# Cosecant Squared Pattern Synthesis for Reflector Antenna using a Stochastic Method

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***Abstract*** **—** A novel method based on invasive weed optimization (IWO) algorithm is described for synthesizing a point source doubly curved reflector antenna to produce a shaped beam in one plane and narrow beam in the perpendicular plane. The whole reflector body can be created by determining two basic curves. The central vertical curve is approximately responsible for producing a shaped beam in the vertical plane. It can be expressed by a well-defined function, and the function coefficients will compose a solution space to be explored by IWO. The transverse section curve is known to be a parabola for producing a narrow beam in that plane. The validity of the proposed method is verified by an example of shaped reflector antenna with cosecant squared pattern and comparing it with previously used methods. The simulation results based on physical optics (PO) further prove the validity and versatility of this technique for solving reflector synthesize problems.

***Index Terms*** **—** Cosecant squared pattern, doubly curved reflector, IWO.

## I. INTRODUCTION

Antennas with the cosecant squared pattern in the elevation plane and pencil beam in the azimuth plane are widely used in surveillance-search radar systems. In comparison with phased array antennas, reflector antennas don't have the calibration and implementation problems. Doubly curved reflector antenna is a classical antenna for this purpose. The synthesis procedure based on geometrical optics (GO) is described in details by many authors [1-4]. Different types of configuration [4-6] and analysis methods for computing its farfield pattern has been presented in [7-11]. The design procedure based on GO, is inflexible to achieve extra desired features such as low sidelobe level or low ripples at the shaped beam region for antenna performance. By knowing that doubly curvature antenna can be synthesized to produce a shaped beam in one of the principle planes, and after the success of optimization algorithms in antenna problems, the authors are guided to create a new synthesis method for doubly curved reflector antennas.

Optimization algorithms in solving antenna problems are always employed for improving antenna performance for example, [12, 13]. In this paper, the authors try to employ an optimization algorithm to create a synthesis method for doubly curved reflector antennas. The algorithms could find the antenna structure independently and by defining a proper goal both synthesize and optimization can be done. Describing central section curvature of reflector surface using a few parameters, and finding the parameters to reach the antenna desired performance, is fundamental to the proposed method. This method uses IWO as a tool to perform the idea, because IWO is an efficient and robust optimization algorithm method in finding global minima. It was first extracted by [14] and successfully has been used in antenna design problems [15-18]. In this paper, the basic concept of GO synthesis is reviewed briefly. Then, the proposed method is described in details so that the procedure can be fully understood and in the next step, a reflector is designed and compared with the reflector designed based on the previous GO based method.

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## II. THEORETICAL ANTENNA DESIGN

Totally for reflector antenna problems and propagation problems [19], geometrical optics analysis can be more useful and efficient than other methods. For doubly curved reflector antennas, the conventional way to synthesize a shaped pattern is based on GO also [1-3]. In this section, we will only review the basic concept of GO synthesis procedure and extract design formulas.

Doubly curved reflector antenna contains two main sections. The central vertical section of the reflector must be designed to provide the desired elevation shaped pattern. The transverse section is required to be a parabola for focusing feed rays in the transverse or azimuth plane to produce the narrow beam in that plane.

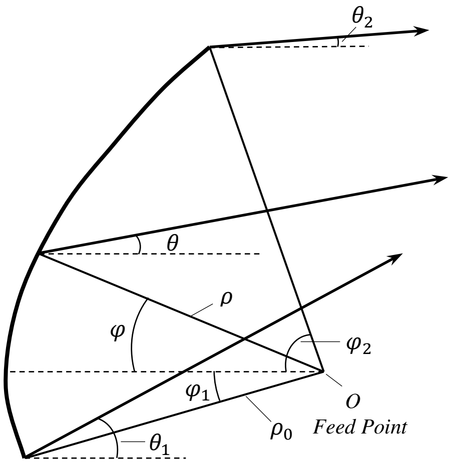


Fig. 1. Central vertical curve parameters.

