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Modelling of current distribution and nonlinearity in superconducting coplanar waveguide (CPW) transmission line

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**ABSTRACT**

There are nonlinear behaviours such as harmonic generation and intermodulation distortion (IMD) in superconductive circuits especially at low temperatures. In this paper, current distribution in superconducting coplanar waveguide transmission line (CPWTL) was modelled. After that, nonlinear circuit model to predict nonlinear behaviours in superconducting CPWTLs was proposed. Current distribution in superconducting CPWTL geometry with finite ground planes was formulated using of numerical method based on 3D-FEM. These formulations can be used to obtain accurate closed-form expressions for nonlinear components in distributed circuit model based on considering both quadratic and modulus nonlinear dependence on current. This model was analysed nonlinearly using Harmonic Balance method as a nonlinear solver. The proposed model can be used to predict nonlinear behaviours such as IMD and harmonic generation in superconducting CPWTL in different input powers and different temperatures. There are good agreement between nonlinear results from our proposed model and measured ones.

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Nonlinear behaviours; superconducting coplanar waveguides; nonlinear modelling; current distribution

1. Introduction

Superconducting materials indicate nonlinear behaviours due to dependence of the superfluid density on the current distribution \([1,2]\). These nonlinear behaviours are also because of Nonlinear Meissner Effect (NLME) that observed in different superconducting materials \([3–5]\). Using Superconducting materials in the telecommunication devices, for instance transmission lines and filters \([6]\) has advantages such as low losses, reduction in dimensions and high quality factor. But nonlinear behaviours cause limitation for usage of the superconductive circuits. One of the main limitations is intermodulation distortion (IMD), especially third order that is closest to original signal and could be a bottleneck in the multi-channel wireless communications. Other limitations are the harmonic generation, spectral regrowth and desensitization which prediction of them is the important point of interest in the communications systems.

There are linear models for superconducting lines and coplanar lines \([7,8]\). There are models to predict nonlinear behaviours in superconducting microstrip transmission...
In similar way in [15] to compute current distribution in superconducting CPWTLs, 3D-FEM was used as a numerical method to solve London equations. In fact, we assumed complex permittivity, see Equation (1), for superconductive regions consist of central superconductor ($S$) and ground planes ($W_g$), see Figure 1, in superconducting CPWTL and then Maxwell equations were solved in all regions. In Equation (1), $\lambda$ and $\sigma_1$ are penetration depth and conductivity respectively.

$$\varepsilon_{sc} = \varepsilon_0 - \frac{1}{\omega^2 \mu_0 \lambda^2} - j \frac{\sigma_1}{\omega}$$ (1)

For instance, the computed current density ($J$) for two considered superconducting CPWTLs with different central superconductor ($S$) which made of thin film YBCO deposited on LaAlO$_3$(LAO) shown in Figure 2(a) and (b). As shown in these figures the surface current density ($J$) normalized to its minimum value in central superconductor that was named $J_0$. According to Figure 2, $j_1$ and $j_2$ were defined as current density peaks in central

![Figure 1. General configuration of the superconducting CPWTL.](image-url)
Figure 2. Normalized current density in superconducting CPWTLs made of 320 nm YBCO, $T_c = 90$ K, at $T = 77$ K, deposited on 0.5 mm LaALO$_3$ (LAO) at $f = 1$ GHz.

superconductor and ground planes edges, respectively. Normalization of minimum current density $J_0$ was named $j_0 = 1$.

From Figure 2(a) and (b) it can be observed that $(j_2/j_1)$ ratio increases in superconducting CPWTL with larger central superconductor width ($S$). The increasing behaviour of $(j_2/j_1)$ for two superconducting CPWTLs with different central superconductor ($S$ varied from 1 to 500 µm) shown in Figure 3(a). As well as, variation of $(j_2/j_0)$ and $(j_1/j_0)$ behaviours vs. different central superconductor width shown in Figure 3(b) and (c) respectively.

