Workshop on
Graphs and Algorithms, June 1 and 2, 2011, IPM, Tehran, Iran

Fault-Tolerant Conflict-Free Colorings

Mohammad Ali Abam
Sharif University of Technology
Iran

Consider a cellular network consisting of a set of base stations, where the signal from a given base station can be received by clients within a certain distance from the base station. In general, these regions will overlap. For a client, this may lead to interference of the signals. Thus one would like to assign frequencies to the base stations such that for any client within reach of at least one base station, there is a base station within reach with a unique frequency (among all the ones within reach). The goal is to do this using only few distinct frequencies. Recently, the conflict-free coloring is introduced to model this problem. Base stations in cellular networks are often not completely reliable: every now and then some base station may (temporarily or permanently) fail to function properly. This leads us to study fault-tolerant CF-colorings: colorings that remain conflict-free even after some stations fail. We show how to model this problem using graphs and apply some existing theorems in the graph theory to solve this problem.

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The Complexity of the Proper Orientation Number of Graphs

Arash Ahadi
Sharif University of Technology
Iran

Graph orientations is a well-studied area of graph theory. A proper orientation of a graph $G = (V, E)$ is an orientation f of $E(G)$ such that for every two adjacent vertices v and u, $d_f^-(v) \neq d_f^-(u)$ where $d_f^-(v)$ is the number of edges with head v in f. The proper orientation number of G is defined as $\overrightarrow{\chi}(G) = \min_{f \in \Gamma} \max_{v \in V(G)} d_f^-(v)$ where Γ is the set of proper orientations of G. We have $\chi(G) - 1 \leq \overrightarrow{\chi}(G) \leq \Delta(G)$. We show that, it is NP-complete to decide $\overrightarrow{\chi}(G) = 2$, for a given planar graph. Also, there is a polynomial time algorithm for determining the proper orientation number of a given 3-regular graph. In sharp contrast, it is NP-complete for 4-regular graphs. Also, we prove that for every tree T, $\overrightarrow{\chi} \leq 4$ and this bound is sharp.

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Rounding by Sampling

Arash Asadpour
New York University
USA

In this talk we present new developments in "Randomized Rounding", a methodology that has been extensively used in designing algorithms for various optimization problems. We will focus on a technique called "Rounding by Sampling" which at the same time deals with the combinatorial structure of the solution and its quantitative properties. We show how this technique can be used in order to design approximation algorithms for some of the most well-known algorithmic and optimization problems such as the Traveling Salesman Problem (TSP) and Fair Allocation of Indivisible Goods.

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New Recursive Methods for Enumeration of Perfect Matching

Afshin Behmaram
University of Tehran
Iran

A k-matching is the set of k independence edges which no two of them share a common vertices.The matching which cover all of the vertices of graph is called perfect matching.This definition implies that perfect matching is $\frac{n}{2}$ matching. Counting perfect matching is very interesting and hard problem in graph theory and have application in chemistry and physics.In this lecture we find recursive formula for the number of perfect matchings in graph by separating a paraph G in to subgraph H and Q and uses this formula to count perfect matching of $P_m \times P_n$ $(m = 2, 3, 4)$ and Q_n and in octagonal latice(c4c8 graphs). Also we apply this theorem to find an interesting relation for the number of perfect matching in edge transitive graph.We prove that if G is an edge transitive graph and $e = uv \in E(G)$ then we have :

$$PM(G) = \frac{2q}{p} PM(G - \{u, v\}) \tag{1}$$

$$[deg(v_1), deg(v_2), ..., deg(v_n), \frac{2q}{(2q, 2q - p)}] | PM(G) \tag{2}$$

An Efficient Algorithm to Recognize Locally Equivalent Graphs

Salman Beigi
IPM
Iran

The local complementation of a graph G at vertex v is defined as the operator that replaces the induced subgraph on the neighbors of v by its complement. For instance the local complementation of the complete graph at any vertex gives a star. Two graphs are called locally equivalent if by a sequence of local complementations on one of them we can reach the other. The local complementation operations naturally appear in the study of quantum stabilizer codes under the action of Clifford group. We will see in this talk that translating the problem of distinguishing locally equivalent graphs to the language of quantum codes provides us with a simple algebraic formulation. That is, we obtain a set of linear and non-linear equations in terms of two given graphs, that have a solution if and only if those graphs are locally equivalent. Then we present an efficient algorithm to find solutions of these equations, and thus to decide whether two given graphs are locally equivalent or not. We also define the local complementation operators on labeled graphs with labels in any finite field, and show that the same algorithm designed for the binary field, works for non-binary fields as well.