### A. Central vertical section

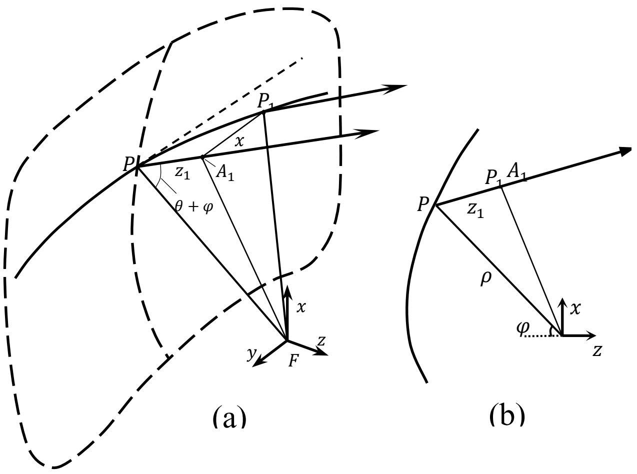
In order to shape the reflector antenna's surface to have a prescribed elevation pattern, the proper central vertical section must be found. It means that the central vertical section has the dominant effect on elevation pattern. The central curve is illustrated in Fig. 1 [4]. Consider that the phase center of feed is located at the origin $O$. $\varphi$ is the angle of incident ray with respect to $z$ and $\theta$ is the angle of reflected ray. The angle between the incident and reflected rays, $\sigma$, can be determined as follows:

$$\sigma = \theta + \varphi, \tag{1}$$

$\rho$ is the distance from origin to central curve. Clearly $\theta$ and $\rho$ are functions of $\varphi$ (i.e. $\sigma(\varphi)$ and $\rho(\varphi)$). The differential equation of the central curve is [2]:

$$\frac{d\rho}{\rho d\varphi} = \tan(\frac{\sigma}{2}), \tag{2}$$

In this equation, $\sigma(\varphi)$ and $\rho(\varphi)$ are both unknown. If we know $\sigma(\varphi)$, $\rho(\varphi)$ can be determined.



(a) (b)

Fig. 2. Ray path in transverse section curve: (a) 3D view, (b) side view.

### B. Transverse section of reflector

The transverse sections of the surface are determined by the requirement that the reflector is to convert a spherical wave into a cylindrical wave. For this purpose, all reflected rays from the reflector in the direction $\theta$ need to be parallel. Therefore, as illustrated in Fig. 2 the following condition must be met [4]:

$$\overline{FP_1} = \overline{FP} + \overline{PA_1}, \tag{3}$$

this leads to

$$\left[\rho^2 + z_1^2 - 2\rho z_1 \cos(\varphi+\theta) + y^2\right]^{\frac{1}{2}} = \rho + z_1, \tag{4}$$

The latter equation gives transverse parabolas of the reflector surface. The desired surface can be achieved by combining both vertical and transverse curves of reflector.

To determine the reflector central curve using (2), it is necessary to derive a more useful differential equation based on energy balance principles of GO.

Let $I(\varphi,\psi)$ be the incident energy of feed. $I(\varphi,0)d\psi d\varphi$ is the power incident on a central element of the reflector. This power is reflected in a wedge with a $\rho d\psi$ width and with a wedge angle $d\theta$. If $P(\theta)$ is the reflected energy of antenna, the following relation can be written based on the GO principle of energy balance [1-4]:

$$P(\theta)\rho d\psi d\theta = I(\varphi) d\psi d\varphi, \quad (5)$$

Taking logarithmic derivatives with respect to $\varphi$ of Eq. (5) and substituting from Eq. (2), we get instead the differential equation which for cosecant squared pattern with

$$P(\theta) = \csc^2(\theta), \theta_1 < \theta < \theta_2 \text{ when } \varphi_1 < \varphi < \varphi_2, \quad (6)$$

it is

$$\frac{d^2\theta}{d\varphi^2} + \left[\tan\left(\frac{\varphi-\theta}{2}\right) - \frac{I'(\varphi)}{I(\varphi)}\right]\frac{d\theta}{d\varphi} - \left(2\cot\theta\right)\left(\frac{d\theta}{d\varphi}\right)^2 = 0. (7)$$

This equation couldn't be solved analytically and has to be discretized and solved numerically. Besides, a proper guess for the initial value of $d\theta/d\varphi$ is required to solve this equation [1]. Finding the proper initial value for $d\theta/d\varphi$ needs an exhausting trial-and-error and different values must be examined to achieve the desired farfield pattern.

After finding $\theta(\varphi)$, $\sigma(\varphi)$ and $\rho(\varphi)$ will be clear. By using Eq. (2) and Eq. (4), whole reflector body is achieved.