By simulating superconducting CPWTLs with different geometries and considering continuity condition of current density we propose following equation for current density in central superconductor as following:

$$J_1(x, z) = \begin{cases} \frac{J_0}{\sqrt{1 - (\frac{x}{S})^2}} & |x| \leq \frac{S\gamma_1\lambda^{2\alpha_1}}{2t^2\beta_1} \\ J_0 \left( S^{1.009}t^1 \exp \left( \frac{0.0078}{S^{0.45}} \right) \right) \leq |x| \leq \frac{S}{2} \end{cases}$$ (2)

where

$$\gamma_1 = \frac{S}{4 \left( S^{1.009} \exp \left( \frac{0.0078}{\ln(S^{0.45})} \right) \right)}$$ (3)

$$\alpha_1 = \frac{12.7617}{\ln(\lambda)}$$ (4)

In Equation (2) $\beta_1$ is equal to 0.5.
Figure 3. Normalized current density for \( j_0 = 1 \) in central superconductor and ground planes edges vs. different central superconductor width \( S \) from 1 to 500 \( \mu \text{m} \) and \( W = 46 \mu \text{m} \), simulations were done with these assumption: superconductor is YBCO with thickness \( t = 320 \text{ nm} \) and penetration depth \( \lambda = 200 \text{ nm} \).

And for current distribution in ground planes we propose Equation (5).

\[
J_2(x, z) = \begin{cases} 
\frac{J_0 t^{\beta_2} S^{1.009}}{z^{\gamma_2}} & \frac{S}{2} + W < |x| \leq \frac{S}{2} + W + \gamma_2 \left( \frac{z^{\alpha_2}}{\lambda_2^{\beta_2}} \right)^{4/3} \\
J_0 \left[ \frac{7.9056 \times 10^{-4}}{4 \sqrt{(x - \frac{S}{2} - W)^3}} \right] & |x| \geq \frac{S}{2} + W + \gamma_2 \left( \frac{z^{\alpha_2}}{\lambda_2^{\beta_2}} \right)^{4/3}
\end{cases}
\]  

(5)

where

\[
\lambda_2 = \frac{7.3099 \times 10^{-5}}{\sqrt{54.036}}
\]

(6)

\[
\alpha_2 = \frac{12.8395}{\text{Ln}(\lambda)}
\]

(7)
Figure 4. Display of current density in central superconductor and ground planes from numerical simulation (solid lines) compared with proposed formulation (dotted lines).

In the Equations (2) and (5), \( t \) is superconducting film thickness (unit: m) and \( \lambda \) is penetration depth (unit: m) and \( \beta_2 \) is equal to 0.5. Figure 4 shows current density in a superconducting CPWTL \((S = 400 \mu m, W = 46 \mu m, \text{made of YBCO deposit on LaALO}_3)\) from proposed formulation in (2) and (5) (shown with dotted lines) compared with numerical simulation, 3D-FEM, (shown with solid lines).

3. Nonlinearity in superconducting CPWTLs

Superconducting materials indicate nonlinear behaviours due to dependence of superfluid density on the current distribution at finite temperatures below the critical temperature \( (j_c) \) [22]. As mentioned in [23] Relation between variation of \( n_s \) and current density can be written by following nonlinearity function:

\[
f(T,j) = \frac{n_s(T,0) - n_s(T,j)}{n_s(T,0)} \tag{8}\]

As mentioned in [22] in the relativity small current levels \((\frac{j}{j_c})\), the nonlinearity function of \( f(T,j) \) has quadratic phenomenological dependence on current density and can be described by following equation:

\[
f(T,j) = b_\theta(T) \left( \frac{j}{j_c} \right)^2 \tag{9}\]

And in relativity high current levels the nonlinearity function has modulus phenomenological dependence on current density and can be described by (10):

\[
f(T,j) = b_\theta(T) \left| \frac{j}{j_c} \right| \tag{10}\]

where \( b_\theta(T) \) and \( b_\theta'(T) \) defined in [13].

As shown in Figure 5, the current density in superconducting CPWTL geometries is considerably increased near the central superconductor and ground planes edges and we
Figure 5. Display of the cross over positions in central superconductor and ground planes in superconducting CPWTL.

have two peaks that were named $J_1$ and $J_2$ in Figure 5. So the both quadratic and modulus nonlinearity dependence may occur simultaneously. According to Yip – Sauls results in [24,25] we suppose Equation (11) for $j_b$, to determination amount of current density that is the crossover between quadratic and modulus states.

$$j_b = 4mj c Ln(2) \left( \frac{T}{\Delta_0} \right)$$

where $\Delta_0$ is gap energy in superconducting materials and $m$ is a constant and we suppose approximately $m = 1.63$. According to Figure 5, $S_{q1}$ and $x_g$ ($x_g = W_g - S_{q2}$), are effective quadratic widths for central superconductor and ground planes, respectively, which the current density along them is smaller than $j_b$.

