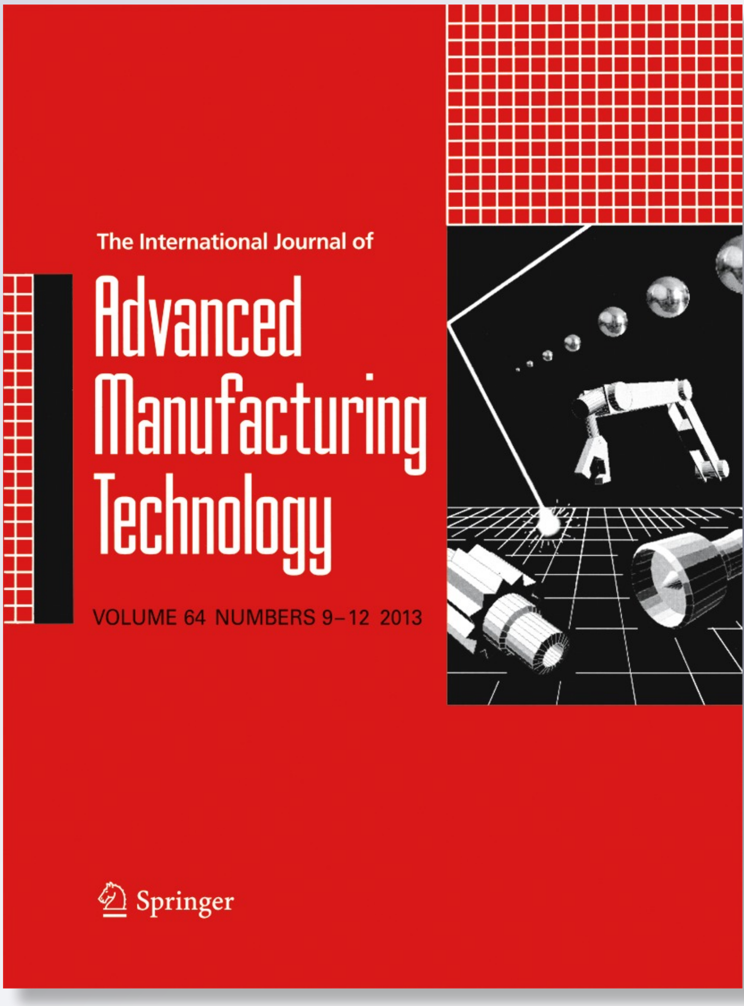
Modifying simple linear profiles monitoring schemes in phase II to detect decreasing step shifts and drifts

Amirhossein Amiri, Marzieh Mehrjoo & Zbigniew J. Pasek

The International Journal of Advanced Manufacturing Technology

ISSN 0268-3768
Volume 64
Combined 9-12

Int J Adv Manuf Technol (2013)
64:1323-1332
DOI 10.1007/s00170-012-4103-6



Springer

Int J Adv Manuf Technol (2013) 64:1323–1332
DOI 10.1007/s00170-012-4103-6

ORIGINAL ARTICLE

Modifying simple linear profiles monitoring schemes in phase II to detect decreasing step shifts and drifts

Amirhossein Amiri · Marzieh Mehrjoo · Zbigniew J. Pasek

Received: 28 August 2011 / Accepted: 22 March 2012 / Published online: 14 April 2012

Abstract In profile monitoring, a relationship between a response variable and one or more explanatory variables is monitored. Different methods were developed for phase II monitoring of simple linear profiles. While some of the methods can be used to detect both increasing and decreasing shifts in the regression parameters, others need to be modified to enable detection of decreasing shifts in a process. In this paper, necessary modifications of the phase II methods for simple linear profile monitoring are proposed to improve their performance in detecting decreasing shifts. The paper also presents a performance comparison of several phase II methods.

Keywords Average run length · Statistical process control · Profile monitoring · Phase II · Step shift · Drift

1 Introduction

In today's competitive life, scientists and practitioners are beginning to realize that efficiency and product or process quality are of higher importance than ever before. As a result, techniques of statistical process control (SPC), an approach long practiced across various industries, are being improved to ensure efficient and effective compliance with a broad range of specifications and requirements. In some SPC applications in which the quality variable is functionally dependent on independent or explanatory variable(s), the quality of corresponding process or product cannot be adequately represented by the standard use of the distribution of a single quality characteristic or a general multivariate quality vector. In these cases, increasingly common in practical applications, monitoring profiles, relationship between the response variable and explanatory variable(s), are highly recommended in the literature.

For phase II monitoring of simple linear profiles, which can adequately model the quality characteristic in many applications, several approaches have been proposed. In phase II, process parameters are assumed to be known, and the aim is to detect shifts as quickly as possible. Kang and Albin [9] introduced the T^2 and exponentially weighted moving average (EWMA)-R methods for phase II monitoring of simple linear profiles. For the same purpose, Kim et al. [10] applied three independent EWMA control charts by using coded x values and introduced EWMA-3 method. Noorossana and Amiri [12] used a multivariate cumulative sum (CUSUM) control chart along with a χ^2 control chart to improve the performance of existing methods. Niaki et al. [11] used generalized linear statistical model and an R control chart for this purpose. Gupta et al. [8] compared a method developed by Croarkin and Varner [3] with the performance of Kim et al. [10]. Zou et al. [23] proposed a multivariate exponentially weighted moving average (MEWMA) control chart. Saghaei et al. [17] proposed CUSUM-3 method which applies three CUSUM control charts to monitor the intercept, the slope, and the standard deviation of a simple linear profile. Zhang et al. [21] proposed a method based on likelihood ratio statistics to monitor simple linear profiles in phase II. Noorossana et al. [13] and Soleimani et al. [18] proposed some methods based on time series approach for the case there is autocorrelation between and within simple linear profiles, respectively. Noorossana et al. [14] also studied the performance of control charts for monitoring simple linear profiles under the