## III. SHAPED REFLECTOR SYNTHESIS USING IWO

Recently, employing optimization algorithms, such as genetic algorithm, particle swarm optimization, ant colony, and etc., for solving an antenna optimization problem is very common. Another algorithm is the invasive weed optimization (IWO) which was first proposed by Mehrabian and Lucas [14]. IWO is an effective and robust algorithm to find global minima as it has been shown in [14]. According to [14, 18], the algorithm process can be summarized as follows:

1. A finite number of seeds spread out randomly on the search area.
2. They grow to flowering weeds and produce seeds. The number of reproduced seeds of each weed depends on its own fitness and better fitness permits more seeds to be reproduced. However, the maximum number of seeds is limited.
3. These reproduced seeds disperse over the search area around their parent weeds. The random dispersion has a normal distribution with mean equal to zero but varying variance (spatial dispersal). The standard deviation (SD) decreases in each time step of the algorithm as [14]

   $$SD_{iter} = \frac{(iter_{\max} - iter)^n}{(iter_{\max})^n}(SD_{initial} - SD_{final}) + SD_{final}, (8)$$

   where $iter$ is the number of current time step, $iter_{max}$ is the maximum number of iterations, $SD_{iter}$ is the SD at the present time step, $SD_{initial}$ and $SD_{final}$ are prescribed constant values for SD at first and last iteration, and $n$ is the nonlinear modulation index which usually is set to 3. This decreasing behavior for SD causes aggregation of seeds around better solutions.
4. There is a maximum for the number of weeds in each time step and only plants with better fitness can survive and produce seeds in the next step (competitive exclusion). The process continues until the maximum number of iterations is reached and finally the plant with the best fitness is closest to the optimal solution. The flow chart of this algorithm is represented Fig. 3. [18].

### A. Antenna synthesis using IWO

In this paper, we employ the ability of IWO for synthesizing a doubly curved reflector antenna. IWO will choose the best design depending on its defined goal. The synthesis method can define the desired pattern and reach to it while in synthesis procedure by the GO method, there is not enough flexibility in defining the desired sidelobe level or limited ripple in the shaped region. The optimization algorithm tries to find the best reflector body which its characteristics are fitted on the desired characteristics.

It was mentioned in Section 3 that the basic curves of the doubly curved reflector antenna can construct the whole body of it. It can be said that the central vertical curve approximately produces the shaped elevation pattern and transverse section is mostly a parabola to emerge feed rays parallel for generating a pencil beam in the transverse or azimuth plane.

In the synthesis procedure based on IWO, the main idea is finding a central section curve which can create a reflector body with the desired shaped pattern. Transverse sections are parabolas and don't need to be found. In order to implement the idea, we recommend the following method.

### B. Central curve synthesis using IWO

In order to determine the central curve by the optimization algorithm, at first, the curve must be expressed by a finite number of parameters. For example, the curve's function can be approximated to an $n$-th order polynomial with $n+1$ coefficient or parameters. Afterward, these parameters need to be determined properly by IWO for creating the central curve. The central curve with the parabolas which are mentioned in Eq. (4) will create the whole body of reflector. The optimization procedure has a goal which is obtaining the desired $(\csc\theta)^2$ pattern in the elevation plane.

In this paper, central curve is expressed indirectly. As it was described in Section 3, if the distribution of $\sigma(\varphi)$ between $\varphi_1$ and $\varphi_2$ is determined, $\theta(\varphi)$ and $\rho(\varphi)$ will be determined, respectively according to equations (1) and (2), and then the central vertical curve will be created. For approximating $\sigma(\varphi)$ distribution, various functions were examined to find a function with fewer parameters and good accuracy. The best choice was the 4-th order polynomial:

$$\sigma(\varphi) = p_1\varphi^4 + p_2\varphi^3 + p_3\varphi^2 + p_4\varphi + p_5, \tag{9}$$

where $p_i, i = 1,2,\ldots,5$ are the parameters which should be determined. The IWO process starts with initial random coefficients of a chosen function for $\sigma(\varphi)$. Random central curves and then random surfaces will be generated according to Eq. (4). So in each time step of the algorithm, we have a number of random reflector surfaces. To obtain the elevation radiation pattern of each reflector it has to be analyzed. Since we have the radiation pattern of the feed, the secondary pattern of reflector can be obtained by PO simulations. The obtained elevation pattern is compared with the ideal sector cosecant squared pattern. Consequently, the error (or fitness) value of weed (produced surface) is the difference between far field vertical plane pattern and a desired sector cosecant squared pattern. The optimization process continues until accomplishing a radiation pattern which is closest to cosecant squared pattern.

