

# A New DOA Estimation Based on Direct Data Domain Algorithm

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**Abstract**—Direct data domain least square ( $D^3LS$ ) approach has been developed so quickly for array signal processing. Because, only a single data snapshot instead of multiple needed to form a covariance matrix in conventional algorithms. Unfortunately it is generally assumed that one knows the direction of arrival (DOA) of the signal of interest (SOI) and the goal is to estimate its complex amplitude in the presence of jammer, clutter and noise. This paper proposes a novel direction of arrival (DOA) estimation algorithm based on  $D^3LS$  approach using the property of minimum of the sum of the norm of the adaptive weights that can be used as an indicator to estimate the DOA of the SOI in adaptive algorithms. Simulation results show that the proposed algorithm can estimate the DOAs of signals and its complex amplitude accurately in the presence of interference, clutter and noise.

**Index Terms**— $D^3LS$ , DOA estimation, adaptive arrays

## 1. Introduction

Adaptive antenna arrays are very useful in radar and communication systems since they can change their patterns automatically to any desired direction while automatically placing deep pattern nulls in the specific direction of interference sources. Direction of arrival (DOA) estimation is an important feature of smart antenna arrays. Several algorithms have been proposed for DOA estimation, including MUSIC, ESPRIT [1] and matrix pencil (MP) [2–4]. These signal processing algorithms have been shown to provide accurate estimates, even in moderate signal to noise (SNR) conditions. Proposed adaptive algorithms are based on the covariance matrix of the interference [5]. However, these statistical algorithms have two problems. First, they need hundreds of snapshots independent identically distributed data to estimate the covariance matrix of the interference and their performance may degrade when signals are correlated. The second is that the estimation of the covariance matrix requires the storage and processing of the secondary data [9]. This is computationally intensive, requiring many

calculations in real time. Recently, a direct data domain least squares ( $D^3LS$ ) algorithm for solving adaptive problems [6-8] has been proposed to overcome the drawbacks of a statistical technique. In this approach one adaptively minimizes the interference power while maintaining the gain of the antenna array along the direction of the signal of interest (SOI). Not having to estimate a covariance and with one snapshot makes it possible to carry out an adaptive process in real time. Unfortunately  $D^3LS$  have two disadvantages, first it is generally assumed that one knows the direction of arrival (DOA) of the signal of interest (SOI) and the goal is to estimate its complex amplitude in the presence of jammer, clutter and noise [9], so this algorithm couldn't find DOA of SOI. Second due to mechanical vibrations, calibration errors, DOA will change and the adaptive algorithm treats the SOI as an interferer and nulls it out. Sarkar and Choi showed that in the actual angle of SOI the sum of norm of the weights will be minimum and can be used to refine the estimate of the DOA and thereby prevent the problem of signal cancellation for an adaptive algorithm. This paper propose a novel technique for direction of arrival (DOA) estimation algorithm based on  $D^3LS$  approach using the minimum property of the sum of norm of the adaptive weights and can be used as an indicator to estimate the DOA of the SOI in adaptive algorithms.

This paper is organized as follows. In Section II we provide a brief review of the direct data domain least squares approach for the one-Dimensional (1-D) adaptive problems. In Section III we show when we search all the angles, norm of the optimum weights at the actual DOA will be minimized and we can estimate DOA for both SOI and Jammer. In Section IV, numerical simulations illustrate this property. Finally, in Section V we present some conclusions.

## II. Review of the $D^3LS$ approach

Consider an array of  $N$  uniformly spaced isotropic point sensors separated by a spacing of  $d$  shown in Fig. 1.

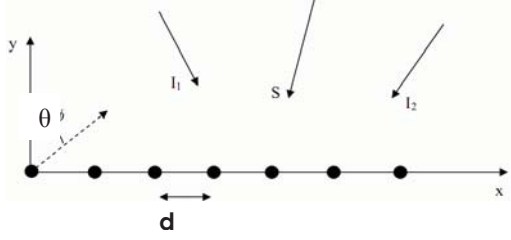


Figure 1. Linear uniform array

The array receives a signal  $S$  from an assumed direction  $\theta_s$  and some interference sources  $J_i$  from unknown directions. Therefore, the voltage at the  $i$ th element due to the incident fields is:

$$X_n = \alpha_s e^{\frac{j2\pi nd \cos \theta_s}{\lambda}} + \sum_{p=1}^P j_m e^{\frac{j2\pi nd \cos \theta_p}{\lambda}} + n_n \quad (1)$$

Our objective is to estimate its complex amplitude while simultaneously rejecting all other interferences and noise. We make the narrowband assumption for all the signals including the interferers. Therefore, by suppressing the time dependence in the phasor notation, complex vectors of phasor voltages  $[X]$  is

