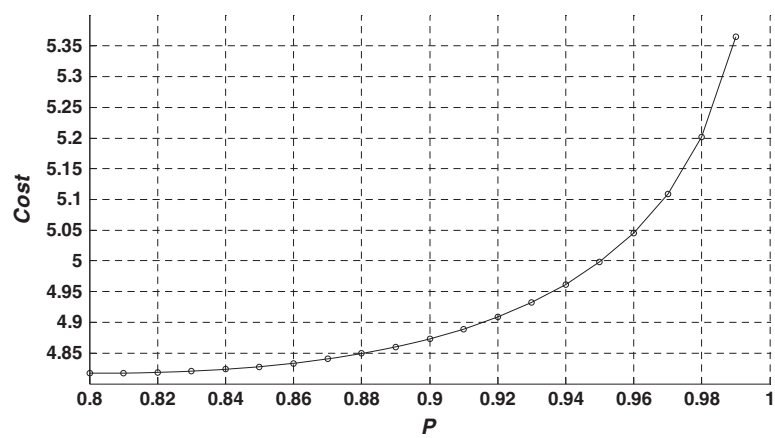

Figure 4. The effect of changes in right-hand-side value of power constraint on the expected cost per hour in economic-statistical design of ACC

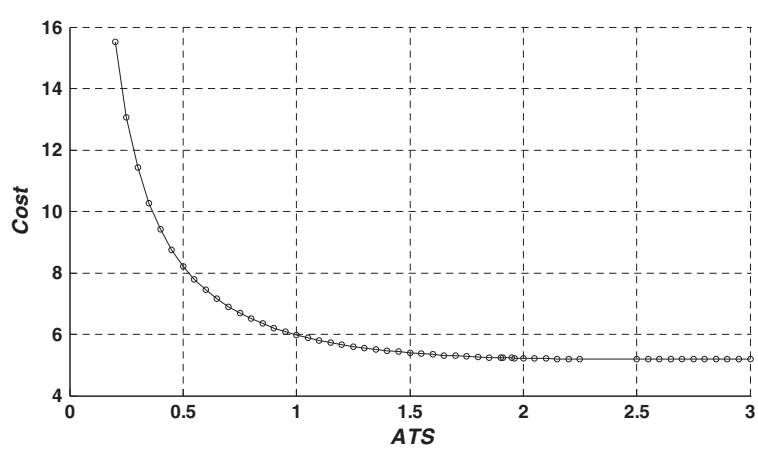

Figure 5. The effect of changes in right-hand-side value of ATS constraint on the expected cost per hour in economic-statistical design of ACC

parameters of the illustrative example in Section 4. Table II presents the expected cost values, the probability of Type I and Type II errors of statistical, economic, and economic-statistical design of ACC for the experiments mentioned in Table I. Similar to the economic-statistical design of ACC, we used $\alpha = 0.004$, $\beta = 0.02$, and ATS = 4 for statistical design of ACC. These values are bounds used in the constraints of economic-statistical design of ACC. Hence, the sample size (n) can be determined by Equation (8) proposed by Freund[1]) for each experiment considered in Table I,

$$n = \left[\frac{(z_\beta + z_\alpha)\sigma'_w}{\mathrm{RPL} - \mathrm{APL}}\right]^2 \tag{8}$$

Table II. Performance comparison among statistical, economic, and economic-statistical design of ACC

