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Analysis of Demand Satisfaction Probability and Network Costs in Warehouse Relocation Using Probabilistic Knapsack

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Abstract

Nowadays, the warehouse costs include considerable proportion in supply chain's costs. The competitive markets, uncertainty in demands and actualization of costs for energy consumption lead to decide about relocating the current situation of supply chain. In the reconfiguration of warehouses, the best tactical decisions can be taken for determining the future situations of the network. In this regard, deterministic relocation models have been evaluated in the literature; however, in this paper, probabilistic analysis for the relocation of warehouses in a supply chain has been investigated. In the proposed approach, desirable confidence level is integrated with the relocation model, so that the manager can analyze the demand satisfaction probability in comparison with costs. A hypothetical numerical example shows how the proposed MILP (mixed integer linear programming) model works on a supply chain network. Finally, the results demonstrate that relocation cost is less than current system's; hence, using the proposed comprehensive analysis which describes the tradeoff between the possible budgets of the network and the corresponding confidence levels of satisfying the demands, can be suggested to the management.

Keywords
Supply chain, Warehouse relocation, Demand satisfaction probability, Knapsack problem

1. Introduction

A supply chain network comprises echelons such as plant, warehouse, distribution center and customer. This network flows the manufactured products from the plant to customers considering the customer satisfaction and optimum cost (Drezner and Hamacher 2004). There are researches for locating the facilities in the candidate regions of supply chain networks but the managers wish to check the efficacy of their system and other less costly configurations. For this purpose, some of the related questions for redesigning the facilities in each echelon are given as: "Which facilities should be retained, established, eliminated or consolidated?"

All of the above questions and related concepts are integrated in a novel research area named relocation models. In this paper, there are probabilistic scenarios for the demands. By applying one of the chance constraint approaches, we propose a probabilistic model by considering the confidence level for demand satisfaction so that customer satisfaction is related to the probability of unmissed demands.

As will be shown, we want to investigate the approach that provides an analytical tool for evaluating the relation between increasing the risk and decreasing the costs while some forced constraints for demand satisfaction probability and available budget are considered.

The rest of this paper is organized as follows: The literature of supply chain location and relocation is discussed in section 2. The proposed model for reconfigured warehouses in a supply chain is presented in section 3. Then, results of a hypothetical example are illustrated in section 4, moreover, problem solutions and method analysis are explained in section 4 and finally, conclusion is discussed in the last section.

2. Literature Review

The most recent comprehensive review for facility location and supply chain network design (SCND) demonstrated that most of the literature deals with deterministic models rather than stochastic ones (approximately 82% against 18%) (Melo et al. 2009) while uncertainty is more applicable in real environment. On the other hand, as mentioned before, the managers want to evaluate their supply chain's efficiency and productivity. Consequently, relocation models are capable approaches for proposing the best configuration of the SCND. The advantage of relocation models is in considering the whole relocation costs, which have been ignored in the location models. These costs consist of moving costs, consolidating, extending, etc.

Uncertainty in facility location especially in SCND has received significant attention in recent studies. In the most of them, customer demand is considered as an uncertain parameter. In this regard, the researchers try to conquest uncertainty situation and propose a robust and reliable solution (see Aghezzaf 2005, Chan et al. 2001, Goh et al. 2007, Bidhandi and Yusuff 2011).

By surveying the related literature, we can conclude that a high proportion of methods are based on expected value, chance constraint (Charnes and Cooper 1962) and recently, two-stage stochastic programming (Santoso et al. 2005).

In addition, relocation models have begun to appear in the literature in the past decade because of its necessity. According to Ballou and Masters's investigation (1993) of 200 logistics executives, 65% of the respondents indicated that they decided to evaluate their current warehouse network and consider relocating it in the near future. In this area, Melachrinoudis and Min (2007) have presented a relocation model on warehouse design for reducing the cost and assessing the current situation of the system. Also, Melachrinoudis and Min (2000) and Min and Melachrinoudis (1999) focused on the relocation of facility with deterministic parameters. Melachrinoudis and Min (2000) was presented a relocation model by considering cost and time as objectives. Melo et al. (2006) have presented dynamic planning horizon, by considering availability of capital for investments, and storage limitations in their model. In their model, external capacities cannot be added in the problem. Although all of the relocation models have been introduced in deterministic environment, real world cases usually are uncertain based.