This talk is based on a joint work with Mohsen Bahramgiri.

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On the f-coloring of Graphs

Maliheh Chavooshi
University of Tehran
Iran

Let G be a graph. The minimum number of colors needed to color the edges of G is called the *chromatic index* of G and is denoted by $\chi'(G)$. It is well-known that $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$, for any graph G, where $\Delta(G)$ denotes the maximum degree of G. A graph G is said to be *Class* 1 if $\chi'(G) = \Delta(G)$ and *Class* 2 if $\chi'(G) = \Delta(G) + 1$. Also, G_Δ is the induced subgraph on all vertices of degree $\Delta(G)$. An f*-coloring* of a graph G is a coloring of the edges of $E(G)$ such that each color appears at each vertex $v \in V(G)$ at most $f(v)$ times. The minimum number of colors needed to f-color G is called the f*-chromatic index* of G and is denoted by $\chi'_f(G)$. It was shown that for every graph G, $\Delta_f(G) \leq \chi'_f(G) \leq \Delta_f(G) + 1$, where $\Delta_f(G) = \max_{v \in V(G)} \lceil \frac{d_G(v)}{f(v)} \rceil$. A graph G is said to be f*-Class* 1 if $\chi'_f(G) = \Delta_f(G)$, and f*-Class* 2, otherwise. Also, G_{Δ_f} is the induced subgraph of G on $\{v : \frac{d_G(v)}{f(v)} = \Delta_f(G)\}$. Hilton and Zhao showed that if G_Δ has maximum degree two and G is Class 2, then G is critical, G_Δ is a disjoint union of cycles and $\delta(G) = \Delta(G) - 1$, where $\delta(G)$ denotes the minimum degree of G, respectively. In this talk, we generalize this theorem to f-coloring of graphs. Also, we determine the f-chromatic index of a connected graph G with $|G_{\Delta_f}| \leq 4$.

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Upward Embedding Testing on $\mathbf{T_h}$ is Np-Complete

Ardeshir Dolati
Shahed University
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An upward embedding of a directed graph on a surface is an embedding of its underlying graph so that all arcs are monotonic and point to a fixed direction. It has been shown that upward embedding testing on the plane and sphere are Np-Complete problems. In this paper, we shall characterize all horoidal directed graphs. We shall also prove that the decision problem whether a directed graph has an upward embedding on the horizonal torus is Np-Complete.

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Note on the Erdös-Gyárfás Conjecture on Generic Graphs

Hossein Esfandiari
Sharif University of Technology
Iran

Erdös and Gyárfás conjectured that, every graph with minimum degree at least three has a cycle whose length is a power of 2, [1]. There seems to be a few publication on the Erdös-Gyárfás Conjecture. In This paper we obtain two results about this conjecture, stating that; the minimal counter example for this conjecture has at most one cut vertex. Also, assume that the conjecture holds, and let e be an arbitrary edge, there exist a cycle with length a power of 2 which is do not contain e, or there exists two cycles on e with lengths 2^s and $2^t - 1$ for some positive integer s and t.

References

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On the Edge Coloring of a Graph G with $\Delta(G_\Delta) \leq 2$

Maryam Ghanbari
K. N. Toosi University of Technology
&
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Let G be a graph. The core of G, denoted by G_Δ, is the subgraph of G induced by the vertices of degree $\Delta(G)$, where $\Delta(G)$ denotes the maximum degree of G. A *k-edge coloring* of G is a function $f : E(G) \to L$ such that $|L| = k$ and $f(e_1) \neq f(e_2)$ for all two adjacent edges e_1 and e_2 of G. The *chromatic index* of G, denoted by $\chi'(G)$, is the minimum number k for which G has a k-edge coloring. A graph G is said to be *Class* 1 if $\chi'(G) = \Delta(G)$ and *Class* 2 if $\chi'(G) = \Delta(G) + 1$. A graph G is called overfull if $|E(G)| > \lfloor \frac{|V(G)|}{2} \rfloor \Delta(G)$. In 1996, Hilton and Zhao conjectured that if G is a connected graph such that $\Delta(G_\Delta) \leq 2$ and G is not the Petersen graph with one vertex removed, then G is Class 2 if and only if G is overfull. In this talk, we prove this conjecture for some graphs of even order.