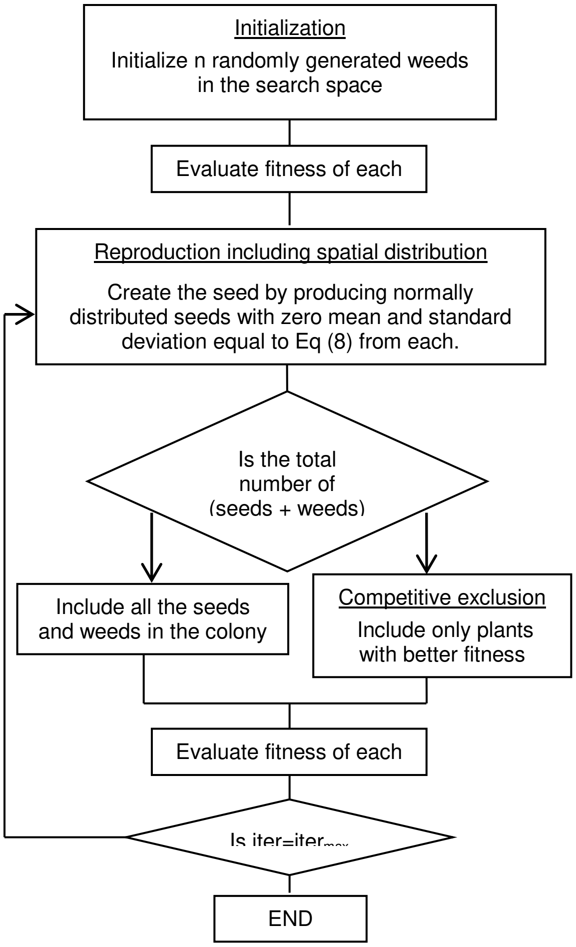


Fig. 3. A flow-chart representation of the IWO algorithm.

In this paper, the focus is on achieving a low ripple in the shaped region. To do this a proper fitness function is needed to be calculated for every produced reflector, which focuses on shaped region error. Consider that the obtained elevation pattern of each reflector varies from $-180^\circ$ to $180^\circ$. We sample $N$ point of this elevation pattern (the value of gain $G_i$) and calculate the fitness function as follows:

$$f = \frac{1}{N}\left(\sum_{\theta_1^\circ}^{\theta_2^\circ}(\left|G_i(\theta) - (\csc\theta)^2\right|)^2 + \sum_{o.w.}(\frac{1}{2}(X + |X|))^2\right), \tag{10}$$

where

$$X = \left[G_i(\theta) - (-25)\right]; \quad \& \quad o.w. \equiv \begin{cases} -180^\circ \le \theta \le \theta_0 \\ \theta_2 \le \theta \le 180^\circ \end{cases}$$

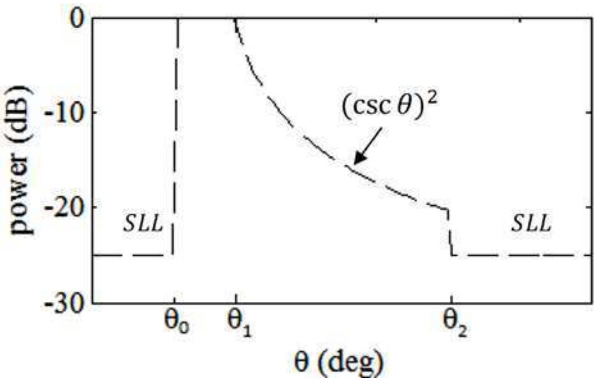


Fig. 4. Specifying $\theta_0$, $\theta_1$, and $\theta_2$ in goal boundaries.

As it is mentioned in the fitness function if the side lobe level goes upper than -25 dB, another term, which is the relative error of the unwanted and desired sidelobe level, will be added to the error due to the cosecant region. The region between $\theta_0$ and $\theta_1$ does not affect the total error.

