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The Integer-valued Characters of Some Mathieu Groups

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Abstract

Using the concept of markaracter tables proposed by S. Fujita, we are able to discuss characters and marks concerning a group of a finite order on a common basis. He also introduced tables of integer-valued characters that are obtained for finite groups. In this paper, all the integer-valued characters for some Mathieu groups(M_{22}, M_{23} and M_{24}) are successfully derived for the first time.

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1 Introduction

Shinsaku Fujita suggested a new concept called the markaracter table, which enables us to discuss marks and characters for a finite group on a common basis, and then introduced tables of integer-valued characters, which are acquired for such groups [4].
Fujita's theory was further developed and utilized for a variety of enumeration problems of chemical species eventually [3, 4]. A dominant class is defined as a disjoint union of conjugacy classes corresponding the same cyclic subgroups, which is selected as a representative of conjugate cyclic subgroups. Moreover, the cyclic (dominant) subgroup G_i selected from a non-redundant set of cyclic subgroups of G denoted by SCS_G is

used to compute the integer-valued character of G [4]. Probing all the integer-valued characters for some Mathieu groups (M_{22}, M_{23}, M_{24}), our aim in this study.

2 Main Results

In this section, respect to Fujita's symbols, we describe some notations that will be kept thoroughly in this paper.

Definition 2.1 *Let G be an arbitrary finite group and $h_1, h_2, \in G$ we say $h_1$and h_2 are Q-conjugate if $t \in G$ exists such that $t^{-1} < h_1 > t =< h_2 >$, which is an equivalence relation on group G and generates equivalence classes that are called dominant classes. Therefore G is partitioned into dominant classes as follows: $G = K_1 + K_2 + ... + K_s$ such that K_i corresponds to the cyclic (dominant) subgroup G_i chosen from a non-redundant set of cyclic subgroups of G denoted SCS_G.*

Now assume that an action P of G on a set X and a subgroup H of G are given. So by considering the set X consist of all H's right cosets, and the partition of G induced by these cosets $G = \bigoplus_i^m Hg_i$, we have an action of G on X and a permutation representation signified by $G(/H)$ correspondingly [4].

Definition 2.2 *If G_i and G_j be any subgroups of G, a subduced representation is known as a subgroup of the coset representation $G(/G_i)$ that contains only the elements associated with the elements of G_j denoted by $G(/G_i) \downarrow G_j$.*

Definition 2.3 *The table of marks of a arbitrary finite group G is a square matrix: $M(G) = (m_{ik})_{1 \leq i,k \leq r}$, where m_{ik} is the number of right cosets of G_k in G which are remained fixed under right multiplication by the elements of G_i see [4].*

Definition 2.4 *Let M is a normal subgroup and N is another subgroup of G such that $M \bigcap N = e$ with $G = MN =< M, N >$, then G is called the semi direct product of N by M denoted by $N : M$. As well, if K and H be groups as H acts on the set Γ, then the wreath product of K by H, denoted by $KwrH$ is identified with $K^\Gamma : H$ such that $K^\Gamma = \{f | f : \Gamma \rightarrow K\}$ see [4].*

Theorem 2.1 *(i) The direct product of the matured groups again is a matured group, but the direct product of at least one unmatured group is an unmatured group.*

(ii) The semi direct product of the matured groups again is a matured group, but the semi direct product of at least one unmatured group is an unmatured group.

(iii) The wreath product of the matured groups again is a matured group, but the wreath product of at least one unmatured group is an unmatured group.

Definition 2.5 *Let $C_{u\times u}$ be a matrix of the character table for an arbitrary finite group. Then, C is transformed into a more concise form called the Q-conjugacy character table that we denote its $s \times s$ matrix by C_G^Q containing integer-valued characters where $(s \leq u)$.*

Suppose that H be a cyclic subgroup of order n of a finite group G. Then the maturity discriminant of H, denoted by $m(H)$, is an integer number delineated by $|N_G(H) : C_G(H)|$. In addition, the dominant class of $K \bigcap H$ in the normalizer $|N_G(H)|$ is the union of $t = \frac{\varphi(n)}{m(H)}$ conjugacy classes of G where φ is Euler function, i.e. the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. If $t = \frac{\varphi(n)}{m(H)} = 1$, the group G should be matured but if $m(H) \leq \varphi(n)$ or $t > 2$, the group G is an unmatured concerning subgroup H.

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