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On the Integral Circulant Graphs

Mohsen Molla Haji Aghaei
Amirkabir University of Technology
Iran

A graph is called *integral* if all eigenvalues of its adjacency matrix are integers. A graph is called *circulant* if it is a Cayley graph on the additive group $\mathbb{Z}_n$ (a residue group modulo n), i.e. its adjacency matrix is circulant.

Let D be a set of positive, proper divisors of the integer $n > 1$. Define the gcd-graph $X_n(D)$ to have vertex set $\mathbb{Z}_n = \{0, 1, ..., n-1\}$ and edge set $E(X_n(D)) = \{\{a, b\}; a, b \in Z_n, gcd(a-b, n) \in D\}$. It was proven that a circulant graph is integral if and only if it is a gcd-graph.

It would be interesting to study chromatic, clique number and some other properties of Integral Circulant Graphs.

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On Automorphism Groups of Graph Powers

Moharram N. Iradmusa
Sharif University of Technology
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An automorphism of a graph G is a permutation π of the vertex set of G such that, for any vertices u and v of G, we have $\pi(u) \sim \pi(v)$ if and only if $u \sim v$. The set of all automorphisms of G, with the operation of composition of permutations, is a permutation group on $V(G)$, denoted by $Aut(G)$. The automorphism group of a graph characterize its symmetries, and is therefore very useful in determining certain of its properties.
The *$k-$power of* G, denoted by G^k, is defined on the vertex set $V(G)$ by adding edges joining any two distinct vertices x and y with distance at most k.
In this paper, we investigate the automorphism groups of the graph powers and show that $Aut(G^n) \cong Aut(G)$, provided that G is a connected graph with sufficiently large girth and without terminal vertices. In addition we show that $Aut(G^2) \cong Aut(G)$, when $diam(G) \geq 4$ and $g(G) \geq 7$.

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Large Chromatic Number and Ramsey Graphs

Sogol Jahanbekam
University of Illinois at Urbana-Champaign
USA

The clique number, the chromatic number, and the independence number of a graph $G = (V, E)$ is denoted by $\omega(G)$, $\chi(G)$, and $\alpha(G)$, respectively. Intuitively, large chromatic number must imply large cliques. Define

$$Q(n, c) := \min\{\omega(G) : |V(G)| = n \text{ and } \chi(G) = c\}.$$

It is obvious that $Q(n, n) = n$, it is not difficult to show that $Q(n, n-1) = n-1$ ($n \geq 2$) and that $Q(n, n-2) \leq n-3$. Biró determined $Q(n, n-k)$ for $k \leq 6$, whenever n is sufficiently large.

Based on these values he conjectured that if n is large enough, then $Q(n, n-k) = n - 2k + \lceil k/2 \rceil$. He also showed $Q(n, n-k) \geq n - 2k + 3$ for $k \geq 5$ and n is large enough. On the other hand, Jahanbekam and West observed that $Q(n, n-k)$ is at most the conjectured value whenever $n \geq 5k/2$ and they conjecture that this threshold on n is both sufficient and necessary for equality.

In this note we give an exact formula for $Q(n, n-k)$ for $n \geq 2k + 3$ using Ramsey graphs. Our results also disprove the above conjectures.

Isoperimetry Problems on Trees and its Application to Image Segmentation

Ramin Javadi
Sharif University of Technology
Iran

Isoperimetric (Cheeger) constant is considered as a geometric tool measuring the connectivity of a geometric object. Many relationship with the other parameters such as the second eigenvalue of Laplace operator (spectral gap) has been proved. Discrete analogue of Cheeger constant has been recently at the center of attention. The significance of the isoperimetric problem is not only due to its relation to the central theoretical concepts (e.g. expander graphs, rapid mixing and balanced cut), but also its varied real world applications (e.g. image segmentation, network reliability and graph clustering).

This talk is aimed to investigate some computational aspects of different isoperimetry problems on weighted graphs and specially on weighted trees. Particularly, we will report that for every fixed number $k > 1$, computing the kth isoperimetric number is an NP-hard problem for planar bipartite weighted graphs and for simple (unweighted) graphs, as well. Also, we report that a linear time algorithm is available to computing the kth isoperimetric number on a weighted tree. We apply this algorithm to introduce a new clustering method and deploy it in image segmentation.