A. Amiri
Industrial Engineering Department,
Faculty of Engineering, Shahed University,
Tehran, Iran
e-mail: amiri@shahed.ac.ir

M. Mehrjoo (✉) · Z. J. Pasek
Department of Industrial and Manufacturing Systems Engineering,
University of Windsor,
Windsor, ON N9B 3P4, Canada
e-mail: mehrjoo@uwindsor.ca

Z. J. Pasek
e-mail: zjpasek@uwindsor.ca

assumption of non-normality in the distribution of the regression error terms. The readers are referred to the papers Woodall et al. [19], Woodall [20], and Noorossana et al. [15] for more information about other types of profiles as well as profiles monitoring procedures.

Although both increasing and decreasing shifts have detrimental effects on the process, in all of the proposed methods, only increasing shifts have been studied. EWMA-R, MEWMA, and likelihood ratio test (LRT) are the methods effective for monitoring increasing as well as decreasing shifts. However, the methods T^2, EWMA-3, and CUSUM-3 need to be modified to be able to detect both increasing and decreasing shifts.

On the other hand, all profile monitoring techniques have been focused only on detecting step shifts. However, drift, which is a time-varying change, frequently occurs in industrial applications. Graduate deterioration of equipment, waste accumulation, catalyst aging, or human causes such as operation fatigue or close supervision are some causes of drift, Reynolds and Stoumbos [16]. A lot of authors including Aerne et al. [1], Bissell [2], Davis and Woodall [4], Divoky and Taylor [5], Fahmy and Elsayed [6], Gan [7], Reynolds and Stoumbos [16], and Zou et al.[22] studied the performance of different control charts when there is a drift in the process mean. Bissell [2] presented the results of run length evaluation under linear trend for CUSUM charts. Davis and Woodall [4] showed that the trend rules, which relate to an out of the ordinary long series of consecutive decreases or increases in operational performance, are not effective in detecting a linear shift in the mean of the monitored process. Aerne et al. [1] studied the performance of Shewhart chart with supplementary runs rules, CUSUM, and EWMA charts when the process mean changes as a linear trend. Gan [7] presented a numerical procedure to compute the average run length (ARL) of an EWMA chart under a linear drift in the process mean. Divoky and Taylor [5] simulated process mean drift to evaluate the sensitivity of trend rules when applied jointly with combinations of standard supplementary runs test. They expanded previously presented trend rules to a larger set of rules, as well. Reynolds and Stoumbos [16] monitored the mean and the standard deviation of the process under drifts, jointly. They showed that in detecting slow and moderate rate drifts, the combination of two EWMA control charts for monitoring both mean and variance performs better than I/MR control charts. Fahmy and Elsayed [6] used a statistic based on the deviation between the target mean and the expected mean of the process in their proposed approach. They also compared the performance of their approach with CUSUM, EWMA, Shewhart, and Generalized Likelihood Ratio (GLR) charts. Zou et al. [22] compared five control schemes including EWMA, CUSUM, generalized EWMA, GLR-S, and GLR-L for monitoring drift through presenting both the asymptotic estimation and the numerical simulation of ARL.

As mentioned before, in the literature of drift detection, focus is mainly on the process mean. However, drift can occur when a function between two or more variables is characterizing the process performance. The particular focus of this study is on the phase II methods including EWMA-R, MEWMA, LRT, T^2, EWMA-3, and CUSUM-3. The T^2, EWMA-3, and CUSUM-3 methods are modified to make them effective for monitoring increasing and decreasing shifts in the parameters of simple linear profiles. In addition, the performance of the methods under both increasing and decreasing step shifts as well as drift is evaluated.

The remainder of this paper is structured as follows: In Section 2, the modified methods are explained in detail. Section 3.1 presents the performance of all phase II methods in detecting increasing and decreasing step shifts. The performance of the methods under increasing and decreasing drift is considered in Section 3.2. Concluding remarks are provided in the final section.

2 Modified methods

As it was mentioned previously, MEWMA, LRT, and EWMA-R methods are effective in detecting both decreasing and increasing shifts. Thus, it is not necessary to modify them, and only a short description of these approaches is provided. However, the methods including T^2, EWMA-3, and CUSUM-3 are modified to enable them to detect decreasing shifts in addition to increasing shifts. Later in the paper, a comprehensive comparison of all the methods under both increasing and decreasing step shifts and drift is provided.

Suppose for sample j collected over time, observations (x_i,y_{ij}), i=1,2,…,n are obtained, where the subscript i represents the ith observation within each profile and the subscript j shows the jth profile. It is assumed that a simple linear regression model properly relates the response variable, Y, to the explanatory variable, X. When the process is under statistical control, the model is as Eq. (1):

$$Y_{ij} = A_0 + A_1 X_i + \varepsilon_{ij} \tag{1}$$

where ε_{ij} are independent, identically distributed normal random variables with zero mean and variance σ^2. For simplicity, it is assumed that the x values are fixed and constant from profile to profile. Since the focus is on the phase II case in this paper, the in-control values of the parameters A_0, A_1, and σ^2 are assumed to be known.

