ELectronics EXpress

IEICE Electronics Express, Vol.8, No.6, 318–324

Non- Sensitive Matrix Pencil method against mutual coupling

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Abstract: A new Matrix Pencil (MP) method including mutual coupling effects based on a Uniform Linear Array (ULA) is presented. By setting a group of elements as auxiliary on each side of the ULA, it can accurately estimate the Direction of Arrival (DOA) using a single snapshot of data and the effect of mutual coupling can be eliminated by the inherent mechanism of the proposed method. Theoretical analysis and simulation results demonstrate the effectiveness of the proposed algorithm.

Keywords: matrix pencil, mutual coupling, DOA, ULA

Classification: Microwave and millimeter wave devices, circuits, and systems

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DOI: 10.1587/elex.8.318
Received December 15, 2010
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1 Introduction

The study of adaptive antennas in radar and wireless communications has been an active research topic for several decades. Furthermore, DOA estimation is an important feature of adaptive antenna arrays. MUSIC, ESPRIT [1] and MP [2, 3, 4] are some popular conventional methods of DOA estimation.

The problem is that these adaptive DOA estimation algorithms are sensitive to array Mutual Coupling (MC) and such an effect needs to be removed in order to achieve a high performance in an actual system [5]. Many efforts have been made to reduce or compensate for this effect on Array [5, 6, 7, 8, 9, 10].

Some literatures have referred that using auxiliary elements can reduce the effect of MC [11, 12]. In this paper a simple solution based on MP algorithm is presented in order to combat the effect of MC. Because the matrix pencil method is based on the spatial samples of the data and the analysis is done on a snapshot-by-snapshot basis, therefore non-stationary environments can be handled easily [2]. It is proposed that the array elements on the boundary of ULA should be of auxiliary elements, and only use the output of the rest array to estimate the DOAs. Through this process, the MP algorithm can be directly applied for DOA estimation.

DOI: 10.1587/elex.8.318
Received December 15, 2010
Accepted February 14, 2011
Published March 25, 2011

2 Matrix pencil method

Consider a linear adaptive array consisting of N equally spaced elements with the spacing of d. The array receives M narrow band signals from directions $\theta_1, \theta_2, \ldots, \theta_M$, as shown in Fig. 1. All incident fields propagate perpendicularly to the z-direction.

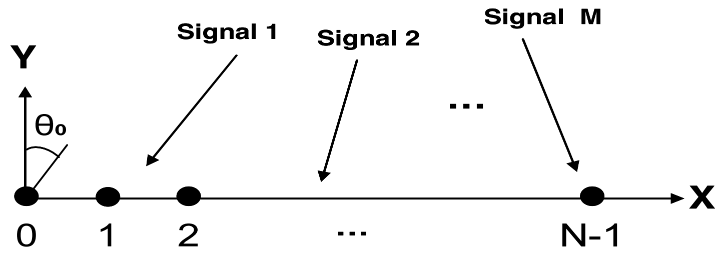

Fig. 1. ULA with N elements

$\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]$ is the vector of noise free voltages measured at the feed point of the antenna elements of the ULA which can be modeled by a sum of complex exponentials, i.e.,

$$y(p) = x(p) + n(p) = \sum_{m=1}^{M} R_m z_m^p + n(p) \quad , p = 0, 1, \ldots, N-1 \tag{1}$$

where R_m is the complex amplitude of m th signal, $n(p)$ is the additive noise and $z_m = \exp(j\frac{2\pi}{\lambda} d \sin\theta_m)$, $m = 1, 2, \ldots, M$. The objective is to find the best estimates for θ_m. Let us consider the matrix Y, which is obtained directly from $x(p)$.

$$\mathbf{Y} = \begin{bmatrix} x(0) & x(1) & \ldots & x(L-1) \\ x(1) & x(2) & \ldots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \ldots & x(N-1) \end{bmatrix}_{(N-L+1)\times(L)} \tag{2}$$

where L is referred to as the pencil parameter. The pencil parameter is very useful in eliminating some effects of noise in the data. Two matrices of $\mathbf{Y_1}$ and $\mathbf{Y_2}$ are defined. $\mathbf{Y_1}$ is obtained from $\mathbf{Y}$ by deleting the last row and $\mathbf{Y_2}$ is obtained from Y by deleting the first row. One can also write

$$\mathbf{Y_1} = \mathbf{Z_1 R Z_2} \quad , \quad \mathbf{Y_2} = \mathbf{Z_1 R Z_0 Z_2} \tag{3}$$