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Longest Paths in Grid Graphs

Fatemeh Keshavarz-Kohjerdi
Islamic Azad University
Iran

A grid graph is a finite node-induced subgraph of the infinite two-dimensional integer grid. It is rectangular if its set of nodes is the product of two intervals. The longest path problem is a well-known NP-hard problem and so far it have been solved polynomially only for few classes of graphs. In this paper, we give a linear-time algorithm for finding a longest path between any two given vertices in a rectangular grid graph.

Weight Choosability of Graphs

Mahdad Khatirinejad
University of British Columbia
Canada

A total weight colouring of a (di)graph is a weight assignment from a set R to the vertices and from a set S to the edges such that the accumulated weights at the vertices yield a proper vertex colouring. If such an assignment exists, we say the graph is (R,S)-total weight colourable. One may also consider the (k, k')-total weight choosability of a graph, that is, the list version of the previous definition where the lists assigned to the vertices and edges have sizes k and k', respectively.

It is conjectured that every graph is (2,2)-total weight choosable and every graph with no isolated edge is (1,3)-total weight chooable. In this talk, I will present an overview of the progress made towards these conjectures.

Part of this work is joint work with Reza Naserasr, Mike Newman, Ben Seamone, and Brett Stevens.

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On $Q-$spectrum of Graphs and their Betweenness

Maryam Mirzakhah
Amirkabir University of Technology
&
IPM
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Associated with the graph G, the *signless Laplacian* matrix is defined as $Q(G) = A(G) + D(G)$ in which $A(G)$ and $D(G)$ are adjacency and degree matrix of G, respectively. Also, let $\sigma_{uv}(w)$ denote the number of shortest paths from vertex u to vertex v that go through w, and σ_{uv} is the total number of shortest paths from u to v. Then the *betweenness* of vertex w is $B_w = \sum_{u,v \neq w} b_w(u, v)$, where $b_w(u, v) = \frac{\sigma_{uv}(w)}{\sigma_{uv}}$ for $u, v, w \in V(G)$.

In this talk, we want to relate some results about $Q-$spectrum of a graph and its betweenness.

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On the Sum of All Distance Between Vertices of Graphs

Mohammad Javad Nadjafi-Arani
University of Kashan
Iran

Let G be a connected graph. $\eta(G) = Sz(G) - W(G)$, The Wiener index $W(G)$ is defined as the sum of all distances between vertices of G. Suppose $e = uv$. Define $n_u(e)$ to be the number of vertices of G lying closer to u than v and $n_v(e)$ is defined analogously. The Szeged index of G is defined as $Sz(G) = \sum_{e=uv\in E(G)} n_u(e)n_v(e)$. Suppose $\eta(G) = Sz(G) - W(G)$, a well-known result of Klavžar, Rajapakse and Gutman states that $\eta(G) \geq 0$ and by a result of Dobrynin and Gutman $\eta(G) = 0$ if and only if each block of G is complete. In this paper a path-edge matrix for the graph G is presented by which it is to possible classify the graphs in which $\eta(G) \leq 5$ and $\eta(G) \neq 1, 3$. It is also proved that there is not a graph G with the property that $\eta(G) = 1$ or $\eta(G) = 3$. Finally, it is proved that for a given positive integer $k, k \neq 1, 3$, there exists a graph G with $\eta(G) = k$.

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Roman Bondage Number of Graphs

Sahar Qajar
Sharif University of Technology
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A *Roman domination function* on a graph $G = (V(G), E(G))$ is a labeling $f : V(G) \to \{0, 1, 2\}$ satisfying the condition that every vertex with label 0 has at least a neighbor with label 2. A *Roman domination number* $\gamma_R(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over such functions. A *Roman bondage number* $b_R(G)$ of G is the minimum cardinality of all sets $E \subseteq E(G)$ for which $\gamma_R(G - E) > \gamma_R(G)$. In this talk we present previous results of Roman bondage number briefly, and introduce some new results.