2.1 T^2

Kang and Albin [9] proposed T^2 method that applies a multivariate T^2 control chart for monitoring the parameters of a simple linear profile. The T^2 control chart is given in Eq. (2) as follows:

$$T_j^2 = (z_j - \mu)^T \sum^{-1} (z_j - \mu), \tag{2}$$

where $\mu = (A_0, A_1)^T, z_j = (a_{0j}, a_{1j})^T$ is the vector of regression parameter estimators and $\sum$ is the covariance matrix of regression parameters estimators. $\chi^2_{\alpha,2}$ is the upper control limit of this statistic. This control chart can detect any shift in the intercept and slope of a simple linear profile. In addition, it is able to detect increasing shift in the variance. However, decreasing shift in the variance leads to the reduction in the difference between mean of estimates and μ. Therefore, T^2 statistic decreases and since there is no lower control limit, T^2 control chart has a poor performance in detecting this type of shift. Hence, this method is modified by adding a lower control limit to the related control chart to improve its performance in detecting decreasing shift in the variance. Consequently, the upper and lower control limits for the statistic in Eq. (2) are as Eq. (3):

$$\mathrm{LCL} = \chi^2_{1-\alpha/2,2} \; and \; \mathrm{UCL} = \chi^2_{\alpha/2,2} \tag{3}$$

2.2 EWMA-R

Kang and Albin [9] proposed another method for monitoring simple linear profiles named EWMA-R. They suggested the combination of two control charts EWMA and R. As mentioned before, this method is not explained in detail since no modification was needed. It should be noted that this method is not able to detect decreasing shift in variance for $n<7$ since the R control chart does not have lower control limit. Thus, one only needs to modify the sample size in the example by Kang and Albin [9] from $n=4$ to $n=7$ to be able to check the performance of this method under decreasing shift in the variance.

2.3 EWMA-3

Kim et al. [10] proposed using three EWMA control charts to detect shifts in the intercept, slope, and variance. They coded the x values to make the estimates of the slope and the intercept parameters independent. Their coded model is in the Eq. (4).

$$Y_{ij} = B_0 + B_1 X_i' + \varepsilon_{ij}, i = 1, 2, \ldots, n, \tag{4}$$

where $B_0 = A_0 + A_1\overline{X}$, $B_1 = A_1$, and $X_i' = (X_i - \overline{X})$.

In the Eq. (5), the estimator of the intercept, b_0 (j), is used to monitor the intercept, B_0:

$$\mathrm{EWMA_I}(j) = \theta\, b_{0j} + (1-\theta)\mathrm{EWMA_I}(j-1), \tag{5}$$

where $(0<\theta<1)$ is a smoothing constant and $\mathrm{EWMA_I}(0) = B_0$. The lower and upper control limits for this statistic are given in the Eqs. (6) and (7), respectively.

$$\mathrm{LCL} = B_0 - L_\mathrm{I}\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \tag{6}$$

$$\mathrm{UCL} = B_0 + L_\mathrm{I}\sigma\sqrt{\frac{\theta}{(2-\theta)n}}, \tag{7}$$

where L_I (>0) is chosen to give a specified in-control ARL. For monitoring the slope, B_1, the estimator of the slope, $b_1(j)$, is used and related EWMA statistic is computed based on the Eq. (8) as follows:

$$\mathrm{EWMA_S}(j) = \theta\, b_{1j} + (1-\theta)\mathrm{EWMA_S}(j-1), \tag{8}$$

where $(0<\theta<1)$ is a smoothing constant and $\mathrm{EWMA_S}(0) = B_1$. The control limits are computed as Eq. (9):

$$\mathrm{LCL} = B_1 - L_\mathrm{S}\sigma\sqrt{\frac{\theta}{(2-\theta)S_{xx}}}, \tag{9}$$

$$\mathrm{UCL} = B_1 + L_\mathrm{S}\sigma\sqrt{\frac{\theta}{(2-\theta)S_{xx}}}, \tag{10}$$

where L_S (>0) is chosen to give a specified in-control ARL. In this method, they used the following statistic in Eq. (11) for monitoring the variance:

$$\mathrm{EWMA_E}(j) = \max\left\{\theta \ln(\mathrm{MSE}_j) + (1-\theta)\mathrm{EWMA_E}(J-1),\; In(\sigma_0^2)\right\}, \tag{11}$$

where $(0<\theta\leq 1)$ is a smoothing constant and $\mathrm{EWMA_E}(0) = \ln(\sigma_0^2)$. The upper control limit is given in Eq. (12) as follows:

$$\mathrm{UCL} = L_\mathrm{E}\sqrt{\frac{\theta \mathrm{var}\left[\ln(\mathrm{MSE}_j)\right]}{(2-\theta)}}, \tag{12}$$

where

$$\mathrm{var}\left[\ln(\mathrm{MSE}_j)\right] = \frac{2}{(n-2)^1} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}, \tag{13}$$

in which L_E (>0) is chosen to give a specified in-control ARL. The EWMA-3 method is not able to detect decreasing shift in the variance. Therefore, besides modifying the related statistic, lower control limit to this control chart needs to be added. The modified EWMA statistic for monitoring variance is given in Eq. (14),

$$\mathrm{EWMA_E}(j) = \theta \ln\left(\mathrm{MSE}_j\right) + (1-\theta)\mathrm{EWMA_E}(j-1) \tag{14}$$

and the lower and upper control limits are introduced in Eq. (15) as follows, respectively.