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \alpha_s \begin{bmatrix} a_1(\theta_s) \\ a_2(\theta_s) \\ \vdots \\ a_N(\theta_s) \end{bmatrix} + \sum_{m=1}^{M-1} j_m \begin{bmatrix} a_1(\theta_m) \\ a_2(\theta_m) \\ \vdots \\ a_N(\theta_m) \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \quad (2)$$

where  $\alpha_s$  is the complex amplitude of the SOI, to be determined. The column vectors in (2) show the signal induced in each of the  $N$  antenna elements.  $a_n$  represents the voltage induced at the  $n$ th antenna element due to signal of unity amplitude arriving from a particular direction  $\theta_s$ . For a conventional adaptive array system, using each of the  $K$  weights  $W_k$ , the relationship between  $K$  and  $N$  can be chosen as  $K = (N + 1)/2$  [9]. Let us define

$$Z = \exp \left[ j2\pi \frac{d}{\lambda} \cos \theta_s \right] \quad (3)$$

Then  $X_1 - Z^{-1}X_2$  contains no components of the SOI. Therefore one can form a reduced rank matrix where the weighted sum of all its elements would be zero [9]. In order to make the matrix full rank, we fix the gain of the subarray by forming the weighted sum  $\sum_{k=1}^K W_k \cdot Z^{k-1}$  along the DOA of the

SOI to a prespecified value. gain of the subarray is  $M$  along the direction of  $\theta_s$ . Then we can write:

$$\begin{bmatrix} 1 & \dots & Z^{K-1} \\ X_1 - Z^{-1}X_2 & \dots & X_K - Z^{-1}X_{K+1} \\ \vdots & \vdots & \vdots \\ X_{K-1} - Z^{-1}X_K & \dots & X_{N-1} - Z^{-1}X_N \end{bmatrix}_{K \times K} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix} = \begin{bmatrix} M \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{K \times 1} \quad (4)$$

we can now estimate the SOI through using the following weighted sum:

$$y = \frac{1}{M} \sum_{k=1}^K W_k \cdot X_k \quad (5)$$

Most DOA estimation methods like MUSIC, ESPRIT and maximum likelihood method estimate the DOAs from the properties of the covariance matrix. However, when there are coherent signals, additional processing needs to be done with these techniques to estimate the DOAs. Also these techniques do not make any distinction between the SOI and the interferers. In this paper, our goal is to determine DOA of all the signals including interferers based on Direct Data Domain. so we can determine DOA of SOI and interefers only with one snapshot.

### 3. THE PROPOSED MINIMUM NORM DOA ALGORITHM

Suppose a solution for the matrix equation in the (4) exist. Sarkar and Choi [10] showed underlying the minimum norm theorem, when the actual direction of arrival of the signal of interest coincides with the assumed direction, the norm of the weights for the vector  $W$  will be minimum. They use from this property for calibration at the array. First review this theorem:

**Theorem:** if for equation  $A\vec{X} = \vec{b}$  one or infinite number solution exist and we want find a solution with the smallest norm, that is minimum norm solution and is unique.

The norm of the weights for the vector  $W$  can be defined by

$$\|W\| = \sqrt{|W_1|^2 + |W_2|^2 + |W_3|^2 + \dots + |W_K|^2} \quad (6)$$

will be minimum. This property is true for both received signals and Jammers, so if we search all angles from 0 to 180 and then we check norm the weights, position of minimum norm of weights exactly coincides within the DOA of the actual signals and interferers. In real life, there are always some uncertainties associated with this assumed DOA for radar problems. This may be due to mechanical vibrations of the antenna array,

incorrect calibration or due to atmospheric refraction of incident electromagnetic wave. Through this minimum norm property of the weights we can refine estimate of the DOA of the SOI to correct our initial estimate. So with this algorithm we can calibrated the array automatically.

#### 4-NUMERICAL EXAMPLES

The first example deals with a conventional adaptive algorithm using a seventeen element antenna array. In addition to the SOI and thermal noise there are also 2 jammers and their parameters are listed in Table I.

Table I -Parameters for the SOI and Interference

	Magnitude	Phase	DOA
<b>Signal</b>	1V/m	0°	15°
<b>Jammer1</b>	100V/m	0°	40°
<b>Jammer2</b>	400V/m	0°	70°

The number of adaptive weights chosen for our simulation is nine [9]. One of the jammer is 40 dB stronger and the other two are 50 dB stronger than the SOI.

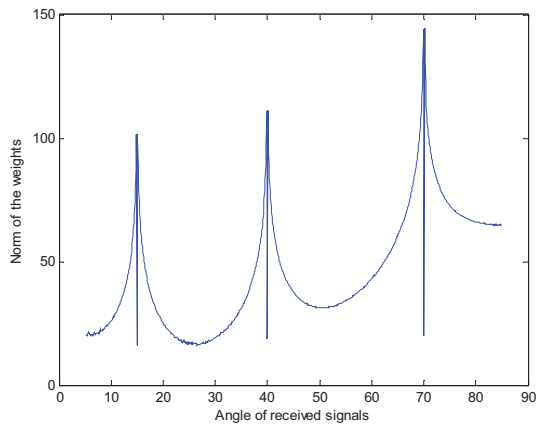


Fig. 2. Norm of the weights for assumed DOA from 0 to 90

We suppose signal and interferers are arriving from different DOA from  $0^\circ$  to  $90^\circ$ . In this case, we solve the problem for any one of the nine hundred angles covering from  $0^\circ$  to  $90^\circ$  at every  $0.1^\circ$  intervals. We compute the sum of the norm of the adapted weights for each simulation, corresponding to each one of the assumed DOA for the SOI.