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On the Zagreb Index of Some Dendrimer-Nanostar Garphs

Alireza Rezaei
Islamic Azad University, Hamedan branch, Hamedan
Iran

Dendrimers have attracted many theoretical and practical attentions. They are interested unusually in transport and have optical properties. They are also promising candidates for a wide variety of applications, including electro-optical organic light sources and nanoscale optical sensors. For a graph G the first Zagreb index $M_1(G)$ is equal to the sum of the sums of the degrees of pairs of adjacent vertices of the graph. The second Zagreb index $M_2(G)$ is equal to the sum of the products of the degrees of pairs of adjacent vertices of the graph. the Zagreb index is one of the important topological indices for describing the boiling point of the molecules. In this paper we present some methods for computing the Zagreb indices of some dendrimer nanostars.

This talk is based on a joint work with Fatemeh Nabii Hossein Abadi from Islamic Azad University, Malayer branch, Malayer.

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A graph is called *integral* if all eigenvalues of its adjacency matrix are integers. A graph is called *circulant* if it is a Cayley graph on the additive group $\mathbb{Z}_n$ (a residue group modulo n), i.e. its adjacency matrix is circulant.

Let D be a set of positive, proper divisors of the integer $n > 1$. Define the gcd-graph $X_n(D)$ to have vertex set $\mathbb{Z}_n = \{0, 1, ..., n-1\}$ and edge set $E(X_n(D)) = \{\{a, b\}; a, b \in Z_n, gcd(a-b, n) \in D\}$. It was proven that a circulant graph is integral if and only if it is a gcd-graph.

It would be interesting to study chromatic, clique number and some other properties of Integral Circulant Graphs.

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On Automorphism Groups of Graph Powers

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An automorphism of a graph G is a permutation π of the vertex set of G such that, for any vertices u and v of G, we have $\pi(u) \sim \pi(v)$ if and only if $u \sim v$. The set of all automorphisms of G, with the operation of composition of permutations, is a permutation group on $V(G)$, denoted by $Aut(G)$. The automorphism group of a graph characterize its symmetries, and is therefore very useful in determining certain of its properties.
The $k-$*power of* G, denoted by G^k, is defined on the vertex set $V(G)$ by adding edges joining any two distinct vertices x and y with distance at most k.
In this paper, we investigate the automorphism groups of the graph powers and show that $Aut(G^n) \cong Aut(G)$, provided that G is a connected graph with sufficiently large girth and without terminal vertices. In addition we show that $Aut(G^2) \cong Aut(G)$, when $diam(G) \geq 4$ and $g(G) \geq 7$.

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Large Chromatic Number and Ramsey Graphs

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The clique number, the chromatic number, and the independence number of a graph $G = (V, E)$ is denoted by $\omega(G)$, $\chi(G)$, and $\alpha(G)$, respectively. Intuitively, large chromatic number must imply large cliques. Define

$$Q(n, c) := \min\{\omega(G) : |V(G)| = n \text{ and } \chi(G) = c\}.$$

It is obvious that $Q(n, n) = n$, it is not difficult to show that $Q(n, n-1) = n-1$ $(n \geq 2)$ and that $Q(n, n-2) \leq n-3$. Biró determined $Q(n, n-k)$ for $k \leq 6$, whenever n is sufficiently large.

Based on these values he conjectured that if n is large enough, then $Q(n, n-k) = n - 2k + \lceil k/2 \rceil$. He also showed $Q(n, n-k) \geq n - 2k + 3$ for $k \geq 5$ and n is large enough. On the other hand, Jahanbekam and West observed that $Q(n, n-k)$ is at most the conjectured value whenever $n \geq 5k/2$ and they conjecture that this threshold on n is both sufficient and necessary for equality.

In this note we give an exact formula for $Q(n, n-k)$ for $n \geq 2k + 3$ using Ramsey graphs. Our results also disprove the above conjectures.

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Isoperimetry Problems on Trees and its Application to Image Segmentation

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Isoperimetric (Cheeger) constant is considered as a geometric tool measuring the connectivity of a geometric object. Many relationship with the other parameters such as the second eigenvalue of Laplace operator (spectral gap) has been proved. Discrete analogue of Cheeger constant has been recently at the center of attention. The significance of the isoperimetric problem is not only due to its relation to the central theoretical concepts (e.g. expander graphs, rapid mixing and balanced cut), but also its varied real world applications (e.g. image segmentation, network reliability and graph clustering).

