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Robust Multi Objective H_2/H_∞ Control of MIMO Nonlinear Uncertain systems via T-S Fuzzy Model

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Abstract: This paper describes robust H_2/H_∞ multi-objective state feedback controller for nonlinear uncertain systems. To apply the H_2/H_∞ multi-objective state feedback method, the nonlinear dynamics is represented by a T-S fuzzy model. First, uncertain parameters and Quantification of uncertainty on physical parameters is defined by affine parameter-dependent systems method. Next, the Takagi and Sugeno's fuzzy linear model is utilized to approximate uncertain nonlinear systems. Then, some states (error of tracking) are augmented to the system in order to improve tracking control. Finally, based on fuzzy linear model with augmented state, a H_2/H_∞ multi-objective state feedback controller is developed to achieve the robustness design of nonlinear uncertain systems. LMI (Linear Matrix Inequality) method and PDC (Parallel Distributed Compensation) are used to design the controller for the whole system. The results show that the proposed method can effectively meet the performance requirements like robustness, disturbance rejection and tracking for the 3-phase permanent magnet synchronous motor (PMSM).

Keywords: Robust control, Nonlinear, Uncertain, MIMO, Multi objective H_2/H_∞, T-S fuzzy model, Augmented state, LMIs, 3-phase permanent magnet synchronous motor

1. INTRODUCTION

Most physical dynamical systems are nonlinear in real world and cannot be represented by linear differential equations. When the system's parameters are uncertain, the problem becomes more complicated. Linearization of systems around and operating point is a common method to design a controller. Utilizing this method rely on some assumptions which restrict application of it. Small range of operation system and be linearizable of the system model are some restricted assumptions. On the other hand, it is necessary that the linear model is precise enough for building up the controller and the system model is well achievable through a mathematical model and the parameters of the system model are reasonably well-known. Firstly, uncertain parameters and Quantification of uncertainty on physical parameters defined by affine parameter-dependent systems method. Then Fuzzy system theory is employed to utilize qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent researches show that a TS fuzzy model can be utilized to approximate global behaviors of a highly complex nonlinear system [1]. The Takagi–Sugeno (T-S) fuzzy system has been represented as a universal approximation of nonlinear dynamic systems[2]. In proposed method, nonlinear plant is represented by Takagi–Sugeno (T-S) fuzzy model. By using this model, local dynamics in different state-space regions are stated by linear models. The whole model of the system is attained by fuzzy combination of these fuzzy models. Then, some states (error of tracking) are augmented to the system in order to improve tracking control. Afterward, based on fuzzy linear model with augmented state, a H_2/H_∞ multi-objective state feedback controller is developed to achieve the robustness design of nonlinear uncertain systems [3, 4]. To design linear feedback control parallel distributed compensation (PDC) is utilized for each local linear model.[4, 5]. The designate controller which is nonlinear in general is fuzzy combination of each particular linear controller. Design conditions for the stability and performance of a system can be stated in terms of the feasibility of a set of linear matrix inequalities (LMI) by using the framework of T-S fuzzy model and PDC control design[6]. The main advantage of the proposed T-S fuzzy control design approach is to deal with stability and performance over the complete operational range of the nonlinear system [7-9].The paper is organized as follows: multi objective H_2/H_∞ framework is presented in Section II, Takagi and Sugeno's fuzzy dynamic model is introduced in Section III. In Section IV, our proposed structure is described. Simulation in section V shows the performance of the method .Concluding remarks are made in Section VI.

2. MULTI OBJECTIVE H_2/H_∞ FRAMEWORK

The objectives of multi objective controller are:

- H_∞ performance (for tracking, disturbance rejection, or Robustness aspects)
- H_2 performance (for LQG aspects)
- Robust pole placement specifications (to ensure fast and well-damped transient responses, reasonable feedback gain, etc.)