	Statistical design												Economic design			Economic-statistical design		
	Actual						Bound											
Experiment no.	α	β	n	h	k	Expected cost	α	β	n	h	k	Expected cost	α	β	Expected cost	α	β	Expected cost
1	0.004	0.016	18	3.94	2.65	5.55	0.004	0.02	17.33	3.92	2.65	5.51	0.0069	0.17	4.79	0.004	0.02	5.2
2	0.004	0.017	15	3.94	2.65	5.34	0.004	0.02	14.55	3.92	2.65	5.51	0.0059	0.16	4.63	0.004	0.02	4.95
3	0.004	0.019	13	3.92	2.65	5.2	0.004	0.02	12.93	3.92	2.65	5.19	0.0052	0.15	4.52	0.004	0.02	4.79
4	0.004	0.018	12	3.93	2.65	5.13	0.004	0.02	11.79	3.92	2.65	5.11	0.0048	0.14	4.45	0.004	0.02	4.68
5	0.004	0.016	18	3.92	2.65	10.08	0.004	0.02	17.33	3.92	2.65	10.01	0.0079	0.2	8.09	0.004	0.02	8.87
6	0.004	0.016	18	3.92	2.65	14.24	0.004	0.02	17.33	3.92	2.65	14.15	0.0085	0.23	11.03	0.004	0.02	12.15
7	0.004	0.016	18	3.92	2.65	0.88	0.004	0.02	17.33	3.92	2.65	0.87	0.0059	0.1	0.44	0.004	0.02	0.9
8	0.004	0.016	18	3.92	2.65	1.31	0.004	0.02	17.33	3.92	2.65	1.28	0.0056	0.12	1.14	0.004	0.02	1.32
9	0.004	0.016	18	3.92	2.65	12.36	0.004	0.02	17.33	3.92	2.65	12.08	0.0144	0.56	6.85	0.004	0.02	11.88
10	0.004	0.016	18	3.92	2.65	19.47	0.004	0.02	17.33	3.92	2.65	19.43	0.0078	0.16	18.99	0.004	0.02	19.31
11	0.004	0.016	18	3.92	2.65	5.33	0.004	0.02	17.33	3.92	2.65	5.28	0.0704	0.15	4.07	0.004	0.02	4.91
12	0.004	0.016	18	3.92	2.65	7.81	0.004	0.02	17.33	3.92	2.65	7.76	0.0007	0.18	7.41	0.0011	0.02	7.77
13	0.004	0.016	18	3.92	2.65	30.32	0.004	0.02	17.33	3.92	2.65	30.28	0.00007	0.16	29.37	0.0001	0.02	29.67
14	0.004	0.016	18	3.92	2.65	6.7	0.004	0.02	17.33	3.92	2.65	6.65	0.013	0.06	6.5	0.004	0.02	6.69
15	0.004	0.016	18	3.92	2.65	9.69	0.004	0.02	17.33	3.92	2.65	9.49	0.0457	0.2	7.08	0.004	0.02	9.52
16	0.004	0.016	18	3.92	2.65	51.01	0.004	0.02	17.33	3.92	2.65	49.28	0.3655	0.23	11.03	0.004	0.02	49.32
17	0.004	0.016	18	3.92	2.65	5.57	0.004	0.02	17.33	3.92	2.65	5.52	0.0072	0.17	4.89	0.004	0.02	5.28
18	0.004	0.016	18	3.92	2.65	5.02	0.004	0.02	17.33	3.92	2.65	4.85	0.071	0.19	0.6	0.004	0.02	4.88
19	0.004	0.017	15	3.93	2.65	1.35	0.004	0.02	14.55	3.92	2.65	1.34	0.0047	0.11	1.25	0.004	0.02	1.37
20	0.004	0.017	15	3.93	2.65	6.65	0.004	0.02	14.55	3.92	2.65	6.61	0.0061	0.17	5.58	0.004	0.02	5.99
21	0.004	0.017	15	3.93	2.65	3.6	0.004	0.02	14.55	3.92	2.65	3.59	0.0005	0.11	3.47	0.0005	0.02	3.65
22	0.004	0.017	15	3.93	2.65	2.49	0.004	0.02	14.55	3.92	2.65	2.48	0.0073	0.03	1.84	0.004	0.02	2.52
23	0.004	0.017	15	3.93	2.65	4.78	0.004	0.02	14.55	3.92	2.65	4.68	0.0392	0.17	2.03	0.004	0.02	4.71
24	0.004	0.019	13	3.92	2.65	0.81	0.004	0.02	12.93	3.92	2.65	0.81	0.004	0.09	0.56	0.004	0.02	0.84
25	0.004	0.019	13	3.92	2.65	10.82	0.004	0.02	12.93	3.92	2.65	10.81	0.0059	0.18	8.4	0.004	0.02	9.01
26	0.004	0.019	13	3.92	2.65	1.96	0.004	0.02	12.93	3.92	2.65	1.96	0.0061	0.023	0.78	0.004	0.02	1.99
27	0.004	0.019	13	3.4	2.65	3.81	0.004	0.03	12.93	3.89	2.65	3.64	0.0408	0.17	0.83	0.004	0.02	3.82

where *APL* and *RPL* are computed by Equations (1) and (2), respectively. n is obtained equal to 17.33 for the first experiment. K and h are computed by using z_α and $ATS \times (1-\beta)$, respectively, and equal to 2.65 and 3.92 for the first experiment. The results of the statistical design of ACC for the first experiment as well as other experiments of Table I are summarized in Table II under the title of statistical design and subtitle of bound. Because the sample size is integer in the real applications, we round up the sample size. This leads to a power more than the bound considered for power in economic-statistical design of ACC. As a result, the values of β and h are recalculated for all experiments and reported under the title of statistical design and subtitle of actual. It is obvious that the expected cost are different for actual and bound statistical design of ACC.

As shown in Table II, for the assumed α and β the expected costs for the pure statistical design of ACC are the greatest among all three designs. In the economic design, only the economical aspects are considered, and minimum expected costs are obtained by different experiments. However, by neglecting the probability of Type I and Type II errors, these values increase respect to the statistical design. To overcome this problem, we also considered the statistical constraints along with the economic loss function, and the model in Equation (7) is applied to the economic-statistical design of ACC. The results show that the expected costs of the economic-statistical design are less than the statistical design and greater than the economic design. Note that although the expected costs of the economic-statistical design of ACC are slightly greater than the economic design of ACC, the economic-statistical design is preferred to the economic design because α and β are controlled by the appropriate constraints.