Where

$$\mathbf{Z_1} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ z_1 & z_2 & \ldots & z_M \\ \vdots & \vdots & & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \ldots & z_M^{N-L-1} \end{bmatrix}_{(N-L)\times M} \tag{4}$$

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Received December 15, 2010
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$$\mathbf{Z_2} = \begin{bmatrix} 1 & z_1 & \dots & z_1^{L-1} \\ 1 & z_2 & \dots & z_2^{L-1} \\ \vdots & \vdots & & \vdots \\ 1 & z_M & \dots & z_M^{L-1} \end{bmatrix}_{M \times L} \tag{5}$$

$$\mathbf{Z_0} = diag[z_1, z_2, \dots, z_M] \tag{6}$$

$$\mathbf{R} = diag[r_1, r_2, \dots, r_M] \tag{7}$$

where diag[$\cdot$] represents a $M \times M$ diagonal matrix. Consider the matrix pencil:

$$\mathbf{Y_2} - \lambda \mathbf{Y_1} = \mathbf{Z_1 R}\{\mathbf{Z_0} - \lambda \mathbf{I}\}\mathbf{Z_2} \tag{8}$$

where $\mathbf{I}$ is the $M \times M$ identity matrix. It has been shown in [3], that z_m will be the eigenvalues of:

$$\mathbf{Y_1^+ Y_2} - \lambda \mathbf{I} \tag{9}$$

where $\mathbf{Y_1^+}$ is the Moore-Penrose pseudo-inverse of $\mathbf{Y_1}$. The DOA is obtained by:

$$\theta_m = \sin^{-1}(\mathrm{Im}(\log z_m)/\pi d) \tag{10}$$

In the presence of noise, some pre-filtering needs to be done. For efficient noise filtering, the parameter L, is chosen between $N/3$ and $N/2$ [13]. For these values of L, the variance in the parameters z_m, due to noise, has been found to be minimum. Noise reduction can be performed via the Singular Value Decomposition (SVD) [13]. First, $\mathbf{Y}$ is decomposed using the SVD, yielding:

$$\mathbf{Y} = \mathbf{U} \quad \mathbf{\Sigma} \quad \mathbf{V^H} \tag{11}$$

Here, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, composed of the eigenvectors of $\mathbf{YY^H}$ and, $\mathbf{Y^H Y}$, respectively. $\mathbf{\Sigma}$ is the singular values of $\mathbf{Y}$. For simplicity, it is assumed that the number of signals is known in this paper. After SVD of data matrix $\mathbf{Y}$ is computed, the matrix space is divided into two subspaces, signal subspace and noise subspace. Here, the matrices $\mathbf{Y_1}$ and $\mathbf{Y_2}$ are constructed from the signal subspace matrix. So, the "filtered" matrix $\mathbf{\tilde{U}}$ is constructed. It consists of the first M columns of $\mathbf{U}$ and the right-singular vectors from $M + 1$ to L, corresponding to the small singular values, are discarded. Therefore

$$\mathbf{Y_1} = \mathbf{\tilde{U}_1} \quad \mathbf{\tilde{\Sigma}} \quad \mathbf{V^H} \quad , \quad \mathbf{Y_2} = \mathbf{\tilde{U}_2} \quad \mathbf{\tilde{\Sigma}} \quad \mathbf{V^H} \tag{12}$$

where $\mathbf{\tilde{U}_1}$ and $\mathbf{\tilde{U}_2}$ are obtained by deleting the last and the first row of $\mathbf{\tilde{U}}$, respectively and $\mathbf{\tilde{\Sigma}}$ is obtained from the M columns of $\mathbf{\Sigma}$ corresponding to the M dominant singular values. It can be shown [3] that, for the noisy case, the eigenvalues of the following matrix is the solution for determining of z_m:

$$\mathbf{\tilde{U}_1^+ \tilde{U}_2} - \lambda \mathbf{I} \tag{13}$$

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Received December 15, 2010
Accepted February 14, 2011
Published March 25, 2011