Denoting by $T_\infty(s)$ and $T_2(s)$ the closed-loop transfer functions from w to z_∞ and z_2, respectively, our goal is to design a state-feedback law u = K x that:

•Maintains the RMS gain (H_∞ norm) of T_∞ below some prescribed value $\gamma_0 > 0$

•Maintains the H2 norm of T_2 (LQG cost) below some prescribed value $\nu_0 > 0$

•Minimizes an H_2/H_∞ trade-off criterion of the form

$$\alpha\|T_\infty\|_\infty{}^2 + \beta\|T_2\|_2{}^2 \quad (1)$$

•Places the closed-loop poles in a prescribed region D of The mixed H_2/H_∞ criterion takes into account both the disturbance rejection aspects (RMS gain from d to e) and the LQG aspects (H_2 norm from n to z_2) [3, 4].

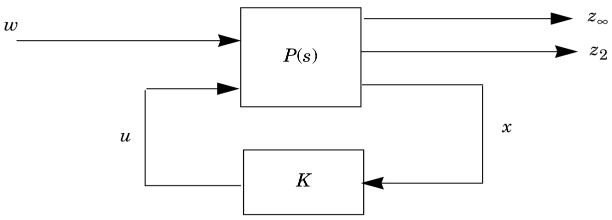


Fig. 1. Control structure.

LMI formulation given a state-space realization

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z_\infty = C_1 x + D_{11} w + D_{12} u \\ z_2 = C_2 x + D_{21} w + D_{22} u \end{cases} \quad (2)$$

of the plant P ,Fig. 1, the closed-loop system is given in state-space form by

$$\begin{cases} \dot{x} = (A + B_2 K)x + B_1 w \\ z_\infty = (C_1 + D_{12} K)x + D_{11} w \\ z_2 = (C_2 + D_{22} K)x + D_{21} w \end{cases} \quad (3)$$

Taken separately, our three design objectives have the following LMI formulation:

H_∞ performance: the closed-loop RMS(H_∞ norm) gain from w to z_∞ does not exceed γ if and only if there exists a symmetric matrix X_∞ such that

$$\begin{bmatrix} (A + B_2K)X_\infty + X_\infty(A + B_2K)^T & B_1 & X_\infty(C_1 + d_{12}K)^T \\ B_1{}^T & -I & d_{11}{}^T \\ (C_1 + d_{12}K)X_\infty & d_{11} & -\gamma^2 I \end{bmatrix} < 0$$

$$X_\infty > 0 \quad (4)$$

H_2 performance: the closed-loop H_2 norm of T_2 does not exceed ν if there exist two symmetric matrices X_2 and Q such that

$$\begin{bmatrix} Q & (C_2 + d_{22}K)X_2 \\ X_2(C_2 + d_{22}K)^T & X_2 \end{bmatrix} > 0$$

$$\begin{bmatrix} (A + B_2K)X_2 + X_2(A + B_2K)^T & B_1 \\ B_1{}^T & -I \end{bmatrix} < 0$$

$$\mathrm{Trace}(Q) < \nu^2 \quad (5)$$

Pole placement: the closed-loop poles lie in the LMI Region specifications (to ensure fast and well-damped transient responses, reasonable feedback gain, etc.)

These three sets of conditions add up to a no convex optimization problem with variables Q, K, X_∞, X_2 and X_{pol}. For tractability in the LMI framework, we seek a single Lyapunov matrix:

$$X \coloneqq X_\infty = X_2 = X_{pol} \quad (6)$$

That enforces all three objectives. With the change of variable Y := KX, this leads to the following suboptimal LMI formulation of our multi-objective state-feedback synthesis problem [3, 5, 7]:

Minimize $\alpha\gamma^2 + \beta$ Trace (Q) over Y, X, Q, and γ^2satisfying

$$\begin{bmatrix} AX + XA^T + B_2Y + Y^TB_2{}^T & B_1 & XC_1{}^T + Y^Td_{12}{}^T \\ B_1{}^T & -I & d_{11}{}^T \\ C_1X + d_{12}Y & d_{11} & -\gamma^2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} Q & C_2X + d_{22}Y \\ XC_2{}^T + Y^Td_{22}{}^T & X \end{bmatrix} > 0$$