Then we plot the sum of the norm of the weights as a function of the assumed DOA. Fig.2 shows the

norm of the weights for different values of angles in the case that we don't have thermal noise. We observe that the norm of the sum of the weights at the some angles has minimum, and this minimums is actual DOA of the SOI and Jammers.

For real, the simulation is repeated for different power of noise at each antenna element. Fig.3 show the norm of the weights for different values of angles in the case that signal to noise ratio is 10dB. The only difference is that the shape of the null is different for different values of the thermal noise level.

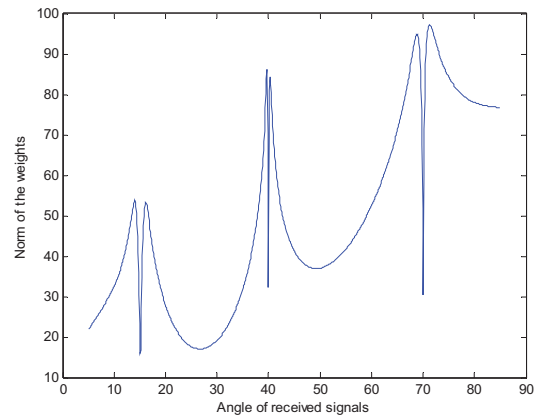


Fig. 3. Norm of the weights for assumed DOA from 0 to 90 with S/N=10dB

In the second example we use the same array as in the first example with the signals bearing at  $15^\circ, 20^\circ$  and parameters of the signals are listed Table 2.

Table II -Parameters for the signals

	Magnitude	Phase	DOA
<b>Signal1</b>	1V/m	0°	15°
<b>Signal2</b>	10V/m	0°	20°

Fig.4 show the norm of the weights for different values of angles in the case that we don't have thermal noise and resolution is good, but as shown in Fig5, when we added noise, performance of resolution decreased.

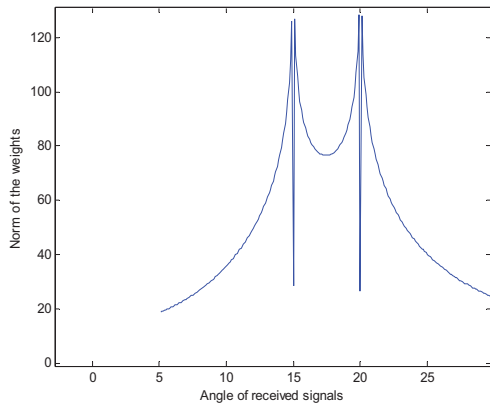


Fig. 4. Resolution of DOA for second example

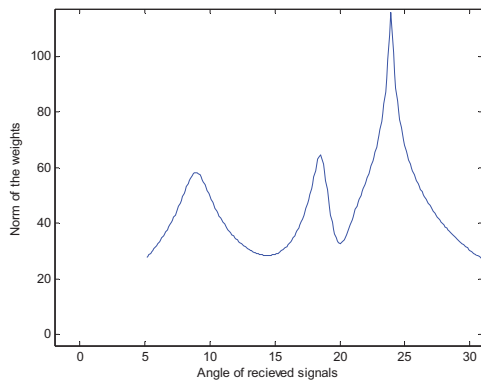


Fig. 5. Resolution of DOA with  $S/N=10\text{dB}$  for second example

with the  $D^3LS$  approach, the norm of weights at the signal incidence direction will be much lower than other directions. We can change the look direction in an angular step from  $0^\circ$  to  $90^\circ$  at every  $0.1^\circ$  interval and try to determine norm of weight vector  $W$  at each angle. When the look direction is equal to the incidence angle, a minimum value will appear and we can detect it. Fig6. shows amplitude of the desired signal will be recovered accurately with the proposed algorithm.

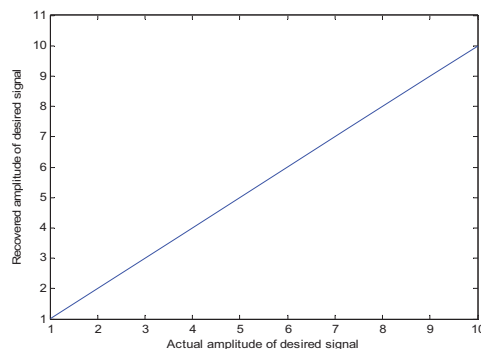


Fig6. Recovered strength of desired signal under different incident strengths of desired signal using proposed algorithm

## 5. Conclusion

This paper proposes a novel simple DOA estimation algorithm based on  $D^3LS$  approach.

With using minimum property we can search and find angles of signal and jammers that it still possesses an advantage of single-snapshot processing as opposed to forming a covariance matrix of voltages at the antennas terminal, but resolution of this algorithm for low signal to noise ratio in comparison with statistical algorithms is not good. Simulation results show that the proposed algorithm can estimate the DOAs of signals accurately. For calibration we can repeat this algorithm every time.

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