For $S_{q1}$ and $S_{q2}$ we have:

$$S_{q1} = S \sqrt{1 - \left( \frac{J_0\Delta_0}{4mj c Ln(2)} \right)^2}$$

$$S_{q2} = \frac{S}{2} + W + \sqrt[4]{\left( \frac{J_0\Delta_07.9056 \times 10^{-4}}{4mj c TLn(2)} \right)^4}$$

As well as, $i_1$ and $i_2$ were defined as integrations of current density over central superconductor and ground planes, respectively. For $i_1$ and $i_2$ we have:

$$i_1 = 2 \times \int_0^t \int_{\frac{S}{2}}^\frac{S}{2} J_1(x,z) dx dz$$

$$i_2 = \int_0^t \int_{\frac{S}{2}+W}^{\frac{S}{2}+W+g} J_2(x,z) dx dz$$
After integration, we can write $i_1$ and $i_2$ as following:

$$i_1 = J_0 A_1 \rightarrow J_0 = \frac{i_1}{A_1} \quad (16)$$

$$i_2 = J_0 B_1 \rightarrow J_0 = \frac{i_1}{B_1} \quad (17)$$

where

$$A_1 = 2\gamma_1^\alpha_1 t^{1-\beta_1} \exp\left(\frac{0.0078}{S_0 0.45}\right) + \frac{St\text{Arcsin} \left(1 - \frac{2\gamma_1^\lambda_2}{St^2\beta_1}\right)}{2\gamma_1^\alpha_1 t^{1-\beta_1}} \quad (18)$$

$$B_1 = 2t \left[\gamma_1^\lambda_2 \frac{\lambda_2^\alpha_2}{t^\beta_2} \right]^3 S^{1.009} + 31.6224 \times 10^{-4} \sqrt{W_g} \right] \quad (19)$$

From (23) and (24) we can write for $S_{q1}$ and $S_{q2}$:

$$S_{q1} = S\sqrt{1 - c_1(T)i_1^2} \quad (20)$$

$$S_{q1} = c_2(T)i_2^4 \quad (21)$$

where

$$c_1(T) = \left(\frac{\Delta_0}{4mA_1c T\ln(2)}\right)^2 \quad (22)$$

$$c_2(T) = \sqrt\left(\frac{\Delta_0 7.9056 \times 10^{-4}}{4mA_1c T\ln(2)}\right)^4 \quad (23)$$

According to [25] ($\frac{\Delta_0}{T_c} = 3$) was choosen. The variation of $c_1(T)$, $c_2(T)$ as functions of normalized temperature ($\frac{T}{T_c}$) shown in Figure 6.

### 4. Accurate nonlinear circuit model

In this section, accurate nonlinear circuit model for superconducting CPWTL based on considering both quadratic and modulus phenomenological nonlinear dependence was

![Figure 6. Temperature dependence of $c_1(T)$ (solid line), $c_2(T)$ (dotted line) for $S = 15 \mu\text{m}$ and $W = 46 \mu\text{m}$.](image)
Figure 7. Nonlinear distributed circuit for an element cell of a superconducting CPWTL with length of \(dy\).

The proposed model given in Figure 7. As shown in this figure, nonlinearity is embedded in inductance and resistance per unit length. For \(L, R\) shown in Figure 7 we have:

\[
L(T, i) = L_{\text{external}} + L_{\text{kinetic}} + \Delta L(T, i_1, i_2) \tag{24}
\]

\[
R(T, i) = R_0 + \Delta R(T, i_1, i_2) \tag{25}
\]

where \(L_{\text{external}}, L_{\text{kinetic}}\) and \(R_0\) were defined in [26,27]. As well as, capacitance per unit length \((C)\) and conductance per unit length \((G)\) can be calculated from [28,29].