$$\left[\lambda_{ij} + \mu_{ij}(AX + B_2Y)X_{pol} + \mu_{ji}(XA^T + Y^TB_2{}^T)\right]_{1\le i,j\le m} < 0$$

$$\mathrm{Trace}(Q) < \nu_0{}^2 \qquad \gamma^2 < \gamma_0{}^2 \quad (7)$$

Denoting the optimal solution by (X*, Y*, Q*, γ*) the corresponding state-feedback gain is given by

$$K^* = Y^*(X^*)^{-1} \quad (8)$$

And this gain guarantees the worst-case performance:

$$\|T_\infty\|_\infty < \gamma^* \qquad \|T_2\|_2 < \sqrt{\mathrm{Trace}(Q^*)} \quad (9)$$

3. TAKAGI AND SUGENO'S FUZZY DYNAMIC MODEL

The fuzzy dynamic model is proposed by Takagi and Sugeno. In this chapter, we generalize the TS fuzzy system to represent a T-S fuzzy system with parametric uncertainties[1]. Is described by fuzzy IF-THEN rules, which locally represent linear input-output relations for the given plant. The ith rule of this fuzzy dynamic model in multi objective H_2/H_∞ framework is of the following form [1, 3, 8]:

Plant rule i.

IF $v_1(t)$ is M_{i1} and ... and $v_p(t)$ is M_{ip} THEN

$$\dot{x}(t) = \left[[A_i + \Delta A_i]x(t) + [B_{1_i} + \Delta B_{1_i}]w(t) + [B_{2_i} + \Delta B_{2_i}]u(t)\right], \; x(0) = 0$$

$$z_\infty(t) = \left[[C_{1_i} + \Delta C_{1_i}]x(t) + [D_{11_i} + \Delta D_{11_i}]w(t) + [D_{12_i} + \Delta D_{12_i}]u(t)\right]$$

$$z_2(t) = \left[[C_{2_i} + \Delta C_{2_i}]x(t) + [D_{21_i} + \Delta D_{21_i}]w(t) + [D_{22_i} + \Delta D_{22_i}]u(t)\right]$$

$$i = 1,2,\dots,r \quad (10)$$

Where, M_{ip} is the fuzzy set and r is the number of IF THEN Rules and $v_1(t) \rightarrow v_p(t)$ are the premise variables. We examine final output of TS fuzzy system with parametric uncertainties as follows , Fig.2 (a):

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(v(t))\left[[A_i + \Delta A_i]x(t) + [B_{1_i} + \Delta B_{1_i}]w(t) + [B_{2_i} + \Delta B_{2_i}]u(t)\right], x(0) = 0$$

$$z_\infty(t) = \sum_{i=1}^{r} \mu_i(v(t))\left[[C_{1_i} + \Delta C_{1_i}]x(t) + [D_{11_i} + \Delta D_{11_i}]w(t) + [D_{12_i} + \Delta D_{12_i}]u(t)\right]$$

$$z_2(t) = \sum_{i=1}^{r} \mu_i(v(t))\left[[C_{2_i} + \Delta C_{2_i}]x(t) + [D_{21_i} + \Delta D_{21_i}]w(t) + [D_{22_i} + \Delta D_{22_i}]u(t)\right] \quad (11)$$

Where:

$$v(t) = [v_1(t) \; \dots \; v_p(t)] \quad (12)$$

And weighting function is

$$\mu_i(v(t)) = \frac{\varpi_i(v(t))}{\sum_{i=1}^{r} \varpi_i(v(t))} \quad (13)$$

$$\varpi_i(v(t)) = \prod_{k=1}^{p} M_{ik}(v_k(t)) \quad (14)$$

And it should be noted that

$$\varpi_i(v(t)) \geq 0\ , i = 1,2,\dots,r\ ;\ \sum_{i=1}^{r} \varpi_i(v(t)) > 0$$

$$\mu_i(v(t)) \geq 0\ , i = 1,2,\dots,r\ ;\ \sum_{i=1}^{r} \mu_i(v(t)) = 1 \quad (15)$$