$$LCL = -L_E\sqrt{\frac{\theta \mathrm{var}\left[\ln\left(MSE_j\right)\right]}{(2-\theta)}} \quad \text{(and)}$$

$$\mathrm{UCL} = L_\mathrm{E}\sqrt{\frac{\theta \mathrm{var}\left[\ln\left(\mathrm{MSE}_j\right)\right]}{(2-\theta)}} \tag{15}$$

2.4 CUSUM-3

Saghaei et al. [17] proposed using three CUSUM control charts for detecting shifts in the intercept, slope, and variance. Similar to the EWMA-3 method, x values were coded. The paper presented three statistics for detecting increasing shifts and three ones for decreasing shifts. Yet, authors ignored detecting both increasing and decreasing shifts simultaneously and also designed the control chart in a way that it is able to detect either increasing or decreasing shifts.

In the approach presented here, the authors propose to use all six control charts to detect both increasing and decreasing shifts, concurrently. Since the statistics and control limits of CUSUM-3 are not modified, the method is not explained in detail.

2.5 MEWMA

For monitoring general linear profiles, Zou et al. [23] proposed a MEWMA control chart. The performance of the method is compared with EWMA-3 under increasing step shifts in their study. As formerly mentioned, this approach can detect both increasing and decreasing shifts, and no modification is needed.

2.6 LRT

Zhang et al. [21] presented a method based on likelihood ratio. This method is also able to detect both increasing and decreasing shifts, and no change is needed. Therefore, we skip to explain it in detail.

3 Simulation studies

The performance of the methods discussed in the previous section is compared in this section. For this purpose, the most popular benchmark example from the literature [9], $Y_{ij}=3+2X_i+\varepsilon_{ij}$ is used where x=2, 4, 6, 8, 10, 12, and 14 for each profile and $\varepsilon_{ij}\sim N(0,1)$ without loss of generality. Ten thousand simulation runs are used to calculate the in-control and out-of-control ARLs by MATLAB software. To do the comparison, the values of parameters in the methods, which are summarized in Table 1, are designed to have the same in-control ARL of roughly 200. Then, the method with the lowest out-of-control ARL has the best performance.

The shifts in the intercept, A_0; the slope, A_1; and the standard deviation, σ, are studied in this paper, and out-of-control ARL values are reported in the sections below. The best method is highlighted in each table.

3.1 Step shift

In this section, performance of all the phase II methods under increasing and decreasing step shifts, respectively,

Table 1 The values of the parameters in the methods

EWMA-3	EWMA-R	CUSUM-3	LRT	MEWMA	T^2
θ=0.2	EWMA	K_I=0.5	h=1.7447	L=11.84	α=0.005
L_I=3.027	L=3.12	K_S=0.05	λ=0.2	λ=0.2	UCL=11.98
L_s=3.017		K_E^-=0. 25			LCL=0.005
L_E=4.073	R	K_E^+=2			
	θ=0.2	H_I^+=H_I^-=0.7697			
	d_2=2.704	H_S^+=H_S^-=0.470			
	d_3=0.833	H_E^+=1.59			
	L_{LCL}=2.32	H_E^-=0.17			
	L_{UCL}=3.375				

Table 2 Comparisons of out-of-control ARL under increasing step shift in the intercept (from A_0 to $A_0+\lambda\sigma$)

Method	Shift size (λ)									
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
T^2	139.06	56.92	20.26	7.88	3.85	2.25	1.51	1.22	**1.08**	**1.02**
EWMA-R	40.73	10.35	5.44	3.7	2.85	2.35	2.06	1.86	1.68	1.47
EWMA-3	37.77	**9.84**	**5.21**	3.56	2.78	2.29	2.02	1.82	1.61	1.4
MEWMA	**37.32**	10.5	5.61	3.84	2.95	2.46	2.14	1.95	1.79	1.6
CUSUM-3	85.28	18.53	6.2	**3.29**	**2.23**	**1.69**	**1.39**	**1.19**	**1.08**	1.03
LRT	40.11	11.21	5.9	3.93	2.89	2.28	1.89	1.6	1.36	1.19

are compared. The out-of-control ARL values under increasing step shifts in the intercept are summarized in Table 2. As shown in this table, CUSUM-3 has the best performance in detecting medium to large shifts except the largest ones in which T^2 performs roughly the same as CUSUM-3. However, MEWMA is the best method for detecting the smallest shift, and EWMA-3 outperforms the other methods in detecting other small shifts.

Table 3 shows simulation results for case where there are increasing step shifts in the slope parameter. These results show that MEWMA performs better than the other methods in detecting small shifts. Furthermore, CUSUM-3 and T^2 approaches have the best performance in detecting medium and large shifts, respectively.

Regarding increasing step shifts in the standard deviation, which are summarized in Table 4, it can be seen that CUSUM-3 approach outperforms in detecting almost all of the shifts except the two smallest shift sizes. The method LRT has the best performance in detecting small shifts.

To evaluate the performance of the methods under decreasing shifts, the magnitude of the slope and intercept shifts are considered the same as increasing shift sizes. The results reached are roughly the same as the values reported in the Tables 2 and 3 (they are not reported here). Since the same shift sizes for standard deviation lead to negative standard deviations, which are unfeasible results, different shift sizes are chosen to investigate the performance of the methods under decreasing shift.