For nonlinear terms in (24), (25) we have:

\[
\Delta L(T, i_1, i_2) = \Delta L_q \left( \frac{S_q^1}{S} \right) i_1^2 - \Delta L_m \left( 1 - \frac{S_q^1}{S} \right) |i_1| + \Delta L_q \left( \frac{x_g}{W_g} \right) i_2^2 + \Delta L_m \left( 1 - \frac{x_g}{W_g} \right) |i_2| \tag{26}
\]

\[
\Delta R(T, i_1, i_2) = \Delta R_q \left( \frac{S_q^1}{S} \right) i_1^2 + \Delta R_m \left( 1 - \frac{S_q^1}{S} \right) |i_1| + \Delta R_q \left( \frac{x_g}{W_g} \right) i_2^2 + \Delta R_m \left( 1 - \frac{x_g}{W_g} \right) |i_2| \tag{27}
\]

As mentioned in [28], for quadratic and modulus inductance terms in (26) we have:

\[
\Delta L_q = \frac{\mu_0 \lambda^2(T, 0)}{j_q^2(T)} \Lambda_q(T) \tag{28}
\]

\[
\Delta L_m = \frac{\mu_0 \lambda^2(T, 0)}{j_m(T)} \Lambda_m(T) \tag{29}
\]

where

\[
 j_q = \frac{j_c}{\sqrt{b_q(T)}}, j_m = \frac{j_c}{\sqrt{b_m(T)}} \tag{30}
\]

In Equation (27) for quadratic and modulus resistance terms we have [28]:

\[
\Delta R_q = \sigma_1(T, 0) \omega^2 \mu_0^2 \lambda^4(T, 0) \frac{2 + \alpha(T)}{j_q^2(T)} \Lambda_q(T) \tag{31}
\]

\[
\Delta R_m = \sigma_1(T, 0) \omega^2 \mu_0^2 \lambda^4(T, 0) \frac{2 + \alpha(T)}{j_m(T)} \Lambda_m(T) \tag{32}
\]
From (16) and (17), $i_1$ and $i_2$ are related to each other as following:

$$
\frac{i_1}{A_1} = \frac{i_2}{B_1} \rightarrow i_2 = \frac{B_1}{A_1} i_1
$$

(33)

By replacing (33) in (26), (27) and using Taylor expansion for nonlinear inductance in proposed circuit model we have:

$$
\Delta L(T, i_1) = \Delta L_0 + \Delta L_1|i_1| + \Delta L_2 i_1^2 + \Delta L_3 |i_1|^3 + \Delta L_4 i_1^4 + \Delta L_5 |i_1|^5 + \Delta L_6 i_1^6
$$

(34)

where

$$
\begin{align*}
\Delta L_0 &= \frac{14}{9} c_2(T) \Delta L_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} - \frac{2}{9} c_2(T) \Delta L_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} \\
\Delta L_1 &= -\frac{40}{9} c_2(T) \Delta L_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} + \frac{7}{9} c_2(T) \Delta L_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} + \Delta L_m \left| \frac{B_1}{A_1} \right| \\
\Delta L_2 &= \frac{35}{9} c_2(T) \Delta L_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} - \frac{14}{9} c_2(T) \Delta L_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} + \Delta L_q
\end{align*}
$$

(35)

And for nonlinear resistance in this model we have:

$$
\Delta R(T, i_1) = \Delta R_0 + \Delta R_1|i_1| + \Delta R_2 i_1^2 + \Delta R_3 |i_1|^3 + \Delta R_4 i_1^4 + \Delta R_5 |i_1|^5 + \Delta R_6 i_1^6
$$

(36)

where

$$
\begin{align*}
\Delta R_0 &= \frac{14}{9} c_2(T) \Delta R_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} - \frac{2}{9} c_2(T) \Delta R_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} \\
\Delta R_1 &= -\frac{40}{9} c_2(T) \Delta R_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} + \frac{7}{9} c_2(T) \Delta R_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} + \Delta R_m \left| \frac{B_1}{A_1} \right| \\
\Delta R_2 &= \frac{35}{9} c_2(T) \Delta R_q \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{10}{3}} - \frac{14}{9} c_2(T) \Delta R_m \left( \left| \frac{B_1}{A_1} \right| \right)^{\frac{7}{3}} + \Delta R_q
\end{align*}
$$

(37)

$$
\begin{align*}
\Delta R_3 &= 0.5 \Delta R_m c_1(T) \\
\Delta R_4 &= -0.5 \Delta R_m c_1(T) \\
\Delta R_5 &= 0.125 \Delta R_m c_1^2(T) \\
\Delta R_6 &= -0.125 \Delta R_q c_1^2(T)
\end{align*}
$$
In above expressions, \( \Lambda_q(T) \) and \( \Lambda_m(T) \) are the quadratic and modulus Geometrical Nonlinear Factors (GNFs) respectively and can be calculated from Equations (38) and (39):

\[
\Lambda_q(T) = \frac{\int j^4 ds}{(\int j ds)^4} \tag{38}
\]

\[
\Lambda_m(T) = \frac{\int j^3 ds}{(\int j ds)^3} \tag{39}
\]

