Evaluating forecast performance of asymmetric GARCH models on Tehran stock exchange

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Abstract

This paper considers a new kind of asymmetric GARCH structure where the conditional variance reacts differently to small and big shocks. A Bayesian strategy through Gibbs and griddy Gibbs sampling is used to estimate the parameters. We illustrate the efficiency of the model by fitting the model to some parts of Tehran stock market. The new structure outperforms the competing models for in-sample fit and out-of-sample one day ahead volatility forecasts.

Keywords: Bayesian estimation, GARCH, Volatility forecast, Out-of-sample, In-sample.

Mathematics Subject Classification [2010]: 62M10, 62F15

1 Introduction

In the past three decades, there has been a growing interest in using non linear time series models in finance and economy (see Granger, He and Terasvirta). For financial time series, the ARCH model and GARCH model , introduced by Engle and Bollerslev, are surely the most popular classes of volatility models. Although these models have been applied extensively in the modeling of financial time series, the dynamic structure of volatility can not be captured passably by such models. The conditional variance will then differ depending on whether the market is relatively volatile or not. Also financial markets become more volatile in response to bad news (negative shocks) than to good news (positive shocks). For more flexible volatility modeling, many different types of models are introduced to capture asymetry property of conditional variance to positive and negative or big and small shocks, for example the Exponential GARCH (EGARCH) model developed by Nelson [6], GJR-GARCH model proposed by Glosten et al, the smooth transition GARCH models presented by Lubrano [4], Ardia [2] and Medeiros, Veiga [5] and [1]. These models have been applied extensively in the modeling of financial time series.

In this paper we study a new class of GARCH models that introduced a smooth transition between two regimes, one regime for small shocks and another one for big shocks to react differently to small and big returns. The parameters of our model are estimated by applying MCMC methods through Gibbs and griddy Gibbs sampling. Using Tehran stock exchange indices for some part, we illustrate the out-of-sample forecasting performance of one day ahead volatility and in-sample fit of the proposed model. The new proposed structure outperforms the competing models for in-sample fit and out-of-sample volatility forecasting.

The organization of this paper is as follows: the new structure smooth transition GARCH model is presented in section 2. Estimation of the parameters of the model are studied in section 3. Section 4 is dedicated to the main results of the proposed model by applying the Tehran stock exchange. Section 5 concludes.

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2 Size asymmetric smooth transition GARCH model

The size asymmetric smooth transition GARCH model, in summary SA-STGARCH, for time series $\{y_t\}$ is defined as

$$y_t = \varepsilon_t \sqrt{H_t}, \tag{1}$$

where $\{\varepsilon_t\}$ are iid standard normal variables and H_t (the conditional variance at time t) is driven as

$$H_t = a_0 + a_1 y_{t-1}^2 (1 - w_{t-1}) + a_2 y_{t-1}^2 (w_{t-1}) + bH_{t-1} \tag{2}$$

where the weight w_{t-1} is the exponential smooth transition function of the past observation as

$$w_{t-1} = \frac{1 - \exp(-\gamma |y_{t-1}|)}{1 + \exp(-\gamma |y_{t-1}|)}, \gamma_j > 0 \tag{3}$$

which is bounded, $0 < w_t < 1$. The parameter γ is called the slope parameter, that explains the speed of transition from one regime to the other one: the higher γ, the faster the transition. The weight function w_{t-1} goes to one when $y_{t-1} \to \pm\infty$ so the impact of a_2 increases. It tends to zero when $y_{t-1} \to 0$ and consequently the influence of a_1 grows. Therefore the effect of small shocks are mainly described by a_1 and of big shocks by a_2. As often big shocks have greater effect on volatilities than small ones, one could assume that $a_2 > a_1$. Sufficient conditions to guarantee strictly positive conditional variance are that a_0 to be positive and a_1, a_2, b being nonnegative.

3 Estimation

We consider Bayesian MCMC method using Gibbs and griddy Gibbs algorithm for estimation of the parameters.
Let $Y_t = (y_1, \cdots, y_t)$ and $\theta = (a_0, a_1, a_2, b, \gamma)$. The posterior density of θ given the prior $p(\theta)$ is calculated by:

$$p(\theta|Y) \propto p(\theta) \prod_{t=1}^{T} f(y_t|\theta, Y_{t-1}) = p(\theta) \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi H_t}} \exp(-\frac{y_t^2}{2H_t}), \tag{4}$$

To sample from the $p(\theta|Y)$ we apply the Griddy Gibbs algorithm. Given samples at iteration r the Griddy Gibbs at iteration $r+1$ proceeds as follows:

1. Select a grid of points, such as $a_0^1, a_0^2, \cdots, a_0^G$. Use (4) to evaluate the kernel of conditional posterior density function of a_0 given all the values of Y and θ except a_0 $k(a_{0|Y_t,\theta_{-a_0}})$ over the grid points to obtain the vector $G_k = (k_1, \cdots, k_G)$.
2. By a deterministic integration rule using the G points, compute $G_\Phi = (0, \Phi_2, \cdots, \Phi_G)$ with

$$\Phi_j = \int_{a_0^1}^{a_0^j} k(a_0|\theta_{-a_0}^{(r)}, Y_t) da_0, \quad i = 2, \cdots, G.$$

3. Simulate $u \sim U(0, \Phi_G)$ and invert $\Phi(a_0|\theta_{-a_0}^{(r)}, Y_t)$ by numerical interpolation to obtain a sample $a_0^{(r+1)}$ from $p(a_0|\theta_{-a_0}^{(r)}, Y_t)$.
4. Repeat steps 1-3 for other parameters.