The out-of-control ARL values for decreasing step shifts in the standard deviation are reported in Table 5. It can be seen that LRT method performs better than the others in detecting small shifts. EWMA-3 has the best performance in detecting medium shifts and CUSUM-3 outperforms in detecting large shifts. According to the results in Tables 2, 3, 4, and 5, the best methods in detecting increasing and decreasing step shifts for various operational circumstances are summarized in Table 6.

Generally, the results under the step shifts showed that the performance of the methods under increasing shifts in the intercept and the slope are the same as their performance under decreasing shifts. Furthermore, in detecting small step shifts in the intercept, the method EWMA-3 by Kim et al. [10] had the best performance. CUSUM-3 approach by Saghaei et al. [17] outperformed the others in detecting both medium and large step shifts in the intercept. In detecting small, medium, and large step shifts in the slope, MEWMA by Zou et al. [23], CUSUM-3 by Saghaei et al. [17], and T^2 by Kang and Albin [9] performed better in comparison with the other methods, respectively. However, the performance of the methods in detecting increasing and decreasing shifts in standard deviation was

Table 3 Comparisons of out-of-control ARL under increasing step shift in the slope (A_1 to $A_1+\beta\sigma$)

Method	Shift size (β)									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
T^2	127.67	43.77	14.19	5.44	2.69	**1.65**	**1.25**	**1.09**	**1.02**	**1**
EWMA-R	40.59	10.42	5.42	3.7	2.84	2.33	2.03	1.83	1.63	1.44
EWMA-3	32.44	9.25	5	3.49	2.72	2.28	2	1.81	1.61	1.39
MEWMA	**29.86**	**8.63**	**4.79**	3.36	2.64	2.22	1.99	1.79	1.6	1.36
CUSUM-3	62.21	14.29	5.66	**3.17**	**2.19**	1.7	1.38	1.19	1.08	1.02
LRT	32.66	9.35	5.03	3.36	2.5	1.97	1.64	1.36	1.17	1.07

Table 4 Comparisons of out-of-control ARL under increasing step shift in the standard deviation (σ to $\gamma\sigma$)

Method	Shift size ($\gamma\sigma$)									
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
T^2	57.45	20.28	10.22	6.37	4.43	3.42	2.85	2.41	2.11	1.95
EWMA-R	39.58	11.25	5.09	3.02	2.17	1.7	1.46	1.31	1.21	1.15
EWMA-3	75.99	29.71	12.98	7.71	5.55	4.44	3.69	3.2	2.86	2.58
MEWMA	19.31	7.05	4.24	3.11	2.51	2.1	1.85	1.64	1.49	1.37
CUSUM-3	28.13	7.51	**3.56**	**2.3**	**1.75**	**1.45**	**1.31**	**1.21**	**1.14**	**1.1**
LRT	**19.21**	**6.45**	3.73	2.65	2.09	1.76	1.52	1.37	1.27	1.2

Table 5 Comparisons of out-of-control ARL under decreasing step shift in the standard deviation (σ to $\gamma\sigma$)

Method	Shift size ($\gamma\sigma$)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
T^2	272.57	254.22	197.02	146.38	98.97	64.70	37.33	16.36	4.481
EWMA/R	246.92	150.06	76.954	34.853	14.55	5.670	2.144	1.102	**1**
EWMA-3	104.43	32.412	13.186	**7.280**	**4.810**	3.440	2.593	1.997	1.508
MEWMA	149.70	44.644	16.266	8.4316	5.489	3.970	3.090	2.433	1.999
CUSUM-3	333.22	194.21	64.561	18.864	5.847	**2.489**	**1.558**	**1.062**	1
LRT	**92.203**	**24.073**	**11.267**	7.514	5.876	5.081	4.590	4.002	4

Table 6 The best method in detecting increasing (+) and decreasing (−) step shifts under different magnitudes of the shifts

Magnitude of the shift	Parameter			
	Intercept (±)	Slope (±)	Standard deviation (+)	Standard deviation (−)
Small	EWMA-3	MEWMA	LRT	LRT
Medium	CUSUM-3	CUSUM-3	CUSUM-3	EWMA-3
Large	CUSUM-3	T^2	CUSUM-3	CUSUM-3

Table 7 Comparisons of out-of-control ARL under increasing drift in the intercept (A_0)

Method	High small to small rate									
	Rate(r_{a0})									
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
T^2	151.7	122.05	102	87.18	77.78	69.4	64.02	58.27	54.11	50.82
EWMA-R	112.18	79.6	63.48	54.06	47.5	42.44	38.64	35.64	32.99	30.89
EWMA-3	108.3	78.39	62.86	53.65	46.92	41.81	**37.76**	**34.85**	**32.36**	**30.54**
CUSUM-3	133.64	99.15	80.46	68.56	59.82	54.04	48.52	44.76	41.32	38.51
MEWMA	**108.06**	**76.74**	**62.21**	**52.78**	**46.49**	**41.59**	37.98	35.22	32.56	30.64
LRT	110.85	78.55	63.7	54.25	47.41	42.65	38.95	35.86	33.63	31.19

Table 8 Comparisons of out-of-control ARL under increasing drift in the intercept (A_0)

