# Robust multi objective H2/H∞ Control of nonlinear uncertain systems using multiple linear model and ANFIS

Vahid Azimi, *Member, IEEE*, Peyman Akhlaghi, and Mohammad Hossein Kazemi

*Abstract*— **This paper considers the multi objective robust control for nonlinear systems using multiple model and adaptive neuro fuzzy inference system (ANFIS). Nonlinear system divided to multiple linear model based on piecewise linearization around set points. For each linear model multi objective robust controller is designed to guarantee both system performance and robust stability. To design the robust controller for each linear model, nonlinearity part of system consider as uncertainty of linear system. The advantages of proposed method are that it can reduce the parameter perturbation and nonlinear uncertainty as well as increased the performance of the system. In this work ANFIS is used to make decision and combine the designed controller for each linear model. Simulation result on a nonlinear benchmark plant shows the effectiveness of the proposed method.**

Key words: Robust, Nonlinear, Multi objective, ANFIS, H infinity, $H_2$

## I. Introduction

In real applications model uncertainty and disturbances are inevitable parts of systems. Robustness is one of the most important requirements for a control system. Recently, extensive research has been done in the problems of robust stabilization and robust performance of uncertain dynamical systems. However, very few studies consider nonlinear system in this field. In this paper, we propose the multi objective H2/H∞ robust control for nonlinear systems.

For nonlinear systems, extended from linear methods, the H∞ filtering problems are solved by applying some methods such as game theoretic approach [1], dynamic programming approach [2], etc. However, all these Methods have not provided clear demonstration and systematic tuning procedures for general nonlinear systems.

In many applications, the controller should consider the immunity of whole system in counter of disturbances. In many cases, controllers design around an operating point. So, they cover only a limited range of operation and cannot perform properly in other operating range. When the operating space is very large, it is necessary to employ an approach to adjust the controller for each desired operating state. Robust controller techniques of linear system are well developed. As a result, in this paper, the non-linear system is linearized by piecewise linearization and un-modeled non-linear part converted into parameter uncertainty.

In our scheme, Piece-Wise linearization is used. By using this method, the analysis of nonlinear system can be done by the analysis of several linear systems. The basic of this technique is model each of the system's nonlinearities with a suitable piecewise linear model that covers the dynamic range of the state variables. By using the first two terms of the Taylor series for the nonlinear function, it can be modeled as f(x(t)) = ax(t)+b. Therefore the nonlinear term can be express as a multiple linear models.

In multiple-model framework, a local controller should have enough robustness to guarantee global stability. Consequently, for each local linear model, a multi objective H2/H∞ robust control is designed as local controller to deal with uncertainties of the model. We consider three uncertainties in our work. 1. The un-modeled non-linear part which converted into parameter uncertainty. 2. Inherent uncertainty of system which in this paper we consider it as an uncertainty of the actuators. 3. Measurement uncertainties which emerge from inaccuracy of the sensors .After obtaining the local controllers, in order to form complete multi-model controller, an ANFIS is used to make decision and switch between local controllers based on the condition of the system.

ANFIS is a hybrid method it uses both ANN and FIS capabilities for modeling the nonlinear systems and also classification. ANN is using in tuning the rule-based fuzzy systems to approximate the way human processes information. The ANFIS learning algorithm is a hybrid algorithm consisting of the gradient descent and the least-squares estimation. After training the ANFIS, it can select appropriate controller based on its inputs [3].

An example shows that the proposed structure using multiple linear model of a system is simple and practical controller design method for nonlinear systems and excellent robust performance can be achieved.

The organization of this paper is as follows. Section II shows the piecewise linearization of a nonlinear system and creation of a multiple linear model. Section III outlines the multi objective H2/H∞ framework and stability arguments. Section IV describes ANFIS method and its application in our scheme. Section V discusses in detail the proposed

V. Azimi is graduate student of Islamic Azad University, South Tehran Branch,Tehran, Iran email: vahid.azimi@ieee.org.

P. Akhlaghi was graduate student of Islamic Azad University, South Tehran Branch,Tehran, Iran and member of Young Researchers Club (corresponding author to provide phone: +98-281-3336395; e-mail: peyman.akhlaghi@gmail.com).

