Analyzing project cash flow by a new interval-valued fuzzy queuing model

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Abstract

According to the literature, the methods used in the cash management project were often based on economic theory and accounting. In this paper, queue systems theory is considered, and a new uncertain queuing model is presented based on interval-valued fuzzy sets for the cash flow management in projects. This paper presents an interval-valued fuzzy-Erlang model for proposing liquidity requirements at various stages of projects to be determined so that the deficits of projects are at least. Also, project managers can provide financial requirements and a lack of liquidity and prevent related problems. After explaining the proposed model on the basis of the interval-valued fuzzy queuing theory, how to use it with an application example in the project management is shown.

Keywords: Project cost, cash flow analysis, queuing theory, interval-valued fuzzy sets, Erlang model.

1. Introduction

A few researchers have perceived the significance and need of applying fuzzy sets theory (possibility theory) or probability theory in project cash flow content and analysis. [1] have shown the use of fuzzy averaging methods in the cash flow management. Their strategy depended on creating membership functions of cash flow and characterizing their linguistic variables at a few valuation periods. Vellanki et al. [2] introduced an approach for evaluating working capital requirements, utilizing fuzzy sets theory for the analysis of different quantitative and subjective variables in which data is subjective under uncertain conditions. Yao et al. [3] exhibited a fuzzy, stochastic, single-period model for cash management to give financial decision markers more knowledge into genuine cash management issues. Cheng et al. [4] utilized artificial intelligence methodologies including fuzzy logic, K-means clustering, a genetic algorithm, and neural network to increase key control over project cash flows.

Cheng and Roy [5] introduced the evolutionary fuzzy support vector machine inference model for time series data as an option way to deal with foreseeing cash flow. Mohagheghi et al. [6] proposed a project cash flow assessment technique, according to project scheduling in different stages of project. Interval-valued fuzzy sets (IVFSs) are taken into account to cope with the uncertainty of activity durations and costs. Han et al. [7] focused on project cash flow forecasting at the initial stage of a project, closely related to the payment conditions and financing schedules. Risk factors are considered that affect the soundness of cash flow; these factors can be categorized into two main groups: financial risks and project-specific risks.

Modern uncertain management needs advanced instruments. A large portion of the previous fuzzy-based approaches were by fuzzy sets theory [8]. In fuzzy sets theory, the project decision maker (DM) or expert faces difficulty when expected to give an exact opinion in a number in interval [0, 1]. Communicating this level of uncertainty by an interval is a possible solution. The IVFSs presented for fitting instruments of this mean [9].

In this paper, project cash flow is optimized based on IVF-queuing theory. This approach simultaneously gives the advantages of IVF-sets and queuing theory. In the proposed model, the performance measures are presented by membership functions rather than crisp values. This approach causes the fuzziness of input information to be thoroughly conserved in real-world situations where the data is vague. Moreover, applying IVFSs results in expressing membership functions in intervals rather than crisp values. Therefore, the introduced approach for ambiguous situations is able to model the system more accurately. Moreover, more data is provided to design queuing systems in real-world conditions. In fact, this paper contributes to the literature of project cash flow by suggesting a new and practical model based on queuing theory. The probabilistic tools provided by queuing theory have numerous conventional applications in operations management. Finally, a practical example is presented and solved to illustrate the application of the IVF-queuing assessment model and the computational results are reported.

The rest of this paper is organized as follows. In Section 2, we propose a new IVF-queuing model based on uncertain queuing theory for project cash flow analysis. In Section 3, the proposed IVF-queuing assessment model is used to solve a practical example. The paper is finalized in Section 4.