Method	Small to medium rate					Medium to large rate					Large to very large rate					
	Rate (r_{a0})															
	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	31.8	19.43	14.33	11.53	9.76	5.78	3.4	2.5	2.04	**1.76**	**1.26**	**1.02**	**1**	1	1	1
EWMA-R	19.94	12.97	10.09	8.46	7.4	4.94	3.36	2.71	2.27	2.03	1.88	1.46	1.07	1	1	1
EWMA-3	**19.69**	**12.77**	**9.95**	**8.33**	**7.34**	4.86	3.31	2.69	2.24	2.02	1.85	1.4	1.05	1	1	1
CUSUM-3	24.05	14.74	11.12	9.06	7.74	**4.82**	**3.04**	**2.34**	**1.96**	**1.76**	**1.26**	**1.02**	**1**	1	1	1
MEWMA	19.9	13	10.08	8.52	7.47	5.02	3.43	2.8	2.35	2.06	1.94	1.59	1.15	1	1	1
LRT	20.22	13.32	10.36	8.73	7.59	5.08	3.39	2.7	2.23	1.98	1.65	1.18	1.01	1	1	1

Table 9 Comparisons of out-of-control ARL under increasing drift in the slope (A_1)

Method	High small to small rate									
	Rate(r_{a1})									
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
T^2	54.35	34.4	26.1	21.18	17.96	15.54	13.98	12.62	11.54	10.62
EWMA-R	35.73	23.11	17.64	14.83	12.84	11.53	10.47	9.62	8.98	8.38
EWMA-3	33.74	22	17.06	14.3	12.5	11.16	10.12	9.32	8.74	8.16
CUSUM-3	40.83	26.02	19.94	16.42	14.06	12.46	11.2	10.2	9.47	8.8
MEWMA	**32.78**	**21.26**	**16.51**	**13.89**	**12.11**	**10.83**	**9.86**	**9.09**	**8.48**	**8**
LRT	33.68	21.78	16.88	14.16	12.42	11.05	10.09	9.28	8.68	8.16

different for both step shifts and drift. LRT method by Zhang et al. [21] was the best one in detecting small increasing step shifts in standard deviation, and for medium and large increasing step shifts, CUSUM-3 by Saghaei et al. [17] outperformed the others. However, LRT by Zhang et al. [21], EWMA-3 by Kim et al. [10], and CUSUM-3 by Saghaei et al. [17] were the best methods in detecting small, medium, and large decreasing step shifts in the standard deviation, respectively.

Table 10 Comparisons of out-of-control ARL under increasing drift in the slope (A_1)

Method	Small to medium rate					
	Rate (r_{a1})					
	0.02	0.04	0.06	0.08	0.1	0.2
T^2	6.27	3.71	2.72	**2.21**	**1.88**	**1.08**
EWMA-R	5.59	3.77	3.03	2.62	2.26	1.81
EWMA-3	5.45	3.72	2.99	2.57	2.25	1.74
CUSUM-3	5.52	**3.51**	**2.71**	2.26	1.96	1.18
MEWMA	**5.33**	3.64	2.94	2.54	2.18	1.79
LRT	5.42	3.63	2.86	2.4	2.08	1.36

3.2 Drift

The performance of the mentioned methods under increasing and decreasing drift is studied in this section. The drifts in the regression parameters are defined as follows:

$$\begin{aligned} A_0(t) &= A_0 + \mathrm{r}_{a0}t \quad \mathrm{r}_{a0}: \text{ rate of drift in } A0 \\ A_1(t) &= A_1 + \mathrm{r}_{a1}\mathrm{t} \quad \mathrm{r}_{a1}: \text{rateofdrift in } A_1 \quad , \\ \sigma(t) &= \sigma + r_\sigma t \quad \mathrm{r}_\sigma: \text{ rate of drift in } \sigma \end{aligned} \tag{16}$$

where $A_0(t)$, $A_1(t)$, and $\sigma(t)$ are the values of the regression parameters and t unit after an assignable cause occurs in a process leading to a drift. It is assumed that the drift occurs at a constant rate. Since the parameters of control charts are determined under in-control conditions and they do not depend on the type of shifts, the parameters of control charts under drift are the same as the parameters determined under step shifts in the Table 1.

In the rest of this section, the performance of the methods under increasing and decreasing drift is compared. The magnitude of rates is classified into four different groups, high small to small rate, small to medium rate, medium to large rate, and large to high large rate. The out-of-control ARLs for increasing drifts in the intercept are given in Tables 7 and 8. The results show that MEWMA is the best method in detecting

Table 11 Comparisons of out-of-control ARL under increasing drift in the standard deviation (σ)

Method	High small to small rate									
	Rate (r_σ)									
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
T^2	111.66	85.16	70.7	60.86	54.45	49.02	45.22	41.72	38.94	36.86
EWMA-R	102.73	75.24	61.23	52.78	46.28	41.86	38.46	35.41	33	30.96
EWMA-3	124.47	96.62	79.1	69.62	61.49	55.55	51.44	47.8	44.28	41.43
CUSUM-3	92.47	67.4	55.04	47.46	41.82	37.14	33.98	31.46	29.26	27.49
MEWMA	**84.16**	**59.39**	**48.22**	**41.43**	**36.23**	**32.76**	**30.03**	**27.9**	25.95	24.42
LRT	86.62	61.3	49.2	41.77	36.96	32.95	30.34	27.94	**25.92**	**24.32**

Table 12 Comparisons of out-of-control ARL under increasing drift in the standard deviation (σ)