Dr. M. H. Kazemi, is from electrical engineering department of Shahed University, Tehran, Iran e-mail: kazemi@shahed.ac.ir.

strategy, and finally, section VI exhibits the simulations that were carried out to validate our approach.

## II. MULTIPLE LINEAR MODEL

The linearization process presented in this work proceeds by replacing the non-linear curves with piece-wise linear representations, with each curve being made up of one or more straight-line segments. These straight-line segments are then used to design robust controller for each local linear model. The accuracy of this method depends on the number of piece-wise linear segments that are used to approximate a nonlinear function. In other hand, increasing the number of the piece-wise will increase complication of the linearization equation and computing time.

The nonlinear function can be modeled as a series of piecewise connected lines. If the line segments are close enough together, there will be very little error between the original function and the piecewise linear representation. Using the first two terms of the Taylor series for the nonlinear function, we can model the function as f(x(t))=ax(t)+b.

However, the Taylor series would have evaluated the derivative of the function at the point we choose to evaluate the function. This, then, would be used as the slope term p.

Instead of doing this, we will basically connect points on the nonlinear curve with line segments as shown in the Fig. 1.

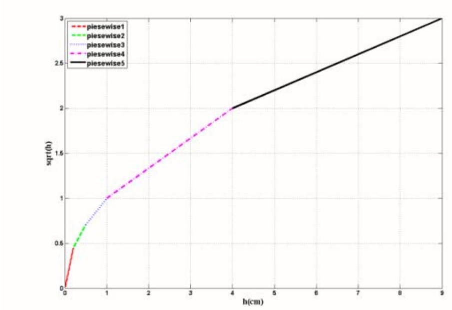

Fig. 1. Piecewise linear model of the nonlinear curve.

Using the additional degrees of freedom, the system capable of dealing with nonlinearly parameterized systems, by treating them as piece-wise linearly parameterized ones and treating the error between the piece-wise linear function and the original nonlinear function over the unknown parameter region as a disturbance.

## III. MULTI OBJECTIVE H2/H∞ FRAMEWORK

The objectives of multi objective controller are:

- $H_\infty$ performance (for tracking, disturbance rejection, or robustness aspects)
- $H_2$ performance (for LQG aspects)
- Robust pole placement specifications (to ensure fast and well-damped transient responses, reasonable feedback gain, etc.)

Denoting by $T_\infty(s)$ and $T_2(s)$ the closed-loop transfer functions from w to $z_\infty$ and $z_2$, respectively, our goal is to design a state-feedback law u = Kx that:

•Maintains the RMS gain ($H_\infty$ norm) of $T_\infty$ below some prescribed value $\gamma_0 > 0$

•Maintains the $H_2$ norm of $T_2$ (LQG cost) below some prescribed value $\nu_0 > 0$

•Minimizes an $H_2/H_\infty$ trade-off criterion of the form $\alpha\|T_\infty\|_\infty{}^2 + \beta\|T_2\|_2{}^2$

•Places the closed-loop poles in a prescribed region $D$ of the open left-half plane[4].

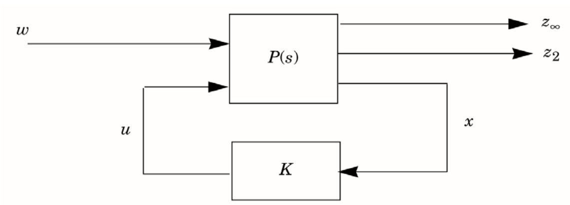


Fig. 2. Control structure.