For our study, the integration (38) and (39) must make over the superconducting CPWTL cross section. According to proposed formulation of current distribution for CPWTL in Equations (2) and (5), for GNFs in superconducting CPWTL structures, we have:

\[
\Lambda_q(T) = \frac{l_1}{l_2^4}, \Lambda_m(T) = \frac{l_3}{l_2^3} \tag{40}
\]

where

\[
l_1 = J_0 \left[ 2\gamma_1 s^{4.036} \frac{t^{1+2\beta_1}}{\lambda^{2\alpha_1}} \exp \left( \frac{0.0312}{s^{0.45}} \right) + t \frac{S^3}{S^2 - 4} \frac{\gamma_1}{t^{2\beta_1}} \right] + \frac{S}{2} \cdot \text{Arctan} \left( 1 - \frac{2\gamma_1}{St^{2\beta_1}} \right) + 2t \gamma_2 s^{4.036} \frac{t^{2\beta_2}}{\lambda^{\alpha_2}} + 3.9061 \times 10^{-13} t \left( \gamma_2 \left( \frac{t^{2\beta_2}}{\lambda^{\alpha_2}} \right)^{\frac{8}{3}} W^{-2} \right) \right] \tag{41}
\]

\[
l_2 = J_0 \left[ 2\gamma_1 \lambda \alpha_1 t^{1-\beta_1} s^{1.009} \exp \left( \frac{0.0078}{0.45} \right) + St \sin^{-1} \left( 1 - \frac{2\gamma_1}{St^{2\beta_1}} \right) + 2t \left( \frac{\lambda^{\alpha_2}}{t^{2\beta_2}} \right)^{\frac{1}{3}} s^{1.009} + 31.6224 \times 10^{-4} \sqrt{W} \right] \tag{42}
\]

\[
l_3 = J_0 \left[ 2\gamma_1 s^{3.027} \frac{t^{1+\beta_1}}{\lambda^{\alpha_1}} \exp \left( \frac{0.0243}{s^{0.45}} \right) + 2t \frac{S^2}{S^2 - 4} \frac{\gamma_1}{t^{2\beta_1}} + 2t \gamma_2 \left( \frac{t^{2\beta_2}}{\lambda^{\alpha_2}} \right)^{\frac{5}{3}} s^{3.027} + 7.9054 \times 10^{-10} t \left( \frac{2}{5} \frac{t^{2\beta_2}}{\lambda^{\alpha_2}} - W_m^{-2} \right) \right] \tag{43}
\]

Figure 8 shows variation of quadratic and modulus GNFs of superconducting CPWTL as a function of normalized central superconductor width (\( S_n = \frac{S}{W} \)). Both quadratic and modulus GNFs normalized to their value for superconducting CPWTL with characteristic impedance equal to 50 ohms.

### 5. Simulation results and discussion

In previous sections, we considered theory of nonlinearity in superconducting CPWTL and then an accurate nonlinear distributed circuit model was proposed. In this section, we
are going to predict nonlinearity in some examples of superconducting CPWTL based on our proposed model. For each case our proposed nonlinear model was analysed nonlinearly with Harmonic Balance (HB) method using ADS software to obtain nonlinear results. As first example superconducting CPWTLs with central superconductor of width $53 \, \mu m$ and different length $2.06, 3.18, 6.54$ and $11.35 \, mm$ reported in [19] were modelled. The introduced CPWTLs in [19] fabricated on a $320 \, nm$ YBCO thin film deposited on $15 \, mm \times 15 \, mm$ LaALO$_3$ substrates. Figure 9(a) and (b) show the measured power in the third harmonic as a function of incident power at the frequency of $5 \, GHz$ at $T = 76 \, K$ for mentioned superconducting CPWTLs in different lengths [19] compared with results from our proposed nonlinear model. Form Figure 9 it can be concluded that there are good agreement between results from our proposed model and measured ones.