Method	Small to medium rate					Medium to large rate					Large to very large rate					
	Rate (r_σ)															
	0.02	0.04	0.06	0.08	0.1	0.2	0.4	0.6	0.8	1	1.5	2	2.5	3	3.5	4
T^2	24.78	16.14	12.52	10.49	9.12	5.96	3.9	3.06	2.6	2.28	**1.26**	**1.02**	**1**	1	1	1
EWMA-R	20.2	12.85	9.84	8.18	7.09	4.49	2.88	2.22	1.86	1.62	1.88	1.46	1.07	1	1	1
EWMA-3	27.13	17.66	13.68	11.49	10.07	6.74	4.59	3.72	3.2	2.88	1.85	1.4	1.05	1	1	1
CUSUM-3	17.72	11.38	8.78	7.3	6.24	**4**	**2.6**	**2.06**	**1.73**	**1.51**	**1.26**	**1.02**	**1**	1	1	1
MEWMA	16.33	10.92	8.66	7.32	6.55	4.49	3.11	2.52	2.2	1.99	1.94	1.59	1.15	1	1	1
LRT	**16.05**	**10.6**	**8.37**	**7.09**	**6.22**	4.19	2.87	2.28	1.96	1.74	1.65	1.18	1.01	1	1	1

Table 13 Comparisons of out-of-control ARL under decreasing drift shift in the standard deviation (σ)

Method	High small to small rate									
	Rate (r_σ)									
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
T^2	207.92	168.2	137.85	119.25	104.24	92.34	84	76.72	70.36	65.52
EWMA-R	169.79	125.08	99.42	83.2	72.45	64.55	57.54	52.54	47.98	44.4
EWMA-3	108.36	76.14	60.56	50.84	44.38	39.54	35.96	32.82	30.5	28.33
CUSUM-3	185.39	129.28	98.38	81.23	69.34	60.68	54.28	49.06	44.8	41.18
MEWMA	124.64	87.48	68.4	57	49.22	44	39.62	36.17	33.51	31.1
LRT	**104.68**	**72.29**	**56.68**	**47.49**	**41.36**	**36.83**	**33.22**	**30.7**	**28.37**	**26.56**

increasing drift shift for high small rates. However, when the rate increases, the best method changes to EWMA-3, and it remains the best even in medium rate shifts. In detecting the shifts belonged to the third group of shift rates, medium to large rate, CUSUM-3 approach performs better than the other methods. T^2 besides CUSUM-3 perform the best in detecting large to very large shift rates.

In Tables 9 and 10, the out-of-control ARL for increasing drift in the slope A_1 are shown. It can be seen that MEWMA is the best method under high small to small rates. As the rate increases, CUSUM-3 and then T^2 have a better performance than the others. For the groups, medium to large rate and large to high large rate, the performance of all the methods is the same, and their ARLs equal to 1. Thus, they are not shown in a table.

According to the Tables 11 and 12, MEWMA has the best performance under high small to small rate drift in the standard deviation. As the rate increases, LRT performs as the best method. In detecting medium to large rate shifts, CUSUM-3 is the best method among the others. However,

Table 14 Comparisons of out-of-control ARL under decreasing drift shift in the standard deviation (σ)

Method	Small to medium rate					
	Rate (r_σ)					
	0.02	0.04	0.06	0.08	0.1	0.2
T^2	38.4	21.3	14.8	11.41	9.35	4.88
EWMA-R	26.32	15.06	10.68	8.38	6.95	3.82
EWMA-3	17.92	11.33	**8.7**	**7.25**	**6.27**	3.97
CUSUM-3	23.62	13.45	9.64	7.64	6.36	**3.69**
MEWMA	19.48	12.12	9.29	7.69	6.65	4.28
LRT	**17.05**	**11.3**	9.02	7.78	6.97	–

in the last group of rates, both T^2 and CUSUM-3 perform as the best methods.

The magnitude of the intercept and slope decreasing shift rates are considered the same as increasing ones. Since we get roughly the same results, out-of-control ARL, as the ones shown in the Tables 7, 8, 9, and 10, the performance of the methods under decreasing drift in the intercept and slope are not shown here.

The out-of-control ARL values under decreasing drifts in the standard deviation are shown in Tables 13 and 14. In our case, the magnitude of decreasing shift in the standard deviation has to be less than one in order not to have a negative value for standard deviation. Hence, ARLs for the shifts with the rates larger than 1 are not considered in this study. For all methods, ARL cannot be calculated for rates larger than 0.2 because these rates lead to non-positive standard deviation values before a signal is detected. So, the ARLs for rates larger than 0.2 are not reported here, as well. For the LRT control chart, this problem is occurred in rate of 0.2, and hence, the result of ARL is not reported. It can be seen that in detecting high small to small rate shifts, LRT scheme outperforms the others. However, EWMA performs better than other methods in detecting medium rate decreasing shifts in the standard deviation.

Based on the results in Tables 7, 8, 9, 10, 11, 12, 13, and 14, the best methods in detecting increasing and decreasing drift are summarized in Table 15. The sign → in Table 15 shows that in that group as the magnitude of shift increases, the best method changes to the method after the sign →.