The mixed $H_2/H_\infty$ criterion takes into account both the disturbance rejection aspects (RMS gain from d to e) and the LQG aspects ($H_2$ norm from n to $z_2$) [5]. In addition, the closed-loop poles can be forced into some sector of the stable half-plane to obtain well-damped transient responses. The concept of LMI region is useful to formulate pole placement objectives in LMI terms. LMI regions are convex subsets $D$ of the complex plane characterized by

$$D = \{z \in C : L + Mz + M^T\bar{z} < 0\} \quad (1)$$

where M and L = $L^T$ are fixed real matrices. The matrix-valued function

$$f_D(z) := L + Mz + M^T\bar{z} \quad (2)$$

is called the characteristic function of the region $D$. The class of LMI regions is fairly general since its closure is the set of convex regions symmetric with respect to the real axis. More practically, LMI regions include relevant regions such as sectors, disks, conics, strips, etc., as well as any intersection of the above.

Another strength of LMI regions is the availability of a "Lyapunov theorem" for such regions. Specifically, if $\{\lambda_{ij}\}_{1\le i,j\le m}$ and $\{\mu_{ij}\}_{1\le i,j\le m}$ denote the entries of the matrices $L$ and $M$, a matrix A has all its eigenvalues in $D$ if and only if there exists a positive definite matrix $P$ such that [3]

$$\left[\lambda_{ij}P + \mu_{ij}AP + \mu_{ji}PA^T\right]_{1\le i,j\le m} < 0 \quad (3)$$

with the notation

$$[S_{ij}]_{1\le i,j\le m} := \begin{bmatrix} S_{11} & \dots & S_{i1} \\ \vdots & \ddots & \vdots \\ S_{m1} & \dots & S_{mm} \end{bmatrix} \quad (4)$$

Note that this condition is an LMI in P and that the classical Lyapunov theorem corresponds to the special case

$$f_D(z) = z + \bar{z} \quad (5)$$

LMI f given a state-space realization

$$\begin{cases}\dot{x} &= Ax + B_1 w + B_2 u \\ z_\infty &= C_1 x + D_{11} w + D_{12} u \\ z_2 &= C_2 x + D_{21} w + D_{22} u\end{cases} \tag{6}$$

of the plant $P$ (Fig. 2), the closed-loop system is given in state-space form by

$$\begin{cases}\dot{x} &= (A + B_2 K)x + B_1 w \\ z_\infty &= (C_1 + D_{12}K)x + D_{11} w \\ z_2 &= (C_2 + D_{22}K)x + D_{21} w\end{cases} \tag{7}$$

Taken separately, our three design objectives have the following LMI formulation:

•$H_\infty$ **performance[6, 7]:** the closed-loop RMS gain from $w$ to $z_\infty$ does not exceed $\gamma$ if and only if there exists a symmetric matrix $X_\infty$ such that [5]

$$\begin{bmatrix}(A + B_2K)X_\infty + X_\infty(A + B_2K)^T & B_1 & X_\infty(C_1 + D_{12}K)^T \\ {B_1}^T & -I & {D_{11}}^T \\ (C_1 + D_{12}K)X_\infty & D_{11} & -\gamma^2 I\end{bmatrix} < 0$$
$$X_\infty > 0 \tag{8}$$

$H_2$ **performance[6, 7]:** the closed-loop $H_2$ norm of $T_2$ does not exceed $\nu$ if there exist two symmetric matrices $X_2$ and $Q$ such that

$$\begin{bmatrix}(A + B_2K)X_2 + X_2(A + B_2K)^T & B_1 \\ {B_1}^T & -I\end{bmatrix} < 0$$
$$\begin{bmatrix}Q & (C_2 + D_{22}K)X_2 \\ X_2(C_2 + D_{22}K)^T & X_2\end{bmatrix} > 0$$
$$Trace(Q) < \nu^2 \tag{9}$$