As the second example, we considered CPWTL introduced in [21]. The CPWTL patterned in a superconducting YBCO thin-film sample of $400 \, nm$ on a lanthanum aluminate substrate of thickness of $500 \, \mu m$, with $T_C = 90 \, K$. Figure 10 shows prediction of the spurious signals at $2f_1 − f_2, 2f_1 + f_2$ and $3f_1$ for $f_2 = 6 \, GHz$ and $\Delta f = 100 \, MHz$ at $T = 76 \, K$ in a CPWTL of $22 \, \mu m$ width of centeral superconductor, $11.25 \, mm$ long and $42 \, \mu m$ width of gap between central superconductor and ground planes from our proposed model compared with measured ones from [21]. From Figure 10 it can be observed that there are good agreement between results obtained from our proposed model and measured ones.

Now based on our proposed nonlinear model we are going to consider temperature dependence of nonlinearity in superconducting CPWTL in different input powers. For this purpose, equivalence circuit model for a $7.5 \, \lambda$ wavelength superconducting CPWTL at $f = 1 \, GHz$ and 50-ohm line ($S = 15 \, \mu m, W = 46 \, \mu m$) made of $320 \, nm$ YBCO, $T_C = 90 \, K$ deposited on $0.5 \, mm$ LaALO$_3$ (LAO). Figure 11 shows the values of second and third order IMD power at different temperatures of $T = 2, 4, 10, 50$ and $T = 80 \, K$ vs. different input power from 0 to $24 \, dBm$. As shown in Figure 11, our results based on considering both modulus and quadratic nonlinearity dependence in proposed nonlinear model predict increasing in slope of second and third order IMD power by increasing input power value. As well as, changing in slope of IMD occurs at lower input power at low temperatures.
Figure 9. Third harmonic results vs. incident power at frequency of 5 GHz at \( T = 76 \text{ K} \) for superconducting CPWTLs with different lengths of 2.06, 3.18, 6.54 and 11.35 mm from our proposed nonlinear model compared with measured ones from [19].

Figure 10. Prediction of the spurious signals at \( 2f_1 - f_2, 2f_1 + f_2 \) and third harmonic \( 3f_1 \) for \( f_2 = 6000 \text{ MHz} \) and \( \Delta f = 100 \text{ MHz} \) at \( T = 76 \text{ K} \) in a superconducting CPWTL of 22 \( \mu \text{m} \), 11.25 mm long and width of central superconductor and 42 \( \mu \text{m} \) width of gap from our proposed model compared with measured ones from [21].

As another result, we considered nonlinear amplitude of second and third IMD power at \( P_{in} = 0, 10, 15 \) and 24 dBm vs. different temperatures from 0 to 85 K. As shown in Figure 12, at specific input power the amplitude level of second- and third-order IMD power decrease at low temperatures and increase at temperatures near to \( T_c \).
Figure 11. Temperature dependence of Power out vs. input power for 7.5 wavelength superconducting CPWTL at different temperatures $T = 2, 4, 10, 50$ and 80 K. Superconducting CPWTL structure is made of 320 nm YBCO ($T_c = 90$ K, $j_c = 1$ MA/cm$^2$ at $T = 77$ K) deposited on 0.5 mm LaALO$_3$ at $f = 1$ GHz and $\Delta f = 100$ KHz.

Figure 12. Temperature dependence of Power out vs. input power for 7.5 wavelength superconducting CPWTL at different input power $p_{in} = 0, 10, 15$ and 24 dBm. Superconducting CPWTL structure is made of 320 nm YBCO ($T_c = 90$ K, $j_c = 1$ MA/cm$^2$ at $T = 77$ K) deposited on 0.5 mm LaALO$_3$ at $f = 1$ GHz and $\Delta f = 100$ KHz.

6. Conclusion

In this paper, we formulated current distribution in superconducting CPWTL structures using 3D-FEM as a numerical solver. This formulation used to analyse nonlinearity in superconducting CPWTLs based on an accurate nonlinear distributed circuit model. In this accurate model, we simultaneously considered both of quadratic and modulus phe-
nomenologial dependence on the current based on new definition for cross over between quadratic and modulus state. So in this model, parameters have complicate dependency on the values of current density and calculated through proposed closed-form expressions analytically. Extracted equivalence circuit models were analysed by HB algorithm to predict nonlinear behaviours in superconducting CPWTLs.

Based on proposed model, we analysed different superconducting CPWTLs with different geometries made of YBCO and there are good agreement between results from our proposed nonlinear model and reported measured ones. As well as based on the proposed model, we predicted nonlinearity at different temperatures and also different input powers. Our results show that there are unusual behaviours such as changing in slope of IMD and it occurred at lower input power at low temperatures.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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