Finally, a comparison has been performed among the heuristic design and the other designs of ACC. Consider Experiment 1 of Table II. If we use the statistical design, the expected cost will be \$5.51, $\alpha = 0.004$ and $\beta = 0.02$ for $n = 17.33$. These values for the economic design are $\alpha = 0.0069$, $\beta = 0.17$ and $EL = \$4.79$ and for the economic-statistical design are $\alpha = 0.004$, $\beta = 0.02$ and $EL = \$5.20$.

However, if we consider $n = 4$, $h = 1$ and $k = 3$ heuristically for ACC, the expected cost will be equal to \$6.88, and α and β are 0.00135 and 0.77, respectively. Similarly, if we take five samples each hour and $k = 3$, the expected cost is \$5.95, $\alpha = 0.0013$ and $\beta = 0.68$. Thus, the heuristic design of ACC increases α, β, and the expected costs. Because the expected costs of the economic and the economic-statistical designs are less than the statistical design, the least favorable design is the heuristic design.

7. Conclusions

In this article, an economic-statistical model for ACCs was presented. This model aims to calculate the design parameters of the control chart, sample size, sampling interval, and control limits to minimize the expected cost that is subject to statistical constraints. These constraints sustain the probability of a Type I error, power, and ATS at a desired level that can be determined by the user. Then, a numerical example was presented for explaining the model and evaluating the economic model compared with the economic-statistical model. To obtain the optimal solution for the numerical example, we applied a nonlinear programming algorithm. To investigate the effects of model parameters on the expected cost of the economic-statistical model, we performed a sensitivity analysis on the parameters as well as the right-hand-side values of the constraints. This study may help determine how important these parameters are in minimizing the average costs of the process during long periods of time. In addition, a comparison of the economic-statistical model and the economic model showed that a slight increase in costs would be acceptable to obtain an ACC with the desired statistical performance. Finally, the comparison among the three types of ACC designs as well as the heuristic design demonstrated the superiority of the economic-statistical design over the pure statistical, economic, and heuristic designs.

Acknowledgements

The authors gratefully acknowledge Mr Peynabar, PhD candidate of Michigan University, for his valuable helps. In addition, the authors appreciate anonymous referee and the chief editor, Prof. Douglas C. Montgomery, for their valuable and precious comments, which led to improvement in this article.

References

1. Freund RA. Acceptance Control Charts. *Journal of Industrial Quality Control* 1957; **14**:13–23.
2. Duncan AJ. *Quality control and industrial statistics*, (5th edn.), Richard D (ed). Irwin: Homewood IL, 1986.
3. Wadsworth HM, Stephan KS, Godfrey AB. *Modern methods for quality control and improvement*. Wiley: New York, 1986.
4. Wesolowsky GO. Simultaneous acceptance control charts for two correlated processes. *Journal of Technometrics* 1990; **32**:43–48.
5. Wesolowsky GO. Simultaneous acceptance control charts for independent processes. *Journal of Applied Statistics* 1992; **41**:147–158.
6. Steiner SH, Wesolowsky GO. Simultaneous acceptance control charts for product with multiple correlate characteristics. *International Journal of Production Research* 1994; **32**:531–543.
7. Mohammadian F, Paynabar K. Economic design of acceptance control charts. *Proceedings of the 2008 IEEE Industrial Engineering and Engineering Management*, Singapore, 8–11 December 2008.
8. Page ES. Control charts for the mean of a normal population. *Journal of the Royal Statistical Society (VB16)* 1954; **16**:131–135.
9. Prabhu SS, Runger GC, Montgomery DC. Selection of the Subgroup Size and Sampling Interval for a CUSUM Control Chart. *IIE Transactions* 1997; **29**:451–457.
10. Wu Z, Xie M, Tian Y. Optimization Design of the X & S Charts for Monitoring Process Capability. *Journal of Manufacturing Systems* 2002; **21**:83–92.
11. Reynolds MR, Stoumbos ZG. Should Observations Be Grouped for Effective Process Monitoring? *Journal of Quality Technology* 2004; **36**:343–366.
12. Zhao Y, Tsung F, Wang Z. Dual CUSUM control scheme for detecting a range of mean shifts. *IIE Transactions* 2005; **37**:1047–1058.