**Pole placement:** the closed-loop poles lie in the LMI region

$$D = \{z \in C : L + Mz + M^T\bar{z} < 0\} \tag{10}$$

Where

$$L = L^T = \{\lambda_{ij}\}_{1\le i,j\le m}$$
$$M = \{\mu_{ij}\}_{1\le i,j\le m} \tag{11}$$

if and only if there exists a symmetric matrix $X_{pol}$ satisfying

$$\left[\lambda_{ij}X_{pol} + \mu_{ij}(A + B_2K)X_{pol} + X_{ij}X_{pol} + \mu_{ji}X_{pol}(A + B_2K)^T\right]_{1\le i,j\le m} < 0$$
$$X_{pol} > 0 \tag{12}$$

These three sets of conditions add up to a nonconvex optimization problem with variables $Q$, $K$, $X_\infty$, $X_2$ and $X_{pol}$. For tractability in the LMI framework, we seek a single Lyapunov matrix:

$$X := X_\infty = X_2 = X_{pol} \tag{13}$$

that enforces all three objectives. With the change of variable $Y := KX$, this leads to the following suboptimal LMI formulation of our multi-objective state-feedback synthesis problem [4, 3, 2]:

Minimize $\alpha\gamma^2 + \beta$ Trace($Q$) over $Y$, $X$, $Q$, and $\gamma^2$ satisfying

$$\begin{bmatrix}AX + XA^T + B_2Y + Y^T{B_2}^T & B_1 & X{C_1}^T + Y^T{D_{12}}^T \\ {B_1}^T & -I & {D_{11}}^T \\ C_1X + D_{12}Y & D_{11} & -\gamma^2 I\end{bmatrix} < 0$$
$$\begin{bmatrix}Q & C_2X + D_{22}Y \\ X{C_2}^T + Y^T{D_{22}}^T & X\end{bmatrix} > 0$$
$$\left[\lambda_{ij} + \mu_{ij}(AX + B_2Y)X_{pol} + \mu_{ji}(XA^T + Y^T{B_2}^T)\right]_{1\le i,j\le m} < 0$$
$$Trace(Q) < {\nu_0}^2$$
$$\gamma^2 < {\gamma_0}^2 \tag{14}$$

Denoting the optimal solution by ($X^*$, $Y^*$, $Q^*$, $\gamma^*$ ), the corresponding state-feedback gain is given by

$$K^* = Y^*(X^*)^{-1} \tag{15}$$

and this gain guarantees the worst-case performances[4]:

$$\|T_\infty\|_\infty < \gamma^*$$
$$\|T_2\|_2 < \sqrt{Trace(Q^*)} \tag{16}$$

## IV. ANFIS AND DECISION MAKING

Adaptive neuro-fuzzy networks are enhanced Fuzzy Logic System with learning, generalization and adaptively capabilities. These networks encode the fuzzy if-then rules into a neural network-like structure and then use appropriate learning algorithms to minimize the output error based on the training/validation data sets[3]. In this paper, the ANFIS structure is used because of its accuracy and flexibility. The ANFIS is a fuzzy Sugeno model of integration where the final fuzzy inference system is optimized via the ANNs training. To present the ANFIS architecture, two fuzzy if-then rules based on a first order Sugeno model are considered:

*Rule 1: If (x is $A_1$) and (y is $B_1$) then ($z_1 = p_1x + q_1y + r_1$),*

*Rule 2: If (x is $A_2$) and (y is $B_2$) then ($z_2 = p_2x + q_2y + r_2$),*

where x and y are the inputs, $A_i$ and $B_i$ are the fuzzy sets, $z_i$(i = 1,2) are the outputs within the fuzzy region specified by the fuzzy rules, $p_i$, $q_i$ and $r_i$ are the design parameters ANFIS architecture to implement these two rules is shown in Fig. II, in which a circle stands for a fixed node, whereas a square indicates an adaptive node (Fig. 3).

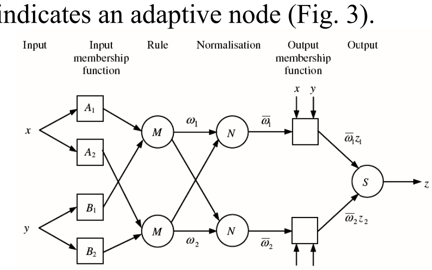


Fig. 3. ANFIS structure used in the proposed method.