2. Proposed IVF-queue model for project cash flow management

The Erlang B and Erlang C models are some of the most important tools of queuing theory that are often applied to express the probability that a user cannot make use of a resource at a given time [10]. In the proposed approach, since fuzzy set theory replaces probability, the concept of probability is converted to possibility. Since projects are unique and lack enough historical data, this approach enhances the applicability of the approach. In the case of project cash flow, this means the possibility of no available cash is equal to:

$$P(Blocking) = p(All\ the\ monetary\ resource\ is\ in\ use) \tag{1}$$

The Erlang *B* is denoted as an M/M/C/C system and the Erlang *C* is expressed as an M/M/C/∞ system. In the notations, the first *M* describes inter-arrival times. The second *M* denotes service time distribution, the first *C* is the number of servers in the queue and the second *C* or ∞ denotes the maximum number of customers who can be in the system. The main difference of these two systems is the fact that the Erlang *C* permits its user to wait to access the resource, therefore, the number of users is not limited.

In the Erlang *B* systems, it is possible to express a continuous system which has discrete observations under stated constraints by employing Markov chains [11]. The main idea in this part is to sample in a time interval (δ), where this time interval is a small positive number.

If N_K or $N(k\delta)$, denotes the unavailable monetary resource at time $k\delta$, then it is possible to express N_K as a discrete Markov chain, and $N_K \in [0, C]$. Consequently, the possibility of state transition $P_{i,j}$ is presented as:

$$P_{i,j} = P\{N_{k+1} = j | N_k = i\} \tag{2}$$

In Fig. 2, the state diagram of this system is displayed. In the state diagram, $[(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)]$ is the IVF-average arrival (demand) rate, $[(\mathrm{H}_1^L, \mathrm{H}_2^L, \mathrm{H}_3^L), (\mathrm{H}_1^U, \mathrm{H}_2^U, \mathrm{H}_3^U)]$ is the average required resource, $\mu = 1/H$, and $A = \lambda H$ is total traffic intensity measured in Erlangs, which are a dimensionless quantity. μ and A are computed as follows:

$$\mu = [(1,1,1), (1,1,1)] \div [(\mathrm{H}_1^L, \mathrm{H}_2^L, \mathrm{H}_3^L), (\mathrm{H}_1^U, \mathrm{H}_2^U, \mathrm{H}_3^U)] = \left[\left(\frac{1}{H_1^L}, \frac{1}{H_2^L}, \frac{1}{H_3^L}\right), \left(\frac{1}{H_1^U}, \frac{1}{H_2^U}, \frac{1}{H_3^U}\right)\right] \tag{3}$$

$$A = [(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)] \times [(H_1^L, H_2^L, H_3^L), (H_1^U, H_2^U, H_3^U)] = [(\lambda H_1^L, \lambda H_2^L, \lambda H_3^L), (\lambda H_1^U, \lambda H_2^U, \lambda H_3^U)] \quad (4)$$

To further illustrate the diagram in Fig. 2, the following is presented. At the initial phase of the system it should be considered that there is no resource is in use (state 0). After a period of time the possibility of remaining in state 0 is equal to $[(1-\lambda\delta_3^L, 1-\lambda\delta_2^L, 1-\lambda\delta_1^L), (1-\lambda\delta_3^U, 1-\lambda\delta_2^U, 1-\lambda\delta_1^U)]$. In the state 1, the possibility of returning to state 0 is equal to $[(\mu\delta_1^L, \mu\delta_2^L, \mu\delta_3^L), (\mu\delta_1^U, \mu\delta_2^U, \mu\delta_3^U)]$ and the possibility of remaining in state 1 is $[(1-\lambda\delta_3^L, 1-\lambda\delta_2^L, 1-\lambda\delta_1^L), (1-\lambda\delta_3^U, 1-\lambda\delta_2^U, 1-\lambda\delta_1^U)]$- $[(\mu\delta_1^L, \mu\delta_2^L, \mu\delta_3^L), (\mu\delta_1^U, \mu\delta_2^U, \mu\delta_3^U)]$. At the moment that the system gets to a state k the possibility that k sources are in use is equal to $k-1$ resources times $\lambda\delta$. Consequently:

$$[(\mu\delta_1^L, \mu\delta_2^L, \mu\delta_3^L), (\mu\delta_1^U, \mu\delta_2^U, \mu\delta_3^U)]P_{k-1} = k[(\mu\delta_1^L, \mu\delta_2^L, \mu\delta_3^L), (\mu\delta_1^U, \mu\delta_2^U, \mu\delta_3^U)]P_k \quad (5)$$