Prior densities of elements of θ can be considered as independent uniform densities over finite intervals.

4 Main results

By applying daily log returns of the Tehran stock exchange for the period of 04/12/2008 to 27/04/2015 (2044 observations) the first 1544 observations are employed to estimate the parameters and the remaining 500 samples are used for forecasting analysis. We compare the performance of our model with the GARCH, GJR-GARCH (Glosten et al), logistic smooth transition GARCH (LST-GARCH) and exponential smooth transition GARCH (EST-GARCH) that introduced by Lubrano. Figure 1 demonstrates the stock market index and the percentage log returns of Tehran market exchange.

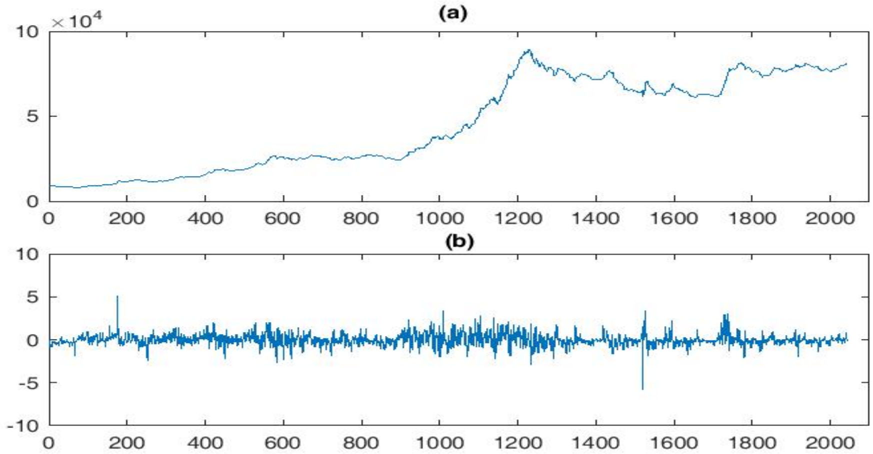

Figure 1: (a): Tehran stock exchange index, (b): Percentage daily log returns

4.1 In-sample and out-of-sample performance analysis

In order to compare the goodness of fit of contending models, we apply the deviance information criterion (DIC) introduced by Spiegelhalter et al that is a Bayesian version or generalization of the reputable Akaike information criterion (AIC).

Table 1: Deviance information criterion (DIC)

Model	DIC
GARCH	4831.9
GJR-GARCH	4956.3
LST-GARCH	4932.3
EST-GARCH	4833.9
SA-STGARCH	4828.3*

The smallest DIC determines the best in-sample fit. According to the results of Table 1 our considered model has the best in-sample fit to the data set among competing models. For appraising the performance of SA-STGARCH to forecast volatility, we apply the Diebold Mariano test. Testing for equal forecast accuracy is an approach to evaluate the predictive capability of competitor models. For evaluating the performance of new proposed model in one-step ahead conditional variance forecast, we compute the DM statistic for pairwise comparison of SA-STGARCH model with GARCH, GJR-GARCH, LST-GARCH and EST-GARCH models. The test results in Table 2 show that the null hypothesis is rejected at the 5% significance level for all cases as all the statistics are less than $Z_{0.05}$. So our presented model has an improvement in the forecasting performance.

Also for specifying the out-of-sample forecast performance of the SA-STGARCH toward the competing models, We compare the forecasting volatility $E(Y_t^2|\mathcal{F}_{t-1})$, or conditional variance, of each model with the squared returns. Table 3 reports the mean squared error (MSE) and mean absolute error (MAE) of the predictions. According to the results of this table, the least values of MSE and MAE are related to the

Table 2: Diebold Mariano statistic

Comparison of SA-STGARCH with	Statistic value
GARCH	-2.522
GJR-GARCH	-2.519
LST-GARCH	-2.335
EST-GARCH	-2.788

SA-STGARCH model that reveals the best forecast compared with the other reviewed models in this paper.

Table 3: Diebold Mariano statistic

Model	Mean square error (MSE)	Mean absolute error (MAE)
GARCH	0.272	0.350
GJR-GARCH	0.389	0.409
LST-GARCH	0.387	0.400
EST-GARCH	0.256	0.349
SA-STGARCH	0.203*	0.326*

5 Conclusion

This paper has presented a new class of GARCH model, SA-STGARCH. The conditional variance of this model reacts differently to small and big shocks. We evaluated the forecast performance of some other asymmetric GARCH models on Tehran exchange index. The results demonstrates that new proposed model has the best in-sample fit and out-of-sample volatility forecast.

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