The ANFIS learning algorithm is then used to obtain these parameters. This learning algorithm is a hybrid algorithm consisting of the gradient descent and the least-squares estimate. Using this hybrid algorithm, the rule parameters are recursively updated until acceptable error is reached. In the defuzzification layer, crisp output is produced from the output of the inference layer. The ANFIS used in this approach employs Gaussian functions for fuzzy sets, constant functions for the outputs rules and Sugeno's inference mechanism. The parameters of the network are the mean and standard deviation of the membership functions (antecedent parameters).

## V. PROPOSED STRUCTURE

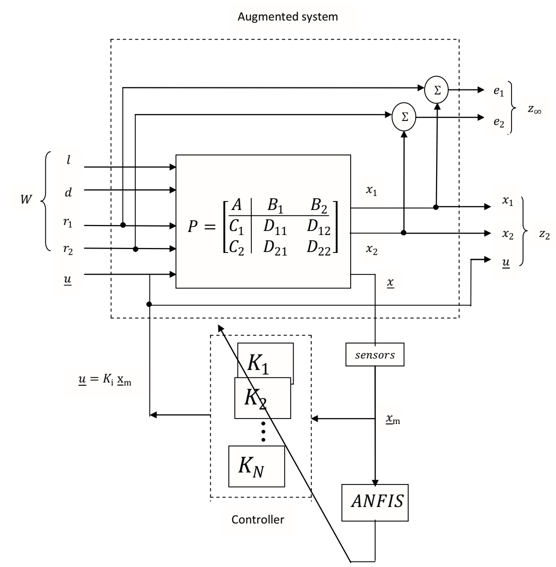


Fig. 4. Block diagram of robust controller.

Gain Scheduling is a widely-used technique for controlling certain classes of nonlinear or LTI systems. Rather than seeking a single robust LTI controller for the entire operating range, Gain Scheduling consists in designing an LTI controller for each operating point and in switching controllers when the operating conditions change. This section presents block diagram for designing Gain Schedule multi objective H2/H∞ framework (Fig. 4). In this structure to switch between different controllers in each operating point, ANFIS as a decision making and supervisor have been used. ANFIS has trained with input data and measurements (liquid levels) to detect that which controller should have been used to guarantee the robust performance of the entire system.

## VI. SIMULATION

In the simulation scenarios, nonlinear differential equations for the water mass balance in the tanks are used to represent the experimental system, which are given in (17)[8](Fig. 5). In this model, Bernoulli's law is used for the flows out of the tanks, $A_i$ is the cross-sectional area, hi is the liquid level, and $a_i$ is the outlet cross-sectional area of tank i; $v_j$ is the speed, $k_j$ the corresponding gain, and $c_j$ is the portion of the flow that goes to the upper tank from pump j [8-10].

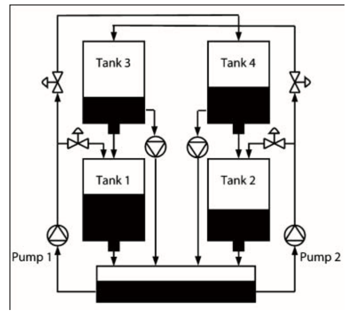


Fig. 5. Four tank system.

In this case study, the system model is expanded to include the pump dynamics between the control signals $u_j$ and the actual speeds $v_j$ as a first order lag with unit gain and time constant $s_i$

$$\dot{h}_1 = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\eta_1 k_1}{A_1}v_1$$
$$\dot{h}_2 = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\eta_2 k_2}{A_2}v_2$$
$$\dot{h}_3 = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\eta_2)k_2}{A_3}v_2$$
$$\dot{h}_4 = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\eta_1)k_1}{A_4}v_1$$
$$\dot{v}_5 = -\frac{v_1}{\tau_1} + \frac{1}{\tau_1}u_1 + \frac{d}{A_1}$$
$$\dot{v}_6 = -\frac{v_2}{\tau_2} + \frac{1}{\tau_2}u_2 + \frac{d}{A_2} \qquad (17)$$