## IV. SIMULATION RESULTS

Table 1: IWO parameter values

| Symbol | Quantity | Value |
|---|---|---|
| N | Number of initial population | 25 |
| $iter_{max}$ | Maximum number of iterations | 50 |
| Dim | Problem dimension | 5 |
| Pmax | Maximum number of plant population | 8 |
| Smax | Maximum number of seeds | 5 |
| Smin | Minimum number of seeds | 1 |
| N | Nonlinear modulation index | 3 |
| SDintial | Initial value of standard deviation | 10 |
| SDfinal | Final value of standard deviation | 0.01 |
| Lini | Initial search area | -5 to +5 |

In order to show the ability of the IWO algorithm in synthesizing this type of antenna, a detailed simulation result is presented. The $\rho_0$ is chosen to be $0.724\,m$. Therefore, the antenna dimension is $1.3\,m \times 0.88\,m$. The feed is a typical horn operating at 9.37 GHZ frequency. Since the feed is placed $15^{\circ}$ offset and has about $92^{\circ}$ 10–dB beam width, $\varphi$ varies from $-61^{\circ}$ to $31^{\circ}$. $\sigma(\varphi)$ is specified according to Eq. (9).

The 10–dB beam width of feed in the transverse plane and the positioning of the central section curve in front of feed, specifies the size of the reflector in that plane.

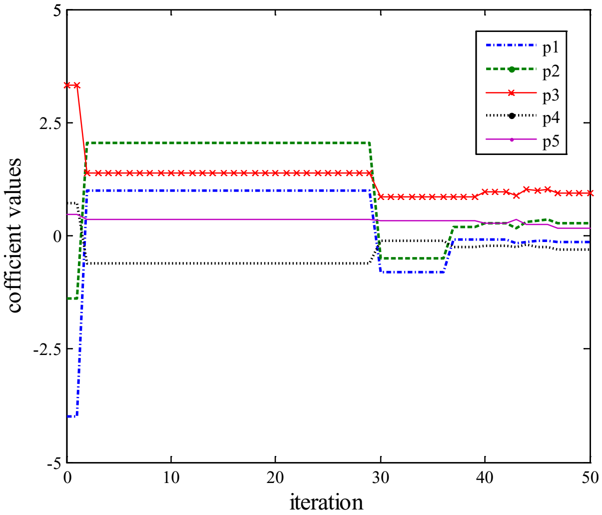


Fig. 5. $\sigma(\varphi)$'s coefficient values as the number of iteration increases.

The optimization procedure is started with the parameters' values which are tabulated in Table 1. The other related graphs for IWO are Fig. 5 and Fig. 6. Figure 5 shows the convergence process of coefficients (which are the algorithm dimensions) versus iteration. After 50 iterations, the optimum solution or lowest fitness is achieved (Fig. 6). The final values of $p_i$ , $i = 1,2, \dots ,5$ are in Table 2.

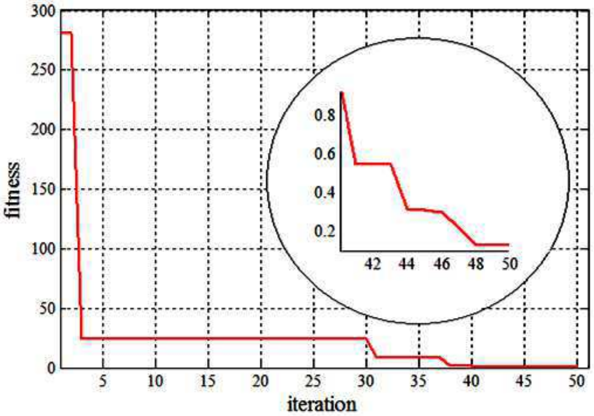


Fig. 6. Convergence diagram of optimization procedure.