The matrices: $\Delta A_i, \Delta B_{1i}, \Delta B_{1i}, \Delta B_{2i}, \Delta C_{1i}, \Delta C_{2i}, \Delta D_{11i}, \Delta D_{12i}, \Delta D_{21i}, \Delta D_{21i}$ represent the uncertainties in the system and satisfy the following assumption:

$$\Delta A_i = F(x(t),t)H_{1i}\ , \Delta B_{1i} = F(x(t),t)H_{2i}\ , \Delta B_{2i} = F(x(t),t)H_{3i}$$
$$\Delta C_{1i} = F(x(t),t)H_{4i}\ , \Delta D_{11i} = F(x(t),t)H_{5i}\ , \Delta D_{12i} = F(x(t),t)H_{6i}$$
$$\Delta C_{2i} = F(x(t),t)H_{7i}\ , \Delta D_{21i} = F(x(t),t)H_{8i}\ , \Delta D_{22i} = F(x(t),t)H_{9i} \quad (16)$$

Where $H_{ji}\ ;\ j = 1,\dots,9$ are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t),t)\| \leq \partial\ , \partial > 0 \quad (17)$$

For a fuzzy controller design, it is supposed that the fuzzy system is locally controllable. Then, the local state feedback multi objective H_2/H_∞ controller [3, 4] is designed as follows:

IF $v_1(t)$ is M_{i1} and … and $v_p(t)$ is M_{ip} THEN

$$u(t) = K_i x(t) \quad i = 1,2,\dots,r \quad (18)$$

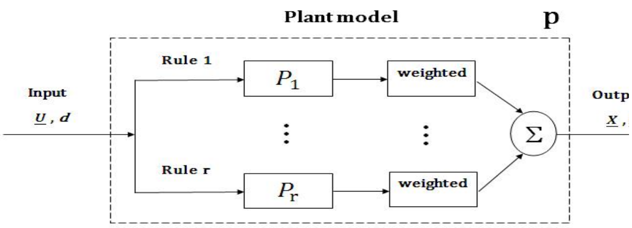
Where K_i is the multi objective H_2/H_∞ controller gain. Then, the final TS fuzzy controller [10-12]is ,Fig.2(b)

$$u(t) = \sum_{j=1}^{r} \mu_j K_j x(t) \quad (19)$$

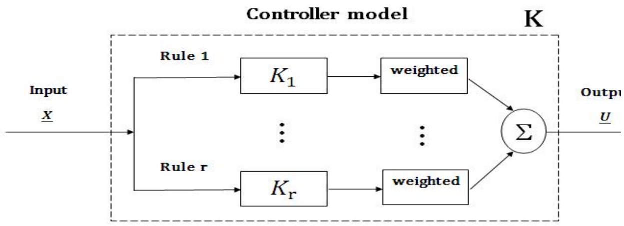
The close loop state space system from of the fuzzy system model, Eq. (11) ,with the controller ,Eq. (19) , is given by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \left[\left((A_i + B_{2_i}K_j) + (\Delta A_i + \Delta B_{2_i}K_j)\right]x(t) + \left[B_{1_i} + \Delta B_{1_i}\right]w(t)\right]\ , x(0) = 0$$
$$z_\infty(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j [[(C_{1_i} + D_{12_i}K_j) + (\Delta C_{1_i} + \Delta D_{12_i}K_j)]x(t) + [D_{11_i} + \Delta D_{11_i}]w(t)]$$
$$z_2(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \left[\left[(C_{2_i} + D_{22_i}K_j) + (\Delta C_{2_i} + \Delta D_{22_i}K_j)\right]x(t) + \left[D_{21_i} + \Delta D_{21_i}\right]w(t)\right] \quad (20)$$

The following theorem provides sufficient conditions for the existence of a robust multi objective H_2/H_∞ fuzzy state-feedback controller.



(a)



(b)

Fig. 2. (a) TS type fuzzy system (b) TS type fuzzy controller

4. PROPOSED STRUCTURE

First of all, nonlinear uncertain system is linearized by T-S fuzzy method , Fig.2.(a), [1, 10]. After linearization, controller is designed for linearized system based on LMI ,Fig.2.(b), [3, 4]. To achieve better tracking [13]performance in presence of disturbance, two states have been augmented to the system. These two states are to reduce the error integrator of tracking in objectives. Then the close loop state space system is considered from of the fuzzy system model with the controller model ,fig. 3.