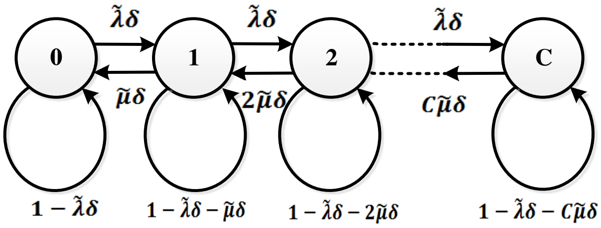

Fig. 2- State diagram for the Erlang B.

Equation (5) is referred to as the global balance equation for the following reason:

$$\sum_{k=0}^{c} P_k = 1 \quad (6)$$

Equation (5) for $k = 1$ equals to:

$$P_1 = \frac{[(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)]P_0}{\left[\left(\frac{1}{H_1^L}, \frac{1}{H_2^L}, \frac{1}{H_3^L}\right), \left(\frac{1}{H_1^U}, \frac{1}{H_2^U}, \frac{1}{H_3^U}\right)\right]} \quad (7)$$

For different values of k the following is yielded:

$$P_k = P_0 \left(\frac{[(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)]}{\left[\left(\frac{1}{H_1^L}, \frac{1}{H_2^L}, \frac{1}{H_3^L}\right), \left(\frac{1}{H_1^U}, \frac{1}{H_2^U}, \frac{1}{H_3^U}\right)\right]} \right)^k \times \frac{1}{K!} \quad (8)$$

The abovementioned equation can be rewritten as follows:

$$P_0 = \left(\frac{\left[\left(\frac{1}{H_1^L}, \frac{1}{H_2^L}, \frac{1}{H_3^L}\right), \left(\frac{1}{H_1^U}, \frac{1}{H_2^U}, \frac{1}{H_3^U}\right)\right]}{[(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)]} \right)^K \times P_k K! = 1 - \sum_{i=1}^{C} P_i \quad (9)$$

Replacing Eq. (8) in Eq. (9) gives the following relation:

$$P_0 = \frac{1}{\sum_{k=0}^{c} \left(\frac{[(\lambda_1^L, \lambda_2^L, \lambda_3^L), (\lambda_1^U, \lambda_2^U, \lambda_3^U)]}{\left[\left(\frac{1}{H_1^L}, \frac{1}{H_2^L}, \frac{1}{H_3^L}\right), \left(\frac{1}{H_1^U}, \frac{1}{H_2^U}, \frac{1}{H_3^U}\right)\right]} \right)^k \times \frac{1}{K!}} \quad (10)$$

Therefore, the probability of blocking for C resources is obtained by the following:

$$P_c = P_0\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^k\frac{1}{k!} \tag{11}$$

Inserting Eq. (10) in Eq. (11) yields the following relation:

$$P_c = \frac{\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^k\frac{1}{k!}}{\sum_{k=0}^{c}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^k\times\frac{1}{K!}} \tag{12}$$

Since the total traffic is obtained by $A = [(\lambda H_1^L,\lambda H_2^L,\lambda H_3^L),(\lambda H_1^U,\lambda H_2^U,\lambda H_3^U)]$, Eq. (12) can be rewritten as:

$$P_c^{Erlang\ B} = \frac{\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^C\frac{1}{C!}}{\sum_{k=0}^{c}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^k\times\frac{1}{K!}} \tag{13}$$

Eq. (13) can be rewritten by applying the concept of IVF-distance as follows:

$$P_c^{Erlang\ B} = \frac{\left(\frac{\sqrt{\frac{1}{6}\sum_{i=1}^{3}[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\left[\sqrt{\frac{1}{6}\sum_{i=1}^{3}\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}\right]}\right)^C\frac{1}{C!}}{\sum_{k=0}^{c}\left(\frac{\sqrt{\frac{1}{6}\sum_{i=1}^{3}[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\left[\sqrt{\frac{1}{6}\sum_{i=1}^{3}\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}\right]}\right)^k\times\frac{1}{K!}} \tag{14}$$

Eqs. (13) and (14) are IVF-Erlang *B* for *C* resources.

