Resource allocation for downlink of multi-user multi-band wireless networks

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The aim of this reported work was investigation of a proper channel assignment method in the downlink channel of a multi-user wireless network in which the transmitter aims at sending individual data streams to certain numbers of users through a limited number of sub-channels. In this regard, considering the network throughput as the performance function, it is demonstrated that the problem can be formulated as an optimisation problem which is not convex in general. However, through relaxing some constraints, the problem can be cast as a convex optimisation problem, leading to a simple power allocation strategy.

Introduction: Resource allocation has been regarded as one of the main challenges in wireless networks. This is due to the limited amount of bandwidth and more demands, arising from emerging wireless technologies. Basically, resource allocation concerns sub-channel assignment as well as a suitable power allocation strategy to meet a desired criterion, such as the network throughput which is employed as the performance metric in the current work.

Basically, there are two main approaches that persuade one's choice as baselines for resource allocation: (i) static resource allocation (SRA) [1] against (ii) dynamic resource allocation (DRA) [2, 3]. The former, regardless the channel gain associated with each user and based on a predetermined scheduling algorithm, attempts to allocate the available resources among existing users. The latter, on the other hand, makes use of the channel state information (CSI) of current users to effectively address this issue. As such, the performance achieved through using DRA outperforms that of using SRA. DRA is more useful when the system works in TDD mode in which each user sends a preamble packet prior to sending information through the uplink channel which can be used for DRA. Although these channel gains may be outdated for the downlink mode, there are some prediction models to alleviate this issue. In fact, the impact of delay can be applied to predict the channel gains.

This Letter concerns dynamic resource allocation in the downlink channel of a multi-user wireless network, where there are a limited number of sub-channels. In this regard, a suitable channel assignment strategy along with a proper power allocation strategy across sub-channels is proposed, showing that the achievable throughput of the proposed method outperforms that of existing works.

Note that the optimisation problem incorporated in the aforementioned problem is not convex, thus finding the optimal solution is too complex to deal with. Although there have been some attempts to address the aforementioned issue, they mostly fail to draw a concrete path towards finding a close to optimal solution, as they basically rely on some heuristic assumptions, thereby falling short of approaching the optimal result (i.e. [2, 3]). For instance, the authors in [2] investigated the same issue through assigning each sub-channel to the user whose corresponding channel gain is the best. Then, the so-called water filling (WF) algorithm [4] is employed to allocate the power across sub-channels. Also, [3] studies the same problem through using a well-established combinatorial problem, called the 'assignment problem' [5]. To this end, it is conjectured that the channel assignment can be approximated with a simpler form in which the objective is to assign sub-channels to the users so that the sum of channel gains associated with the selected users is maximised. However, it is not thoroughly discussed how the original problem can be approximated by this simpler form.

Although the problem studied in the current work is not convex and the proposed approach does not yield the optimal result, it is demonstrated that in some marginal cases, it follows the optimal result, i.e. for the moderate to high SNR regimes.

The rest of this Letter is organised as follows. The following Section presents the problem formulation. The proposed sub-channel assignment and the power allocation strategy is investigated in the subsequent Section. Simulation results are provided in the next Section, and the final Section summarises the findings.

Problem formulation: We consider the downlink channel of a multi-user multi-band network, assuming the transmitter aims at sending individual data streams to N users over K independent sub-channels, each of bandwidth B. Also, it is assumed the CSI associated with each user is perfectly available at the transmitter, owing to power constraint $P_{\max}$. The main objective is to assign sub-channels and allocate the transmit power to increase network throughput. Assuming $\pi(n) \in \{1, \dots, K\}$ is the user which the nth sub-channel is assigned to, thus the problem can be cast as the following optimisation problem,

$$\max_{p_n^{(\pi(n))}} \sum_{n=1}^{N} \frac{1}{N} \log_2\left(1 + \frac{p_n^{(\pi(n))}|h_n^{(\pi(n))}|^2}{N_0 B/N}\right) \quad \text{subject to:} \qquad \sum_{n=1}^{N} p_n^{(\pi(n))} \le P_{\max} \tag{1}$$

where N and N_0 denote, respectively, the number of sub-channels, and noise power density. Also, B specifies the total bandwidth. $p_n^{(\pi(n))}$ is the power assigned to the user indexed by $\pi(n)$ in the nth sub-channel and similarly $h_n^{(\pi(n))}$ is the corresponding channel gain of the user indexed by $\pi(n)$ over the nth sub-channel.