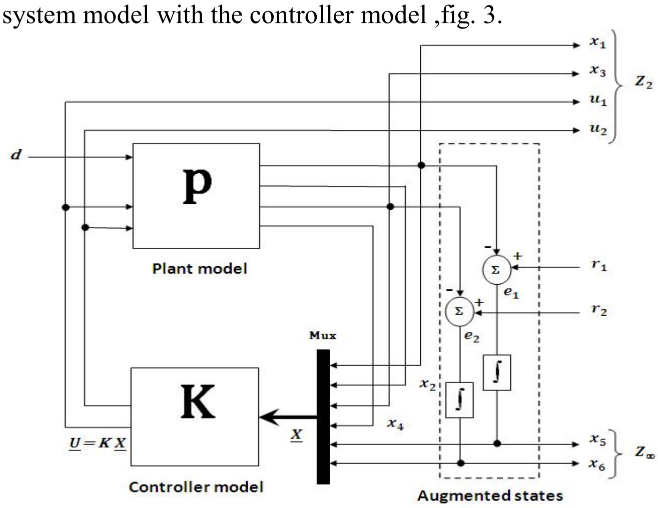


Fig. 3. Block diagram of robust controller and augmented states control loop .

5. SIMULATION

In the simulation scenarios, nonlinear differential equations for the 3-phase permanent magnet synchronous motor are used to represent the experimental system, which are given in , Eq. (21).In this model, the electrical and mechanical equations of the motor are expressed in the (d, q)-frame [14, 15]

$$\frac{d\theta}{dt} = \omega$$
$$\frac{d\omega}{dt} = \frac{P}{J}\left[(L_d - L_q)i_d + \emptyset_f\right]i_q - \frac{f_v}{J}\omega - \frac{C_l}{J}$$
$$\frac{di_d}{dt} = -\frac{R_s}{L_d}i_d + P\frac{L_q}{L_d}\omega i_q + \frac{1}{L_d}v_d$$
$$\frac{di_q}{dt} = -P\frac{\emptyset_f}{L_q}\omega - P\frac{L_d}{L_q}\omega i_q - \frac{R_s}{L_q}i_q + \frac{1}{L_q}v_q \quad (21)$$

In equation, Eq. (21), θ is the angular position of the motor shaft ω is the angular velocity of the motor shaft, i_d is the direct current and i_q is the quadrature current. $\emptyset_f$ is the flux of the permanent magnet, P is the number of pole pairs, R_s is the stator windings resistance, L_d and L_q are the direct and quadrature stator inductances respectively. J is the rotor moment of inertia, f_v the viscous damping coefficient and C_l is the load torque. v_d is the direct voltage and v_q is the quadrature voltage. The parameters R_s and f_v are supposed to differ from their nominal values R_{s_0} andf_{v_0}. The following equation show this differences

$$\eta_1 = \frac{P}{J}(L_d - L_q) \quad \eta_2 = \frac{P}{J}\emptyset_f \quad \eta_3 = -\frac{f_v}{J}$$
$$\eta_4 = -\frac{R_s}{L_d} \quad \eta_5 = P\frac{L_q}{L_d} \quad \eta_6 = \frac{1}{L_d}$$
$$\eta_7 = -P\frac{\emptyset_f}{L_q} \quad \eta_8 = -P\frac{L_d}{L_q} \quad \eta_9 = -\frac{R_s}{L_q}$$
$$\eta_{10} = \frac{1}{L_q} \tag{22}$$

The following equation indicate a state space representation of the synchronous motor

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = (\eta_1 x_3 + \eta_2)x_4 + \eta_3 x_2 - \frac{C_l}{J}$$
$$\dot{x}_3 = \eta_4 x_3 + \eta_5 x_2 x_4 + \eta_6 u_1$$
$$\dot{x}_4 = \eta_7 x_2 + \eta_8 x_2 x_3 + \eta_9 x_4 + \eta_{10} u_2 \tag{23}$$