This talk is aimed to investigate some computational aspects of different isoperimetry problems on weighted graphs and specially on weighted trees. Particularly, we will report that for every fixed number $k > 1$, computing the kth isoperimetric number is an NP-hard problem for planar bipartite weighted graphs and for simple (unweighted) graphs, as well. Also, we report that a linear time algorithm is available to computing the kth isoperimetric number on a weighted tree. We apply this algorithm to introduce a new clustering method and deploy it in image segmentation.

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Longest Paths in Grid Graphs

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A grid graph is a finite node-induced subgraph of the infinite two-dimensional integer grid. It is rectangular if its set of nodes is the product of two intervals. The longest path problem is a well-known NP-hard problem and so far it have been solved polynomially only for few classes of graphs. In this paper, we give a linear-time algorithm for finding a longest path between any two given vertices in a rectangular grid graph.

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Weight Choosability of Graphs

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A total weight colouring of a (di)graph is a weight assignment from a set R to the vertices and from a set S to the edges such that the accumulated weights at the vertices yield a proper vertex colouring. If such an assignment exists, we say the graph is (R,S)-total weight colourable. One may also consider the (k, k')-total weight choosability of a graph, that is, the list version of the previous definition where the lists assigned to the vertices and edges have sizes k and k', respectively.

It is conjectured that every graph is (2,2)-total weight choosable and every graph with no isolated edge is (1,3)-total weight chooable. In this talk, I will present an overview of the progress made towards these conjectures.

Part of this work is joint work with Reza Naserasr, Mike Newman, Ben Seamone, and Brett Stevens.

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On $Q-$spectrum of Graphs and their Betweenness

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Associated with the graph G, the *signless Laplacian* matrix is defined as $Q(G) = A(G) + D(G)$ in which $A(G)$ and $D(G)$ are adjacency and degree matrix of G, respectively. Also, let $\sigma_{uv}(w)$ denote the number of shortest paths from vertex u to vertex v that go through w, and σ_{uv} is the total number of shortest paths from u to v. Then the *betweenness* of vertex w is $B_w = \sum_{u,v \neq w} b_w(u,v)$, where $b_w(u,v) = \frac{\sigma_{uv}(w)}{\sigma_{uv}}$ for $u, v, w \in V(G)$.

In this talk, we want to relate some results about $Q-$spectrum of a graph and its betweenness.

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On the Sum of All Distance Between Vertices of Graphs

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Let G be a connected graph. $\eta(G) = Sz(G) - W(G)$, The Wiener index $W(G)$ is defined as the sum of all distances between vertices of G. Suppose $e = uv$. Define $n_u(e)$ to be the number of vertices of G lying closer to u than v and $n_v(e)$ is defined analogously. The Szeged index of G is defined as $Sz(G) = \sum_{e=uv\in E(G)} n_u(e)n_v(e)$. Suppose $\eta(G) = Sz(G) - W(G)$, a well-known result of Klavžar, Rajapakse and Gutman states that $\eta(G) \geq 0$ and by a result of Dobrynin and Gutman $\eta(G) = 0$ if and only if each block of G is complete. In this paper a path-edge matrix for the graph G is presented by which it is to possible classify the graphs in which $\eta(G) \leq 5$ and $\eta(G) \neq 1, 3$. It is also proved that there is not a graph G with the property that $\eta(G) = 1$ or $\eta(G) = 3$. Finally, it is proved that for a given positive integer $k, k \neq 1, 3$, there exists a graph G with $\eta(G) = k$.

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Roman Bondage Number of Graphs

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A *Roman domination function* on a graph $G = (V(G), E(G))$ is a labeling $f : V(G) \to \{0, 1, 2\}$ satisfying the condition that every vertex with label 0 has at least a neighbor with label 2. A *Roman domination number* $\gamma_R(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over such functions. A *Roman bondage number* $b_R(G)$ of G is the minimum cardinality of all sets $E \subseteq E(G)$ for which $\gamma_R(G - E) > \gamma_R(G)$. In this talk we present previous results of Roman bondage number briefly, and introduce some new results.

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On the Zagreb Index of Some Dendrimer-Nanostar Garphs

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Dendrimers have attracted many theoretical and practical attentions. They are interested unusually in transport and have optical properties. They are also promising candidates for a wide variety of applications, including electro-optical organic light sources and nanoscale optical sensors. For a graph G the first Zagreb index $M_1(G)$ is equal to the sum of the sums of the degrees of pairs of adjacent vertices of the graph. The second Zagreb index $M_2(G)$ is equal to the sum of the products of the degrees of pairs of adjacent vertices of the graph. the Zagreb index is one of the important topological indices for describing the boiling point of the molecules. In this paper we present some methods for computing the Zagreb indices of some dendrimer nanostars.