In detecting drift, MEWMA by Zou et al. [23] had the best performance for very small intercept shifts. For detecting small to medium rate drift in the intercept, EWMA-3 by Kim et al. [10] outperformed the other methods. CUSUM-3 by Saghaei et al. [17] performed better than the other methods for detecting medium to large and very large drift in the intercept, and T^2 by Kang and Albin [9] had the same performance as CUSUM-3 by Saghaei et al. [17] in detecting large and very large drift. Regarding drift in the slope, MEWMA by Zou et al. [23] was the best in detecting very small to small shifts. CUSUM-3 by Saghaei et al. [17] and T^2 by Kang and Albin [9] performed better in detecting small to medium drifts. As the drift rate increased from medium, the performance of all the methods was the same. In detecting increasing drift in the standard deviation, MEWMA by Zou et al. [23], LRT by Zhang et al. [21], and CUSUM-3 by Saghaei et al. [17] outperformed the others under first, second, and third group of drift rates, respectively. In detecting the fourth group, T^2 by Kang and Albin [9] had the same performance as CUSUM-3 by Saghaei et al. [17] as the best methods. However, in detecting first and second group of decreasing drift shifts in the standard deviation, LRT by Zhang et al. [21] and EWMA-3 by Kim et al. [10] were the best, respectively.

4 Conclusions

In this paper, some of the control schemes for phase II monitoring of simple linear profiles were modified in order to improve their performance in detecting decreasing shifts besides increasing ones. We evaluated the performance of the phase II methods under both step shifts and drift through simulation studies in terms of average run length criterion.

Table 15 The best method in detecting increasing (+) and decreasing (−) drift shifts in different rates of the drifts

Magnitude of drift rate	Parameter			
	Intercept (±)	Slope (±)	Standard deviation (+)	Standard deviation (−)
High small to small	MEWMA→EWMA-3	MEWMA	MEWMA→LRT	LRT
Small to medium	EWMA-3	CUSUM-3→T^2	LRT	EWMA-3
Medium to large	CUSUM-3	The same	CUSUM-3	–
Large to high large	CUSUM-3 and T^2	The same	CUSUM-3 and T^2	–

We studied the performance of those methods under increasing shifts as well as decreasing ones. Based on the results and comparisons which are discussed in details in the section of simulation studies, the CUSUM-3 proposed by Saghaei et al. [17] seems to be the best method under both step shifts and drifts.

References

1. Aerne LA, Champ CW, Rigdon SE (1991) Evaluation of control charts under linear trend. Commun Stat-Theory Methods 20 (10):3341–3349
2. Bissell AF (1984) The performance of control charts and Cusums under linear trend. J Appl Stat 33(2):145–151
3. Croarkin C, Varner R (1982) Measurement assurance for dimensional measurements on integrated-circuit photomasks. NBS technical note 1164. U.S. Department of Commerce, Washington, DC
4. Davis RB, Woodall WH (1988) Performance of the control chart trend rule under linear shift. J Qual Technol 20(4):260–262
5. Divoky JJ, Taylor EW (1995) Detecting process drift with combinations of trend and zonal supplementary runs rules. Int J Qual Reliab Manag 12(2):60–71
6. Fahmy HM, Elsayed EA (2006) Detection of linear trends in process mean. Int J Prod Res 44(3):487–504
7. Gan FF (1991) EWMA control chart under linear drift. J Stat Comput Simul 38(1–4):181–200
8. Gupta S, Montgomery DC, Woodall WH (2006) Performance evaluation of two methods for online monitoring of linear calibration profiles. Int J Prod Res 44(10):1927–1942
9. Kang L, Albin SL (2000) On-line monitoring when the process yields a linear profile. J Qual Technol 32(4):418–426
10. Kim K, Mahmoud MA, Woodall WH (2003) On the monitoring of linear profiles. J Qual Technol 35(3):317–328
11. Niaki STA, Abbasi B, Arkat J (2007) A generalized linear statistical model approach to monitor profiles. Int J Eng Trans A: Basics 20(3):233–242
12. Noorossana R, Amiri A (2007) Enhancement of linear profiles monitoring in phase II. AmirKabir J Sci Technol (in Farsi) 18 (66-B):19–27
13. Noorossana R, Amiri A, Soleimani P (2008) On the monitoring of autocorrelated linear profiles. Commun Stat-Theory Methods 37 (3):425–442
14. Noorossana R, Saghaei A, Amiri A (2011) Statistical analysis of profile monitoring. Wiley, New York
15. Noorossana R, Vaghefi A, Dorri M (2011) Effect of non-normality on the monitoring of simple linear profiles. Qual Reliab Eng Int 27 (4):425–436
16. Reynolds MR, Stoumbos ZG (2001) Individual control schemes for monitoring the mean and variance of processes subject to drifts. Stoch Anal Appl 19(5):863–892
17. Saghaei A, Mehrjoo M, Amiri A (2009) A CUSUM-based method for monitoring simple linear profiles. Int J Adv Manuf Technol 45 (11):1252–1260
18. Soleimani P, Noorossana R, Amiri A (2009) Simple linear profiles monitoring in the presence of within profile autocorrelation. Comput Ind Eng 57(3):1015–1021
19. Woodall WH, Spitzner DJ, Montgomery DC, Gupta S (2004) Using control charts to monitor process and product quality profiles. J Qual Technol 36(3):309–320
20. Woodall WH (2007) Current research on profile monitoring. Revista Producão 17(3):420–425
21. Zhang J, Li Z, Wang Z (2009) Control chart based on likelihood ratio for monitoring linear profiles. Computat Stat Data Anal 53 (4):1440–1448
22. Zou C, Liu Y, Wang Z (2008) Comparisons of control schemes for monitoring the mean of processes subject to drifts. Metrika 70 (2):141–163
23. Zou C, Tsung F, Wang Z (2007) Monitoring general linear profiles using multivariate exponentially weighted moving average schemes. Technometrics 49(4):395–408