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3 The proposed algorithm

Most DOA estimation algorithms, including MP assume an ideal, linear array of isotropic sensors. Unfortunately, such an ideal sensor is obviously not realizable. A practical antenna array is composed of the elements of some physical size. The elements sample and reradiate incident fields, causes MC. MC severely degrades the accuracy of the DOA estimator [5]. Any implementation of DOA estimation requires a compensation for the MC. In this paper, a non-sensitive MP algorithm is presented against MC. In order to nullify the effect of MC, the array sensors on the boundary of ULA are set to be auxiliary sensors and only the output of the rest array are used to estimate the DOAs. Utilizing this process, the MP algorithm can be directly applied for DOA estimation.

Assuming that C denotes the MC matrix of the ULA, the array's output can be expressed as $\mathbf{x_c} = \mathbf{Cx}$. Where $\mathbf{x_c} = [x_c(0), x_c(1), \ldots, x_c(N-1)]$ denotes the received signal vector in the presence of MC. It has been shown in [14] that the coupling between neighboring elements of a ULA is almost the same and the magnitude of the coupling parameters decreases very fast by increasing the sensor spacing. Essentially, the MC coefficient between two far apart elements can be approximated to zero. Thus, it is often sufficient to consider the ULA coupling model with only finite non-zero coefficients, and a banded symmetric Toeplitz matrix can be used as a model for the MC. So, the MC is considered among $P+1$ closest sensors only [12, 15]. The corresponding $N \times N$ matrix C is given by:

$$\mathbf{C} = toeplitz\{[c_0, c_1, \ldots, c_P, 0, \ldots, 0]\} \quad 0 < |c_P| < \ldots < |c_1| < c_0 = 1 \quad (14)$$

where the symbol Toeplitz{v} denotes the symmetric Toeplitz matrix constructed by the vector $\mathbf{v}$. In order to eliminate the effect of the MC, the sensors on the boundary of the ULA are set to be auxiliary sensors. Let us consider the matrix $\mathbf{Y_c}$, which is obtained from the output of the middle $N-2P$ in ULA:

$$\mathbf{Y}_c = \begin{bmatrix} x_c(P-1) & x_c(P) & \ldots & x_c(L+P-2) \\ x_c(P) & x_c(P+1) & \ldots & x_c(L+P) \\ \vdots & \vdots & \ddots & \vdots \\ x_c(N-L-P+1) & x_c(N-L-P+2) & \ldots & x_c(N-P) \end{bmatrix}_{(N-L-2P+3)\times(L)} \quad (15)$$

Let us define:

$$\mathbf{C_1} = \begin{bmatrix} c_{P-1} & \ldots & c_1 & 1 & c_1 & \ldots & c_{P-1} & 0 & 0 & \ldots & 0 \\ 0 & c_{P-1} & \ldots & c_1 & 1 & c_1 & \ldots & c_{P-1} & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ldots & \ddots & & \ddots & \ldots & & & \vdots \\ 0 & \ldots & 0 & 0 & c_{P-1} & \ldots & c_1 & 1 & c_1 & \ldots & c_{P-1} \end{bmatrix}_{(N-L-2P+2)\times(N-L)} \quad (16)$$

Similarly, two matrices of $\mathbf{Y_{c1}}$ and $\mathbf{Y_{c2}}$ are defined. $\mathbf{Y_{c1}}$ and $\mathbf{Y_{c2}}$ are obtained from $\mathbf{Y_c}$ by deleting the last row and the first row. One can also

DOI: 10.1587/elex.8.318
Received December 15, 2010
Accepted February 14, 2011
Published March 25, 2011

write

$$\mathbf{Y_{c1}} = \mathbf{C_1 Y_1} \quad , \quad \mathbf{Y_{c2}} = \mathbf{C_1 Y_2} \tag{17}$$