Proposed channel assignment and power allocation method: Using the method of Lagrange multipliers, one can readily verify that (1) simplifies to:

$$\Gamma(p_1, \dots, p_n, \lambda) = \sum_{n=1}^{N} \frac{1}{N} \log_2 (1 + p_n^{(\pi(n))} H_n^{(\pi(n))}) + \lambda\left(P_{\max} - \sum_{n=1}^{N} p_n^{(\pi(n))}\right) \tag{2}$$

where in (2) $H_n^{(\pi(n))}$ is defined as $H_n^{(\pi(n))} = (|h_n^{(\pi(n))}|^2/N_0(B/N))$, indicating the corresponding channel to noise ratio of the user indexed by $\pi(n)$, assuming the nth sub-channel is assigned to this user. Also, λ is the Lagrange multiplier associated with the power constraint. Thus, taking derivation of Γ with respect to $p_n^{(\pi(n))}$, it follows:

$$\frac{\partial \Gamma}{\partial P_n^{(\pi(n))}} = 0 \Rightarrow \frac{H_n^{(\pi(n))}}{N(\ln 2)(1 + H_n^{(\pi(n))} p_n^{(\pi(n))})} - \lambda = 0 \tag{3}$$

This implies $P_n^{(\pi(n))} = \max(1/N\lambda(\ln(2))) - (1/H_n^{(\pi(n))}), 0)$. We simply assume that sub-channels are properly assigned, thus it is less likely to find a situation in which the power allocated to a sub-channel to be zero, as it means this sub-channel is not properly assigned, i.e. the corresponding user has poor channel condition on this sub-channel which is in contrast with the assumption that sub-channels are properly assigned. Thus, one can hardly find a situation in which $p_n^{(\pi(n))} = 0$. In other words, $p_n^{(\pi(n))} = (1/N\lambda(\ln(2))) - (1/H_n^{(\pi(n))})$, meaning with high probability, at optimal point $H_n^{(\pi(n))} \ge N\lambda(\ln(2))$. On the other hand, noting $\Sigma_{n=1}^{N} p_n^{(\pi(n))} \le P_{\max}$, and after some straight manipulations, it follows:

$$p_n^{(\pi(n))} = \frac{p_{\max}}{N} + \frac{1}{N} \sum_{n=1}^{N} \frac{1}{H_n^{(\pi(n))}} - \frac{1}{H_n^{(\pi(n))}} \tag{4}$$

Substituting $p_n^{(\pi(n))}$ in the objective function of (1), the network throughput becomes:

$$T = \sum_{n=1}^{N} \frac{1}{N} \log_2\left[\left(\frac{P_{\max} H_n^{(\pi(n))}}{N}\right)\left(1 + \frac{1}{P_{\max}} \sum_{n=1}^{N} \frac{1}{H_n^{(\pi(n))}}\right)\right] \tag{5}$$

Consequently, noting sub-channels are assigned to the users with good channel conditions and considering we are interested in moderate to high SNR regimes, i.e. $(1/P_{\max})\Sigma_{n=1}^{N}(1/H_n^{(\pi(n))}) \ll 1$, and noting $\ln(1+x) \simeq x$ for $x \ll 1$, it follows:

$$T \simeq \log_2\left(\frac{P_{\max}}{N}\right) + \sum_{n=1}^{N} \frac{1}{N}\left(\log_2 (H_n^{(\pi(n))}) + \frac{1}{\ln(2) P_{\max} H_n^{(\pi(n))}}\right) \tag{6}$$