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Connectivity Measures on Graphs, Digraphs and Hypergraphs: Tree-width, D-width, and Hyper D-width

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Measures of global connectivity like tree-width, clique-width and branch-width play a key role in the area of fixed parameter tractability and could be viewed as an attempt in extending trees. In this talk, we give a brief overview of the state of the art developments on the subject and introduce some relevant works toward generalizing the concept to directed graphs and hyper graphs. This contains the speaker's work on D-Width and Hyper-D-Width.

Social Networks and Graph Theory

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The term "social network" is defined as a structure of social entities connected to other social entities through various types of relations, such as friendship. In a social network, the social environment can be expressed as patterns or regularities in relationships among interacting units. To most people the words social network, refer to web-based Online Social Networks (OSNs) such as Facebook and Twitter. OSNs facilitate direct communication between users. In recent years, a considerable amount of research has been done on analysis of OSNs. However, the study of social networks goes back far father than these modern OSNs. Indeed, among researchers who study networks, sociologists have perhaps the longest and best established tradition of quantitative, empirical work. Previous research has revealed that there exist some common structural properties such as small-world, scale-free, and high clustering attributes in many social networks. These attributes have important effects on the dynamical properties of networks.

In this talk we introduce the basic theoretical tools used to describe and analyze social networks, which come from graph theory, the branch of mathematics that deals with networks. Graph theory is a large field containing many results and we describe a fraction of those results here, focusing on the ones most relevant to the study of social networks.

No-Where Zero Flows and Flow Polynomial

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While (vertex) colorings and chromatic polynomial have been extensively studied over the past 150 year, the concept of flows and flow polynomial are relatively new and are not as well-known. For planar graphs, the two concepts offer a duality through which these ideas are interchangeable. This talk is about nowhere-zero flows and the flow polynomial, which counts the number of nowhere-zero flows of a graph. Following the definitions and properties of the flow polynomial, some examples and calculations are used to illustrate and develop the arithmetic of the flow polynomial. Furthermore, the flow polynomials of some classes of graphs are computed. The discovery of infinite families of flow-equivalent amallamorphs of certain graphs can be presented if time permits.

On the Information Flow Capacity Bounds in Directed Acyclic Graphs

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A key challenge in Network Information Theory is to calculate information flow capacity of directed acyclic graphs representing real world communication network. Capacity bounds in the case of single-source single-destination networks are typically evaluated through solving an optimization formulation. Such optimal solution, which determines the capacity bounds, converges to the network minimum cut value. It has been shown that Ford-Fulkerson algorithm [1] reaches this optimal solution in O(VE2) order of complexity [2].

Recently, Ahlswede et al. [3] have proved that network flow capacity in single-source multi-destination graphs approaches to multicast capacity using in-network additional computation. Subsequently, many algorithms with different complexity orders had been proposed to calculate network capacity aiming to evaluate sources rate region.

Flow capacity in the case of multi-source multi-destination networks is not a function of cut-set value and varies between networks with same minimum cut but different topologies. The idea for overcoming this challenge is to derive upper and lower bound graphs of an ordinary network. In this presentation, we address challenges of deriving upper bound graph of multi-source multi-destination networks. Moreover, we propose an algorithm to evaluate upper bound graph that tries to asymptotically reach the actual value of network capacity and sources rate region.

It is a joint work with Shirin Jalali

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On the Harmonious Colouring of Graphs

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Let G be a simple graph of order n and size m with maximum degree of Δ. A vertex harmonious colouring of G is a proper vertex colouring such that each pair of colours appears together on at most one edge. The harmonious chromatic number $h(G)$ is the least number of colours in such a colouring. Let $L(G)$ be the line graph of G. We define $h'(G)$, the chromatic edge harmonious colouring of G, as $h'(G) = h(L(G))$. In this talk some relations between $h(G)$ and $h'(G)$ have been investigated. It is shown that if $m > \frac{n(\sqrt{n}+1}{2}$, then $h(G) \leqslant h'(G)$. Among other results, an upper bound $h'(G) \leqslant 4\Delta h(G)$ is obtained.