In this paper, a new approach for redesigning the warehouse in a supply chain network with probabilistic demands is presented. Since consideration of the uncertainty in relocation model has been ignored in the recent researches (Melo et al. 2009), it seems that the proposed approach can contribute in developing the literature of redesigning the warehouses in SCND. The proposed approach suggests the optimum relocation of warehouses in a SCND with consideration of tradeoff between desirable confidence level and available costs.

3. Problem description and Model Development

Consider a supply chain network consists of plants, warehouses and customers. The plant manufactures many products and sends them to capacitated warehouses according to requested products. In the current system that is active now, the manager wants to evaluate optimality of his/her system regarding to the probabilistic demands. According to the cost, top manager decides to satisfy a proportion of demands with desirable reliability level. In another words, costly demand scenarios with low probability can be eliminated based on the defined risk interval.

As mentioned before, in this paper, confidence level in customer satisfaction is considered for performing the tradeoff between reliability to observe the demand satisfaction probability and available budget. To reach this aim, we extend the model which has been presented in (Melachrinoudis and Min 2007). Based upon this, knapsack approach is used to define the confidence level. Knapsack approach (for more understanding about probabilistic knapsack approach see Bruni 2009) is characterized by the allocation of limited resources to competing items. Items are associated with resource requirements as well as rewards. In this paper, we have a knapsack with (1-α)% volume. Each scenario has a certain volume and individual effect on minimization of costs. Therefore, this approach adds big M technique and 0-1 decision variable to decide about the constraint, which can be removed from the knapsack based on ignoring α% (as maximum risk-value) of overall knapsack volume. Therefore, the manager accepts α% risk in demand satisfaction for saving the budget.

In this section, the proposed model for redesigning the warehouses configuration in a supply chain network and its extension in a probabilistic environment are introduced as follows:

3.1. Indices and Sets

s=index for scenarios($s \in S$),
p=index for manufacturing plants ($p \in P$),
k=index for customers ($k \in K$),

o=index for product ($o \in O$),
i=index for existing warehouses and new candidate sites for relocation and consolidation ($i \in A$).
$A = E \cup N; (j,i) \in (E \times A)$, Where E is set of existing warehouses and N is set for new candidates. Note that existing warehouses can be consolidated with both N and E sets.

3.2. Parameters

pr_{sko}= occurrence probability for demand of customer k and product o in s^{th} scenario,
d_{sko}= demand of customer k in scenario s for product o,
v_{pio}= unit production cost at plant p for product o (including manufacturing cost and shipment cost between plant p and warehouse i),
w_{iko}= unit warehousing cost at warehouse i and shipment cost between warehouse i and customer k for o^{th} product,
r_{ji}= cost of moving and relocating the capacity and facilities of warehouse j to warehouse i ($j \neq i$) and saved cost achieved (income) from closure of existing warehouse j excluding its relocated equipment,
c_{io}= throughput capacity of warehouse i for product o,
cp_{ik}= Capacity to carry goods between warehouse i and customer k,
q_{po}= production capacity of plant p for product o,
fv_{io}= cost per unit capacity of warehouse i for product o,
fc_i= fixed cost of retaining warehouse i excluding capacity cost,
fs_i= saved cost achieved from closure of existing warehouse i (both of the warehouse and equipment),
fl_{ik}=fixed cost of relation between warehouse i and customer k,
ce_{po}= cost of extra unit for extending the manufacturing capacity for product o,
re_o= Space usage of one unit of product o,
(α_{ko},M)= α_{ko} and M are risk interval and big-M in knapsack problem, respectively,
b_{ik}=coverage matrix (warehouse i and customer k),

3.3. Decision variables

y_{pio}= volume of product o shipped by plant p to warehouse i,
x_{iko}= volume of product o shipped from warehouse i to customer k,
e_{po}= volume of extra production capacity (at plant p, for product o) needed for satisfying customer demands,

$$z_{ji} = \begin{cases} 1, \text{if capacity of warehouse } j \in E \text{ is relocated to site } i \in A, i \neq j \text{ or if existing warehouse } j \in E, i = j \\ \quad \text{remains open} \\ 0, \text{other wise} \end{cases}$$

$$z_{nn} = \begin{cases} 1, \text{if a new warehouse } n \in N \text{ stablishes in candidate site } n \in N \\ 0, \text{other wise} \end{cases}$$

δ_{sko} =1, if s^{th} scenario of customer k and product o is eliminated from decision space.