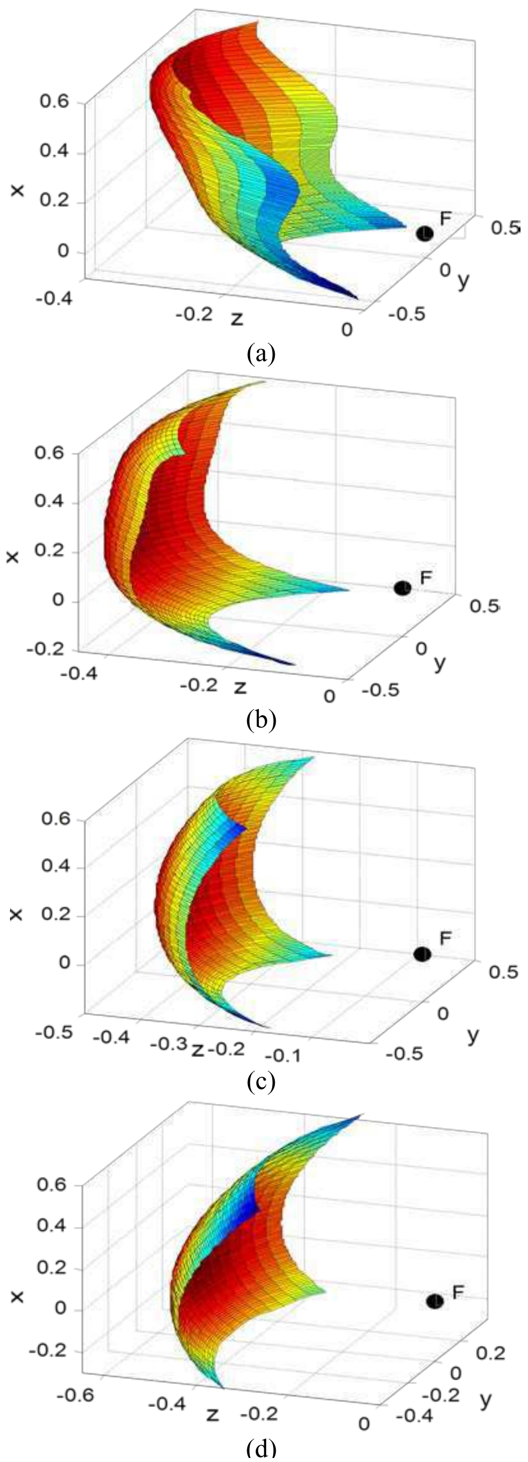


Fig. 7. Convergence procedure of reflector surfaces (3D view). (a) Initial produced reflector, (b) produced reflector surface in second iteration, (c) produced reflector surface in 25-th iteration, and (d) the final optimum reflector surface.

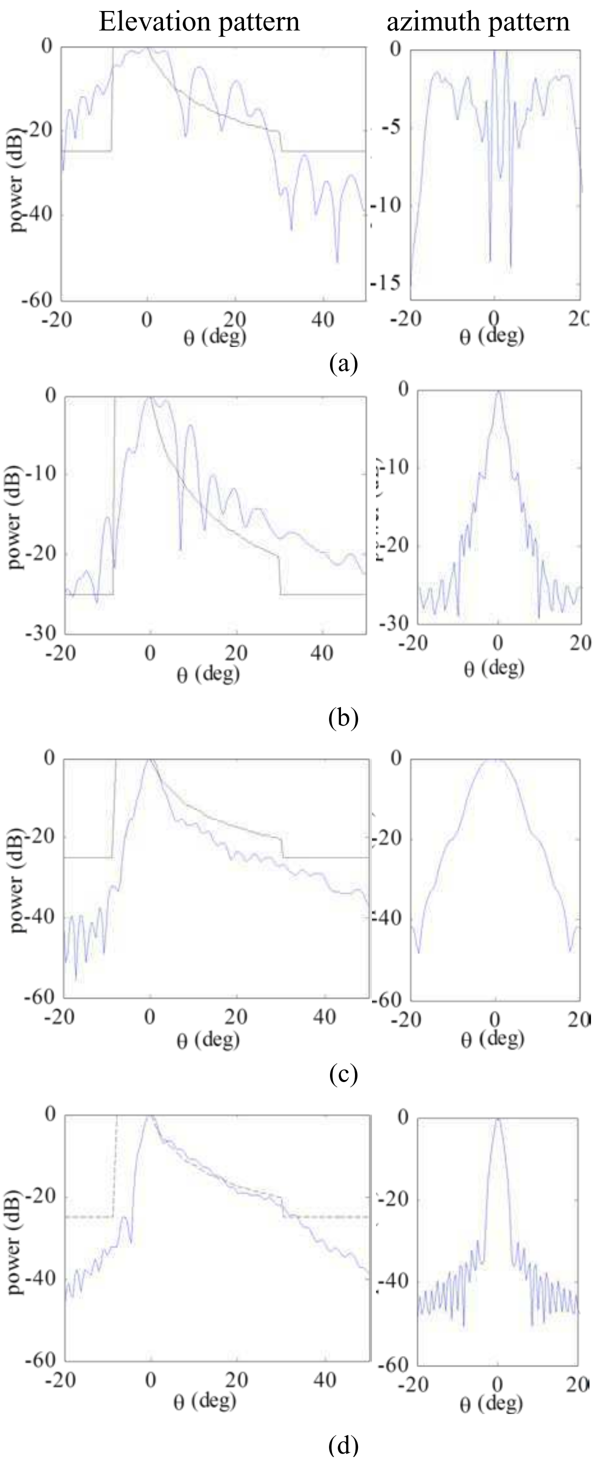
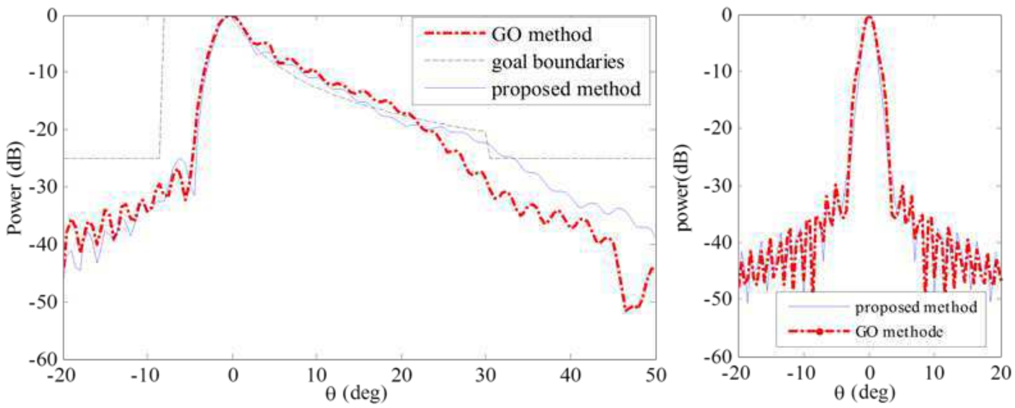