TABLE I
UNITS FOR TANK PROPERTIES

| Symbol | State/Parameters | Value |
|---|---|---|
| $a_i$ | Area of the drain in tank i | 0.2, 0.2, 0.25, 0.25 cm |
| $A_i$ | Areas of the tanks | 15.37 cm$^2$ |
| $\eta_1$ | Ratio of flow in tank 1 to flow in tank 4 | 0.2 |
| $\eta_2$ | Ratio of flow in tank 2 to flow in tank 3 | 0.8 |
| $k_j$ | Pump proportionality constants moment | 7.45, 7.3 cm$^3$/(s%) |
| $g$ | Gravitational acceleration | 981 cm/s$^2$ |
| $\tau$i | Pump response time constants | 2.0, 2.1 s |

In our simulation, disturbance is in the velocity of the pumps.

By using the Taylor series the nonlinear function converted to multiple linear models in deferent operation points. $A_{hi}$ and $b_{hi}$ are the parameter of linearization which they have some uncertainties in each piecewise. These uncertainties occurred because they cannot exactly match to nonlinear model. After some trial and error and considering of nonlinearity of the curve in lower height, the best distance for piecewise linearization is obtained.

$$\sqrt{h_i(t)} = a_{h_i} h_i(t) + b_{h_i}$$

$$\sqrt{h_i(t)} = \begin{cases} 2.2361\, h_i(t) & for \quad 0 \le h_i(t) < 0.2 \\ 0.8663\, h_i(t) + 0.2740 & for \quad 0.2 \le h_i(t) < 0.5 \\ 0.5858\, h_i(t) + 0.4142 & for \quad 0.5 \le h_i(t) < 1 \\ 0.3333\, h_i(t) + 0.6667 & for \quad 1 \le h_i(t) < 4 \\ 0.2\, h_i(t) + 1.2 & for \quad 4 \le h_i(t) < 9 \end{cases}$$

$$a_{h_i} \in [\underline{a}_{h_i} \quad \overline{a}_{h_i}] \qquad b_{h_i} \in [\underline{b}_{h_i} \quad \overline{b}_{h_i}] \qquad 1 \le i \le 4 \qquad (18)$$

After obtaining piecewise linear model, to reduce disturbance effect and tracking, the objectives are defined. The control objectives in the four-tank system is to keep the levels at tanks 1 and 2 at specified reference values in the face of flow disturbances and reduce control signals ($z_\infty, z_2$). W is defined as external inputs which contain d (disturbance), r (reference signals), and l (bias from linearization (19). l is defined as external input to eliminate bias from the control system.

$$z_\infty = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, z_2 = \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \end{bmatrix}, w = \begin{bmatrix} l \\ d \\ r_1 \\ r_2 \end{bmatrix} \qquad (19)$$

System matrix of open loop multi objective framework can be written as following:

$$A_{ij} = \begin{bmatrix} -\frac{a_1}{A_1}\sqrt{2g}a_{h_1}(1 \pm T_1) & 0 & \frac{a_3}{A_1}\sqrt{2g}a_{h_3}(1 \pm T_1) & 0 & \frac{\eta_1 k_1}{A_1}(1 \pm T_2) & 0 \\ 0 & -\frac{a_2}{A_2}\sqrt{2g}a_{h_2}(1 \pm T_1) & 0 & \frac{a_4}{A_2}\sqrt{2g}a_{h_4}(1 \pm T_1) & 0 & \frac{\eta_2 k_2}{A_2}(1 \pm T_2) \\ 0 & 0 & -\frac{a_3}{A_3}\sqrt{2g}a_{h_3}(1 \pm T_1) & 0 & 0 & \frac{(1-\eta_2)k_2}{A_3}(1 \pm T_2) \\ 0 & 0 & 0 & -\frac{a_4}{A_4}\sqrt{2g}a_{h_4}(1 \pm T_1) & \frac{(1-\eta_1)k_1}{A_4}(1 \pm T_2) & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_1}(1 \pm T_2) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_2}(1 \pm T_2) \end{bmatrix}$$