The derivation of the Erlang *C* relation is like the Erlang *B* process; but, the difference is that the user faces no limit after the system has reached its capacity. Fig. 3 illustrates the state diagram for Erlang *C* systems.

For cases where $k \le C$, Eq. (9) changes to:

$$P_k = \frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\times\frac{1}{K}\times P_{k-1} \tag{15}$$

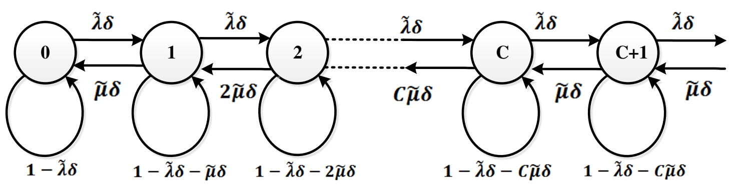

Fig. 3- State diagram for the Erlang C systems

And for cases where $k > C$, Eq. (5) changes to:

$$P_k = \frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\times\frac{1}{C}\times P_{k-1} \tag{16}$$

Therefore,

$$P_k = \begin{cases} \dfrac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}^{k}\times\dfrac{1}{K!}\times P_0, k \le C \\ \dfrac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}^{c}\times\dfrac{1}{c!}\dfrac{1}{c^{k-c}}P_0, k > C \end{cases} \tag{17}$$

Consequently, the global balance equation develops to:

$$\sum_{k=1}^{\infty} P_k = 1 \tag{18}$$

Therefore, we have:

$$P_0\left(1+\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}+\cdots+\frac{1}{C!}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^{c+1}\frac{1}{C^{(c+1)-c}}\right)=1$$

$$P_0 = \frac{1}{\sum_{k=1}^{c-1}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^{k}\frac{1}{k!}+\frac{1}{k!}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^{c}\frac{1}{1-\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]C}}} \tag{19}$$

The possibility that all the resources are in use when a new demand is made is computed by:

$$P_{(all\ the\ resources\ are\ taken)} = P_0\frac{1}{c!}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^{c}\frac{1}{1-\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]C}} \tag{20}$$

Eq. (20) is only valid in cases where $\lambda/\mu C < 1$. $C < A$ would denote that total demand is greater than the quantity of available demand, in other words, the demands will wait for resources indefinitely.

Inserting Eq. (19) in Eq. (20) obtains the following:

$$P_{(\,all\ the\ resource\ in\ use)} = \frac{\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^c}{\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^c + c!\left(1-\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]C}\right)\sum_{k=0}^{c-1}\frac{1}{k!}\left(\frac{[(\lambda_1^L,\lambda_2^L,\lambda_3^L),(\lambda_1^U,\lambda_2^U,\lambda_3^U)]}{\left[\left(\frac{1}{H_1^L},\frac{1}{H_2^L},\frac{1}{H_3^L}\right),\left(\frac{1}{H_1^U},\frac{1}{H_2^U},\frac{1}{H_3^U}\right)\right]}\right)^k} \tag{21}$$

Considering the fact that $A = \lambda/H$ the IVF-Erlang C relation can be proposed as follows:

$$P_c^{Erlang\ C} = \frac{\left(\frac{\sqrt{\frac{1}{6}\sum_{i=1}^3[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\sqrt{\frac{1}{6}\sum_{i=1}^3\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}}\right)^c}{\left(\frac{\sqrt{\frac{1}{6}\sum_{i=1}^3[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\left[\sqrt{\frac{1}{6}\sum_{i=1}^3\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}\right]}\right)^c + c!\left(1-\frac{\sqrt{\frac{1}{6}\sum_{i=1}^3[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\left[\sqrt{\frac{1}{6}\sum_{i=1}^3\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}\right]C}\right)\sum_{k=0}^{c-1}\frac{1}{k!}\left(\frac{\sqrt{\frac{1}{6}\sum_{i=1}^3[(\lambda_i^U)^2+(\lambda_i^L)^2]}}{\left[\sqrt{\frac{1}{6}\sum_{i=1}^3\left[(\frac{1}{H_i}^U)^2+(\frac{1}{H_i}^L)^2\right]}\right]}\right)^k} \tag{22}$$