3.4. Mathematical Model

The objective function and the constraints of the proposed model are presented as follows:

$$\begin{aligned} Min \sum_{p\in P}\sum_{i\in A}\sum_{o\in O} v_{pio} y_{pio} + \sum_{i\in A}\sum_{k\in K}\sum_{o\in O} w_{iko} x_{iko} + \sum_{j\in E,(j\neq i)}\sum_{i\in A} r_{ji} z_{ji} + \sum_{i\in A}\sum_{o\in O} fv_{io} \sum_{j\in E} c_{jo} z_{ji} + \sum_{(i=j)\in E}\sum_{j} fc_i z_{ji} \\ + \sum_{i\in N} fc_i z_{ii} - \sum_{j\in E}\left[fs_j \left(1 - \sum_{i\in A} z_{ji}\right) + fc_j \sum_{i\in E, i\neq j} z_{ji} \right] + \sum_{p\in P}\sum_{o\in O} ce_{po} e_{po} + \sum_{k\in K}\sum_{i\in A} fl_{ik} l_{ik}, \end{aligned} \tag{1}$$

S.t.

$$\sum_{i\in A} y_{pio} \leq q_{po} + e_{po} \quad ,\forall p \in P, \forall o \in O \tag{2}$$

$$\sum_{p\in P} y_{pio} = \sum_{k\in K} x_{iko} \quad ,\forall i \in A, \forall o \in O \tag{3}$$

$$\sum_{k\in K} x_{iko} \leq \sum_{j\in E} c_{jo} z_{ji} \quad ,\forall i \in E, \forall o \in O \tag{4}$$

$$\sum_{k\in K} x_{iko} - c_{io} z_{ii} \leq \sum_{j\in E} c_{jo} z_{ji} \quad ,\forall i \in N, o \in O \tag{5}$$

$$\sum_{i \in A} b_{ik} x_{iko} + \delta_{sko} M \geq d_{sko} \quad, \forall k \in K, o \in O, s \in S \tag{6}$$

$$\sum_{s \in S} pr_{sko} \delta_{sko} \leq \alpha_{ko} \quad, \forall k \in K, \forall o \in O \tag{7}$$

$$\sum_{o \in O} x_{iko} re_o \leq cp_{ik} \; , \forall i \in A, k \in K \tag{8}$$

$$\sum_{j \in E} z_{ji} \leq |E| z_{ii} \quad, \forall i \in E \tag{9}$$

$$\sum_{j \in E} z_{ji} \leq |E| z_{ii} \quad, \forall i \in N \tag{10}$$

$$\sum_{i \in A} z_{ji} \leq 1 \quad, \forall j \in E \tag{11}$$

$$y_{pio} \geq 0 \quad, \forall p \in P, i \in A, o \in O \tag{12}$$

$$x_{iko} \geq 0 \quad, \forall i \in A, k \in K, o \in O \tag{13}$$

$$z_{ji}, z_{ii} \in (0,1) \; , \forall j \in E, i \in A \tag{14}$$

$$\delta_{sko} \in (0,1), \forall s \in S, \forall k \in K, o \in O \tag{15}$$

The objective function minimizes the total supply chain cost comprised of production, transportation and relocation in which, related costs including moving and relocating the consolidated warehouses, maintaining the existing and new warehouses and cost savings resulting from the closure or consolidation of redundant warehouses. Also the cost of extending the capacity and maintaining the warehouses are given in equation (1). Constraints (2) insure that the volume of products shipped to warehouses do not exceed the capacity of a manufacturing plant supplying such products and needed product for extension of capacity. Equations (3) assure that the total volume of products supplied by the plant to each warehouse equals to the total volume of products shipped from that warehouse to its customers. Constraints (4)-(5) insure that the total volume of products shipped to customers after consolidation cannot surpass the throughput capacity of the warehouse serving them. Constraints (6)-(7) emphasize on knapsack problem for consideration of demand satisfaction with at least 1-α % confidence level. Also, we can specify especial confidence level (or 1-risk) for demand satisfaction based on each product and each customer. This relation demonstrates constraints, which should be removed from the MILP problem based on risk interval to have more effect on cost reduction. Moreover, theses constraints ensure that customer demands are satisfied through warehouses inside coverage radius.