Thus, referring to (6), the channel assignment problem can be abstracted to finding the best values for $\pi(n) \in \{1, \dots, K\}$ for $n = 1, \dots, N$ such that $\Sigma_{n=1}^{N} c_n$ is maximised, where c_n is defined as $c_n = (1/N)\log_2(H_n^{(\pi(n))}) + (1/N\ln(2)P_{\max}H_n^{(\pi(n))})$. This problem has it roots in combinatorial optimisation, dubbed the assignment problem [5], which can be tackled through the so-called Hungarian method in polynomial time. This enables effectively assigning sub-channels to existing users. Note that in [3], c_n is set to $c_n = H_n^{(\pi(n))}$, however, as we will see in the following Section, in some cases, it has a sizeable gap compared with the method proposed here.

Simulation results: Throughout the simulations, the channel gains are assumed to be either Rayleigh distributed or with probability $p = 0.3$ fall in deep fade (to have zero gain). Also, the total transmit power is set to one. Fig. 1 compares the achievable rate of the proposed channel assignment method with the methods proposed in [2, 3] for three cases of having 5, 10, and 15 users, while the number of sub-channels is set to 10 in all cases. In [2], each subcarrier is assigned to the user which has the best channel gain on it, then this user is discarded from further investigation for remaining sub-channels. In [3], the channel assignment is performed such that the sum of channel gains over these sub-channels is maximised. Note that this method is based on some heuristic observations. It is worth mentioning that to have a basis for fair comparison, in all of the aforementioned methods the WF power allocation is being used. Referring to Fig. 1, the proposed method ranks first in the aforementioned scenarios.

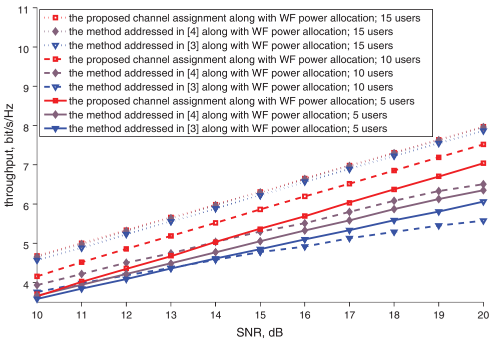

Fig. 1 *Throughput of 10 sub-channels, while having 5, 10 and 15 users for different methods in shadowing environment with fading probability p = 0.3*

Fig. 2 compares the achievable throughput of the proposed method, as well as the methods studied in [2, 3], with that of the optimal for a simple scenario. To this end, it is assumed there are four users and four sub-channels. Note that the optimal solution is derived through an exhaustive search over all possible choices of channel ordering among existing users ($4! = 24$ choices for each channel realisation). As inferred from Fig. 2, the proposed method follows the optimal result from moderate to high SNR regime, confirming the optimality of the proposed method in this regime.

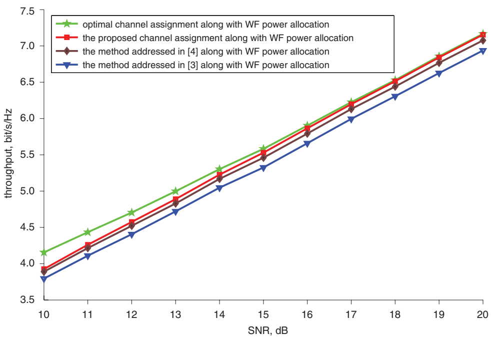

Fig. 2 *Comparison results for four sub-channels and four users*

Conclusion: A new channel assignment method for multi-user multi-band wireless networks is proposed and the achievable throughput is compared to the best existing methods under various conditions, i.e. with and without shadowing, and under various power allocation methods. Simulation results indicate that there is a consistent improvement in all of the aforementioned scenarios, which makes the proposed channel assignment method a versatile technique for use in such networks.

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One or more of the Figures in this Letter are available in colour online.

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