$$B_{ij} = \begin{bmatrix} -\frac{a_1}{A_1}\sqrt{2g}b_{h_1} + \frac{a_3}{A_1}\sqrt{2g}b_{h_3} & 0 & 1 & 0 & 0 & 0 \\ -\frac{a_2}{A_2}\sqrt{2g}b_{h_2} + \frac{a_4}{A_2}\sqrt{2g}b_{h_4} & 0 & 0 & 1 & 0 & 0 \\ -\frac{a_3}{A_3}\sqrt{2g}b_{h_3} & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_4}{A_4}\sqrt{2g}b_{h_4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{A_1} & 0 & 0 & \frac{1}{\tau_1} & 0 \\ 0 & \frac{1}{A_2} & 0 & 0 & 0 & \frac{1}{\tau_2} \end{bmatrix}$$

$$C_{ij} = \begin{bmatrix} 1 \pm T_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 \pm T_1 & 0 & 0 & 0 & 0 \\ 1 \pm T_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 \pm T_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{ij} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_1 = [B_{ij}]_{1 \le j \le 4}, B_2 = [B_{ij}]_{5 \le j \le 6}$$
$$C_1 = [C_{ij}]_{1 \le i \le 2}, C_2 = [C_{ij}]_{3 \le i \le 6}$$
$$D_{11} = [D_{ij}]_{1 \le i \le 2, 1 \le j \le 4}, D_{12} = [D_{ij}]_{1 \le i \le 2, 5 \le j \le 6}$$
$$D_{21} = [D_{ij}]_{3 \le i \le 6, 1 \le j \le 4}, D_{22} = [D_{ij}]_{3 \le i \le 6, 5 \le j \le 6}$$
$$\eta_j \in [\underline{\eta}_j \quad \overline{\eta}_j], 0 \le \eta_j \le 1, \; j = 1,2$$
$$\underline{x} = \begin{pmatrix} h_{k_1} \\ v_{k_2} \end{pmatrix}, \underline{x}_m = \begin{pmatrix} h_{k_1} \pm T1 \\ v_{k_2} \pm T2 \end{pmatrix}, \begin{matrix} 1 \le k_1 \le 4 \\ 1 \le k_2 \le 2 \end{matrix}$$
$$T_1 \in [\underline{T}_1 \quad \overline{T}_1], T_2 \in [\underline{T}_2 \quad \overline{T}_2]$$
$$l = [B_{ij}]_{j=1} \qquad (20)$$

Then by changing the height of the tanks, the system goes to the new operation points. Afterward ANFIS based on the changed on the system, switch to appropriate designed controller (gain scheduled control). Fig. 6 and Fig. 7 show the comparison between step response in our purposed robust structure and ordinary system in tank 1 and tank 2. As it can be seen performance has increased in our proposed method which switches between controllers in different operation point. The controller switch 4 times based on the reference signal 1 ($r_1$=2cm) in tank 1 and 5 times based on reference signal 2 ($r_2$=3cm) in tank 2.

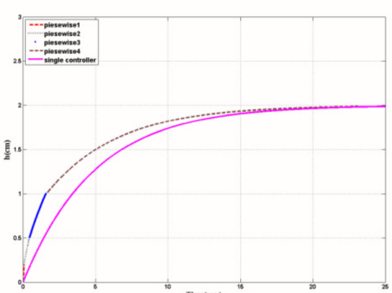


Fig. 6. Comparison between step response of our robust controller structure and ordinary robust controller in tank 1.

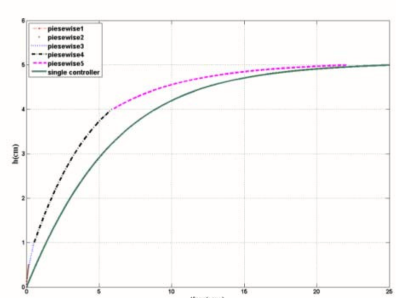


Fig. 7. Comparison between step response of our robust controller structure and ordinary robust controller in tank 2.