13. Lee MH, Khoo MBC. Optimal statistical design of a multivariate CUSUM chart. *International Journal of Reliability, Quality and Safety Engineering* 2006; **13**:479–497.
14. Gadre MP, Rattihalli RN. A side sensitive group runs control chart for detecting shifts in the process mean. *Statistical Methods and Applications* 2007; **16**:27–37.
15. Girshick MA, Rubin H. A Bayes approach to a quality control model. *Annals of Mathematical Statistics* 1952; **23**:114–125.
16. Duncan AJ. The economic design of $\bar{X}$ charts used to maintain current control of a process. *Journal of the American Statistical Association* 1956; **51**:228–242.
17. Montgomery DC. The economic design of control charts: A review and literature survey. *Journal of Quality Technology* 1980; **12**:75–87.
18. Ho C, Case KE. Economic design of control charts: a literature review. *Journal of Quality Technology* 1994; **26**:39–53.
19. Keats JB, Castillo ED, Collani EV, Saniga EM. Economic Modeling for Statistical Process Control. *Journal of Quality Technology* 1997; **29**:144–148.
20. Molnau WE, Montgomery DC, George C. Runger. *Quality and Reliability Engineering International* 2001; **17**:39–49.
21. Baud-Lavigne B, Bassetto S, Penz B. A broader view of the economic design of the X-bar chart in the semiconductor industry. *International Journal of Production Research* 2010; **48**:5843–5857.
22. Xue L, Liu Y, Xu J. Economic Design of EWMA Standard Deviation Charts Using Variable Sampling Policy. *Proceeding of International Conference of Internet Technology and Applications*, 09 September 2010.
23. Mao H, Cheng SW. The Economic Design of the MSE Control Chart. *Communications in Statistics - Simulation and Computation* 2010; **39**:17–27.
24. Sun Q, Wang X. Economic design of integrating SPC and APC with quality constraints. *Proceedings of Control and Decision Conference (CCDC)*, China, May 2010.
25. Saniga EM. Economic statistical control chart designs with an application to $\bar{X}$ and R charts. *Technometrics* 1989; **31**:313–320.
26. Montgomery DC, Torng JC, Cochran JK, Lawence FP. Statistical constrained economic design of EWMA control charts. *Journal of Quality Technology* 1995; **27**:250–256.
27. Zhang G, Berardi V. Economic statistical design of $\bar{X}$ control charts for system with Weibull in-control times. *Computers and Industrial Engineering* 1997; **32**:575–586.
28. McWilliams TP, Saniga EM, Davis DJ. Economic-statistical design of $\bar{X}$ and R or $\bar{X}$ and S charts. *Journal of Quality Technology* 2001; **33**:234–235.
29. Rahim MA, Al-Oraini HA. Economic statistical design of $\bar{X}$ Control charts for system with Gamma (λ,2) in-control times. *Computers and Industrial Engineering* 2002; **43**:645–654.
30. Kim YB, Hong JS, Lie CH. Economic statistical design of 2-of-2 and 2-of-3 runs rule scheme. *Quality and Reliability Engineering International* 2009; **25**:215–228.
31. Torng CC, Lee PH, Liao HS, Liao NY. An economic design of double sampling X charts for correlated data using genetic algorithms. *International Journal of Expert Systems with Applications* 2009; **36**:12621–12626.
32. Chen HU, Pao YI. The joint economic-statistical design of X and R charts for nonnormal data. *Quality and Reliability Engineering International* 2011; **27**:269–280.
33. Celano G. On the economic-statistical design of control charts constrained by the inspection workstation configuration. *International Journal of Quality Engineering and Technology* 2010; **1**:231–252.
34. Faraz A, Saniga E. Economic statistical design of a T^2 control chart with double warning lines. *Quality and Reliability Engineering International* 2011; **27**:125–139.
35. Yin Z, Makis V. Economic and economic-statistical design of a multivariate Bayesian control chart for condition-based maintenance. *Journal of Management Mathematics* 2010; **22**(1):47–63.
36. Montgomery DC. *Introduction to statistical quality control*, (5th edn.). Wiley: New York, 2005.
37. Powell MJD. A Fast Algorithm for Nonlinearly Constrained Optimization Calculations. In *Numerical Analysis*, vol. **630**, Watson GA (ed.). 1978; 144–157.
38. Han SP. A Globally Convergent Method for Nonlinear Programming. *Journal of Optimization Theory and Applications* 1977; **22**:297–309.

Authors' biographies

Fatemeh Mohammadian is a master student in Industrial Engineering in Azad University in Qazvin. She received her Bachelor of Science in Industrial Engineering degree from Azad University in 2007. Her research interests include statistical quality control and healthcare monitoring.

Amirhossein Amiri is an assistant professor at Shahed University. He holds a BS, an MS, and a PhD in Industrial Engineering from Khajeh Nasir University of Technology, Iran University of Science and Technology, and Tarbiat Modares University, respectively. He is a member of the Iranian Statistical Association. His research interests are statistical quality control, profile monitoring, and Six Sigma.