Fig. 8. Radiation pattern of produced reflector surfaces in Fig. 7. (a) Initial produced reflector, (b) produced reflector surface in second iteration, (c) produced reflector surface in 25-th iteration, and (d) the final optimum reflector surface.



(a) (b)

Fig . 9. Comparison between two methods: (a) on the ripple of the shaped region in elevation pattern, (b) pencil beam in azimuth pattern.

Table 2: $\sigma(\varphi)$'s coefficient final optimum values

| $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ |
|---|---|---|---|---|
| -0.1494 | 0.2818 | 0.9416 | -0.3041 | 0.16 |

In Fig. 7, the surfaces, which are produced during IWO iterations, are shown. It can be seen that the surfaces try to find their proper configuration as the number of iteration increases. The far filed patterns of the illustrated surfaces are shown in Fig. 8 respectively. As it is shown, the elevation pattern tries to be fitted to its defined goal as the number of iteration increases. The disorganized azimuth patterns in Fig. 8 are resulted of disorganized surfaces in their corresponding steps of iteration. In order to comprise the simulated pattern of the proposed method and the GO based method, another reflector in the equal conditions, same as feed position and reflector dimension is designed based on the GO method. It is shown in Fig. 9. Actually, the ripple in the cosecant squared band for the designed antenna using IWO, is less than 2dB, But the other pattern has the larger ripple.

## V. CONCLUSION

The invasive optimization algorithm (IWO), a novel stochastic algorithm has been successfully employed to create a flexible method to design doubly curved reflector antennas. Totally, synthesis procedures based on optimization algorithms can reduce the complexity of the problem and improve the flexibility in design goals and antenna performance. In addition, other antenna parameters such as positions, orientations, and feed excitation can be set as other optimization parameters. Furthermore, different types of desired goals can be defined to be optimized. In this paper, the focus was on a lower ripple on the cosecant squared region while other characteristics like the extremely low sidelobe level or elimination ground effect by having a sharp fall in the elevation pattern before shaped region and other definable goals can be defined and achieved. The proposed method is suggested to be employed because of more flexibility in defining a desirable destination pattern. The validity of the proposed technique is verified by designing the fixed antenna set using the two described methods. If a more accurate simulation is employed, the method can be more accurate.

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