Fig 8 shows the disturbance rejection in any piecewise in tank2 (impulse response d → $x_2$ for robust design). Therefore in each operation point the system is robust and it is achieved because in each area, ANFIS as a supervisory system switch the controller to designate robust H2/H∞ multi objective controller.

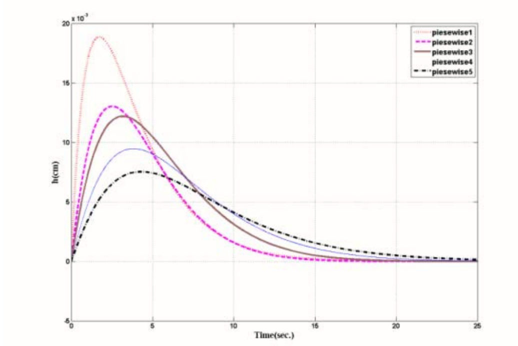


Fig. 8. Impulse responses for the extremal disturbance in each piecewise in tank2.

To investigate the robustness of our proposed structure, impulse response is compared between nominal system and change in the parameter of $\eta_j$ and T in the system (impulse response d → $x_2$ in the presence of uncertainties parameter). (Fig. 8.)
As it can be seen from the following figure (Fig. 9) the variance between plots are very small and it shows that our proposed structure is robust in front of uncertainties of the system.

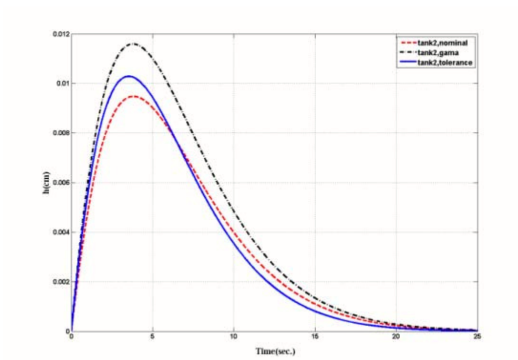


Fig. 9. Impulse responses for the extremal value of $\eta_j$ and T in tank2.

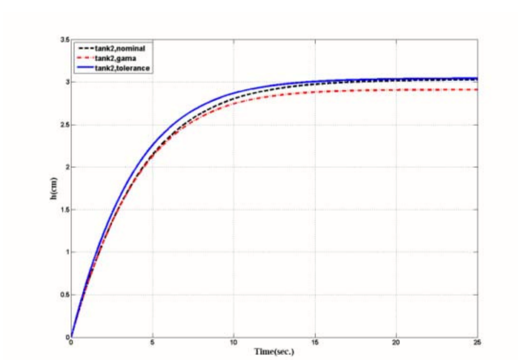


Fig. 10. Step responses for the extremal value of $\eta_j$ and T in tank2.

Fig 10 shows the tracking performance in front of uncertainties. The experimental responses and the simulation results show that when the parameters of system changed, the height of tank 2 which is the objective tracks reference 2. (Step response d → $e_2$ in the presence of uncertainties parameter).

## VII. Conclusion

In this work the problem of robust nonlinear controller is investigated. The multi objective robust control for nonlinear systems using multiple model and adaptive neuro fuzzy inference system (ANFIS) is served. Nonlinear system divided to multiple linear model based on piecewise linearization around set points. For each linear model multi objective robust controller is designed to guarantee both system performance and robust stability. To design the robust controller for each linear model, nonlinearity part of system consider as uncertainty of linear system. The advantages of proposed method are that it can reduce the parameter perturbation and nonlinear uncertainty as well as increased the performance of the system. In this work ANFIS is used to make decision and combine the designed controller for each linear model. Simulation result on a nonlinear benchmark plant shows the effectiveness of the proposed method.

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