Using of (3), the following can be obtained:

$$\mathbf{Y_{c1}} = \mathbf{C_1 Z_1 R Z_2} \quad , \quad \mathbf{Y_{c2}} = \mathbf{C_1 Z_1 R Z_0 Z_2} \tag{18}$$

Similarly, the parameters z_m may be found as the generalized eigenvalues of the matrix pair $\{\mathbf{Y_{c1}}; \mathbf{Y_{c2}}\}$.

$$\mathbf{Y_{c1}^{+} Y_{c2}} - \lambda \mathbf{I} \tag{19}$$

Hence, for the noisy case, similar to (13) the eigenvalues of the following matrix is the solution for determining of z_m:

$$\mathbf{\tilde{U}_{c1}^{+} \tilde{U}_{c2}} - \lambda \mathbf{I} \tag{20}$$

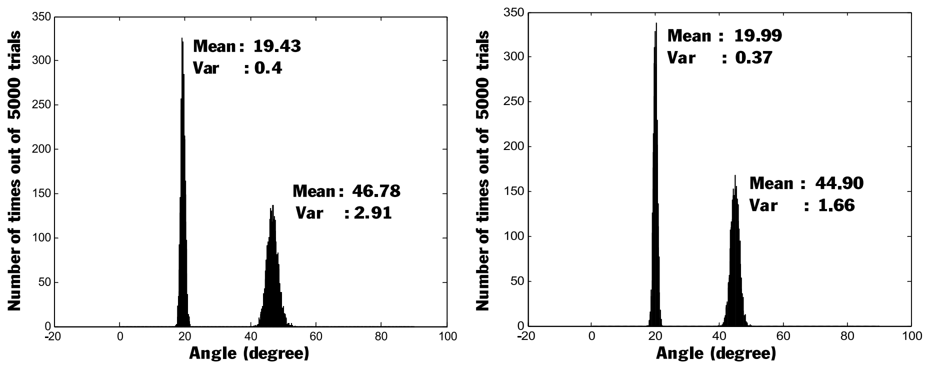
where $\mathbf{\tilde{U}_{c1}}$ and $\mathbf{\tilde{U}_{c2}}$ are obtained by deleting the last and the first row of $\mathbf{\tilde{U}_c}$, respectively.

4 Numerical simulations

In this section, eleven z-direction parallel identical dipoles are used, which are equally spaced with the spacing of 0.5λ, where λ is the wavelength. Each dipole is 0.5λ long and $\lambda/200$ in radius. All the elements are loaded with a terminal load of $Z_L = 50\,\Omega$. The Method of Moments (MOM) is used to accurately model the interactions between antenna elements and the MC between three closest sensors is considered ($P = 2$). The array receives two signals from 20° and 45°. The MP algorithm and the proposed algorithm use only a single snapshot. Table I shows the accuracy of DOA estimation using the new propose algorithm in the presence of MC.

Table I. Comparing Accuracy of MP and the Proposed Algorithm

	The MP method in the presence of MC	The proposed algorithm
Signal 1	19.45°	20°
Signal 2	46.76°	45°



a) MP algorithm in the presence of MC b) the proposed algorithm in the presence of MC

Fig. 2.

DOI: 10.1587/elex.8.318
Received December 15, 2010
Accepted February 14, 2011
Published March 25, 2011

In the next example, the noisy data are used. The signal-to-noise ratio (SNR) was set at 20 dB. The optimum value for the pencil length, L, is chosen to be 4 for efficient noise filtering. 5000 independent trials are used. Fig. 2 shows histograms of MP and non-sensitive MP estimation in the presence of MC. As can be seen, using the proposed algorithm, the bias and variance are less than the conventional MP method and very close to ideal.

5 Conclusion

The problem of DOA estimation is studied for the ULA in the presence of MC. By setting the sensors on the boundary of the ULA as auxiliary sensors, the robustness of the proposed algorithm is proved to be against sensor coupling. Without using MC coefficient calculation, this method can accurately estimate the DOAs.

Acknowledgement

The authors want to acknowledge the Iran Telecommunication Research Centre (ITRC) for their kindly supports.

DOI: 10.1587/elex.8.318
Received December 15, 2010
Accepted February 14, 2011
Published March 25, 2011