Constraints (8) denote that the volume of products shipped from warehouse to a customer do not exceed the maximum transit capacity. Constraints (9) states that an existing warehouse cannot be consolidated into another existing one, unless such consolidated warehouse remains open. Also, $|E|$ is the cardinality of set E resulted from aggregation of constraints over set E with the equal right hand side (RHS) constraint (9). Similarly, constraints (10) have the same concept of previous constraint but for consolidation of existing warehouses into new warehouses. Constraints (11) denote that each warehouse can merge with only one of the destination warehouses. For more understanding, we describe the whole possibilities for z_{ii}. For $j \in E$, $z_{jj} = 1$ if the existing warehouse j remains open. Also, for $i \in A$ and $z_{ji} = 1$ ($i \neq j$), existing warehouse j is consolidated into warehouse i. Note that for $n \in N$ and $z_{nn} = 1$, the new warehouse n is established in nth candidate site. Also, $\sum_{i \in A} z_{ji} = 0$ demonstrates that warehouse j has been redundant and should be eliminated from the supply chain network. Constraints (12) and (13) assure decision variables positivity. Constraints (14)-(15) states that variables are in binary type.

4. Numerical Example

In this section, a hypothetical example is evaluated to show the proposed model more clearly. The example contains two manufacturing plants, three existing warehouses, two candidate sites for establishing the new warehouses and three customers (totally ten facilities). Note that demands occur in discrete scenarios with certain probability. The demands of the first and second customers occur in twenty and eight scenarios, respectively. In addition, third customer has a deterministic demand. The considered example is solved by GAMS software (CPLEX solver). Demand scenarios are expressed in table 1.

In this section, we consider three situations for studying the problem. At the first one, we suppose that the current system continues to work with the three existing warehouses. At the second situation, the relocation costs can be compared to that first cost attained by continuing the current status. After that, risk analysis respected to cost can be evaluated. Therefore, decision maker can select the best decision variable based on demand satisfaction probability (or confidence level) and available budget constraint. As it can be seen in Table 2, increasing the risk of shortfall leads to decreasing the needed budget. This cost reduction is due to ignoring the demand with the least occurrence probability in the risk interval that has more effect on increasing the total cost.

For more understanding, suppose that we want to relocate our supply chain considering 75% demand covering probability and maximum budget equals to 30000 units. According to sensitivity analysis illustrated in figure 1, we can state this situation is possible. Moreover, this figure confirms that we can improve demand satisfaction probability up to (around) 90% by increasing the available budget up to 30950 units.

Table 1: Demand of each customer and its occurrence probability

Scenarios(s)	Customer Demand				
	d_{s1}	pr_{s1}	d_{s2}	pr_{s2}	d_{s3} (Deterministic)
1	380	0.04	100	0.1	60
2	350	0.05	103	0.04	
3	300	0.19	109	0.03	
4	390	0.02	105	0.01	
5	320	0.01	115	0.02	
6	400	0.2	108	0.23	
7	500	0.04	101	0.07	
8	317	0.04	120	0.5	
9	410	0.04			
10	405	0.01			
11	295	0.005			
12	319	0.002			
13	389	0.003			
14	379	0.01			
15	356	0.01			
16	329	0.005			
17	377	0.01			
18	367	0.005			
19	387	0.3			
20	549	0.05			