To adopt the presented equations to project cash flow management, it is necessary to overcome the difference of monetary resource and telephone lines that were the original inspiration of Erlang relations. The main difference is project cash flow a user can demand different amounts of resources at the same time. To overcome this difference in this paper the total amount of cash is divided by the average withdrawals of cash in cash flow, and then each lump of cash is treated as a single resource unit.

3. Applying the proposed IVF-Queue theory model in project cash flow management

To illustrate the applicability of the model, an application example in project cash flow management is provided and solved in this section. The account of a project is displayed in Table 1, and the characteristics of costs are presented in Table 2. For the stated amount of money on short-term investment, how much cash is allowed to be spent so that the possibility that a demand is blocked from getting cash is less than one percent? The results are presented in Table 3.

Table 1- Characteristics of project account

Project account	m$
Assets	100 (m$)
Kept as reserved	10 (m$)
Available for short term investment and spending	90 (m$)

Table 2- Characteristics of costs

Project parameters	Values
λ	$[(0.01, 0.015, 0.02), (0, 0.015, 0.025)]$
H	$[(1.5, 2, 2.5), (1, 2, 3)]$
Number of demands	1000000
The average withdrawals of cash	100$

Table 3- Computational results of proposed IVF-queue model and deterministic model in the project cash flow problem

	Proposed IVF-queue model	Proposed deterministic queue model
Assets	100 (m$)	100 (m$)
Kept as cash	10 (m$)	10 (m$)
Available for spending	82.45 (m$)	82.50 (m$)
Short term investment	7.55 (m$)	7.50 (m$)

The computational results of proposed IVF-queue model and deterministic model in Table 3 indicate the same results for solving the project cash flow problem. It illustrates the validation of the proposed IVF-queue model to cope with uncertain conditions based on interval-valued fuzzy situations in the projects.

This extension of the fuzzy logic is more suitable than conventional uncertainty modeling to represent this degree of certainty for project cash flow problem by an interval form. The fuzzy extension can lead to expand the reliability and lessen the risk and low confidence in the projects regarding the investment; in the case of financial problems in projects' life-cycle, like inability to pay the cost of the project, continuation of the project is difficult; also, project manager for the optimal management of assets should not be excessive amount of liquidity under uncertain conditions in each separate section of project resources. Hence, this paper provides a way for project managers to find the optimal amount of liquidity available at each stage of the project under uncertainty.

4. Conclusions

Among the most widely used methods for the project cash flow management, we can mention the methods that are based on the cumulative cost curve per time, in which the cumulative costs to the project are analyzed with respect to during the implementation of the project. Attempting to use the queue theory in problem of cash flow management projects has not been done in the literature. In this paper, a new interval-valued fuzzy-queuing model is presented to manage the cash flow of the projects. By using benefits of queue theory, the proposed model determines the amount of cash and short-term investments in the projects. In this model, changes in the queuing are taken into account with limited capacity and unlimited Erlang by regarding the activities on the critical path and other activities. The amount of capital in the cash flow must be prepared to pay in cash, and the extent to which it can be maintained for short-term investments, are calculated. The aim is to gain the maximum benefit from the investment of short-term projects and also is to provide the ability to pay the full cost of its project commitments. Finally, a practical example of this approach was demonstrated in the project. Moreover, computational results of the proposed IVF-queue model and deterministic model are discussed and showed that the same results for solving the project cash flow problem under uncertain conditions.

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