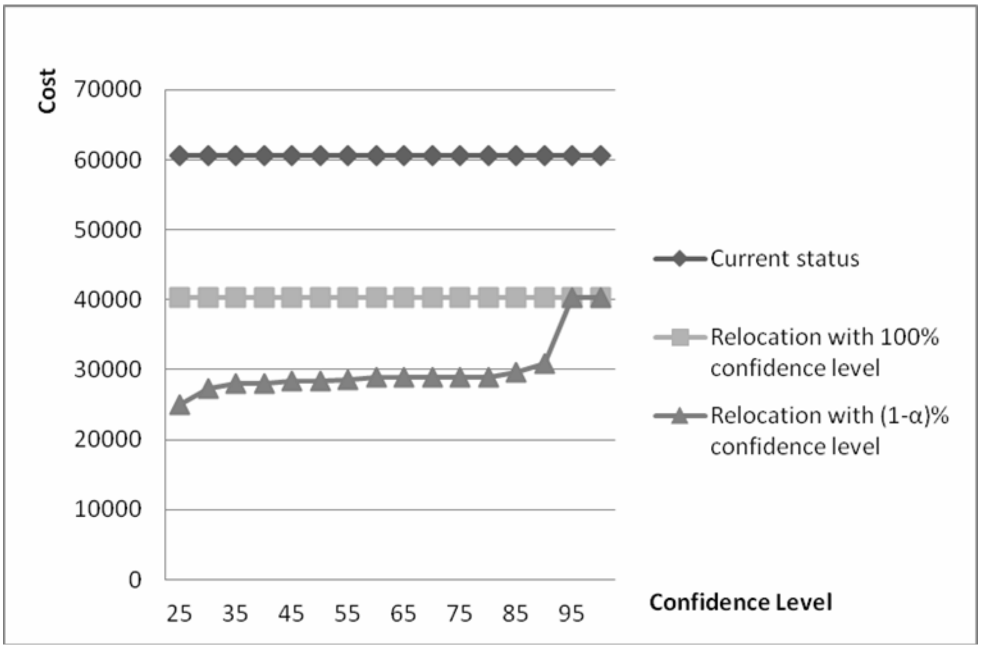

Figure 1: Analysis of Confidence level versus costs based on proposed approch

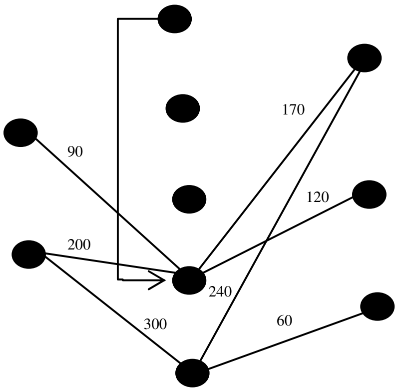

Figure 2: Optimum SCND for numerical example considering 90% confidence level

Table 2: Comparison of current and other network configurations' cost for the hypothetical example

	Current system	Relocation with 100% reliability	Relocation with 95% reliability	Relocation with 90% reliability
Total Costs	60565	40394	40394	30905

Figure 2 shows the active facilities and consolidated warehouses considering 90% confidence level for demand satisfaction. Moreover, this figure indicates the optimum volume of products for shipment between echelons.

Suppose that existing warehouse are labeled with numbers 1 to 3 and new warehouses are labeled 4 and 5. In figure 2, we can state that existing warehouse 1 is consolidated to new warehouse 4. Furthermore, warehouses 2 and 3 are redundant.

In this numerical example, we considered one product and unique risk value for all customers but in the real cases some customers and products have significant and strategic role in SCM, so according to proposed model, we can allocate different confidence level for each product and customer to present more precise analysis on producing in the future.

5. Conclusion

In this paper, a probabilistic approach on redesigning the warehouse in a supply chain network was studied. We proposed MILP model with combination of relocation model and knapsack approach to propose the procedure that provides an analytical tool for evaluating the relation between increasing the risks and decreasing the costs while we have some constraints for demand satisfaction probability and available budget. In this regard, a numerical example was considered with scenario-based demands. We compared cost of three strategies. The results show that relocation with 100% reliability has the less cost for future configuration in comparison with the recent status. Also, according to the proposed approach, top manager can analyze relocation costs by tracing the effect of reducing the demand satisfaction probability on costs and then, he/she can determine the new configuration of supply chain. As a future research, mentioned model can be resolved by other stochastic programming approaches such as two stage stochastic programming. Moreover, for continuous probability distribution, simulation and approximation methods can be applied for transforming the distribution to discrete scenarios.

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