In this equation

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\theta \quad \omega \quad i_d \quad i_q]^T \tag{24}$$

$$[u_1 \quad u_2]^T = [v_d \quad v_q]^T \tag{25}$$

To guarantees robust performance in presence of parameters and load torque variations, a proper control has to be designed. There are two control objectives. First, the rotor angular position $x_1 = \theta$ must track a reference trajectory r_1. Secondly, the direct current $x_3 = i_d$ to track a constant reference $r_2 = 0$ [14]. Therefore to achieve these objectives, the outputs of the integrator are considered as extra state variables.

$$x_5 = \int_0^t e_1(\delta)\, d\delta \qquad e_1 = r_1 - x_1 \tag{26}$$
$$x_6 = \int_0^t e_2(\delta)\, d\delta \qquad e_2 = r_2 - x_3 \tag{27}$$

Consequently the four states in, Eq. (23), is increased to six states.

$$\dot{x}_5 = r_1 - x_1 \;,\; \dot{x}_6 = r_2 - x_3$$
$$x_{aug} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{28}$$

Table1 shows the parameters of a permanent magnet synchronous motor. Parameters of systems are supposed to vary as following equations:

$$R_s = R_{s_0} \pm 50\% \qquad R_s \in [\underline{R}_s \quad \overline{R}_s]$$
$$\underline{R}_s = 1.65\,\Omega \quad , \quad \overline{R}_s = 4.95\,\Omega$$
$$f_v = f_{v_0} \pm 20\% \qquad f_v \in [\underline{f}_v \quad \overline{f}_v]$$
$$\underline{f}_v = 0.00272\ \mathrm{N.m.s} \quad , \quad \overline{f}_v = 0.00408\ \mathrm{N.m.s}$$
$$\underline{C}_l = -2\ \mathrm{N.m} \quad , \quad \overline{C}_l = 9\ \mathrm{N.m}$$
$$C_l \in [\underline{C}_l \quad \overline{C}_l] \tag{29}$$
$$\eta_3 = -2702.702 f_v \qquad \eta_4 = -37.037 R_s$$
$$\eta_9 = -29.498 R_s \tag{30}$$

Symbol	Parameters	Value/Unit
R_s	stator windings resistance	$3.3 \pm 50\%/\ \Omega$
f_v	viscous damping coefficient	0.0034 $\pm 20\%/\ \mathrm{N.m.s}$
L_d	direct stator inductance	0.027/ H
L_q	quadrature stator inductance	0.0339/ H
$\emptyset_f$	flux of the permanent magnet	0.341/ Wb
J	rotor moment of inertia	$0.00223/\ \mathrm{kg.m^2}$
P	number of pole pairs	3

Table 1. The Parameters of PMSM

The following specific state space is represented in despite of this fact that the combination of, Eq. (23), and, Eq. (28), is parameter-dependent. As a result affine parameter-dependent systems[3, 5] are defined as:

$$\begin{cases} E(\rho)\dot{x} = A(\rho)x + B_1(\rho)w + B_2(\rho)u \\ z_\infty = C_1(\rho)x + d_{11}(\rho)w + d_{12}(\rho)u \\ z_2 = C_2(\rho)x + d_{12}(\rho)w + d_{22}(\rho)u \end{cases} \tag{31}$$

$$B(\rho) = [B_1(\rho) \quad B_2(\rho)] \qquad C(\rho) = [C_1(\rho) \quad C_2(\rho)]^T$$
$$D(\rho) = \begin{bmatrix} d_{11}(\rho) & d_{12}(\rho) \\ d_{12}(\rho) & d_{22}(\rho) \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} A(\rho) + jE(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix} = \begin{bmatrix} A_0 + jE_0 & B_0 \\ C_0 & D_0 \end{bmatrix}$$
$$+\rho_1 \begin{bmatrix} A_{\rho_1} + jE_{\rho_1} & B_{\rho_1} \\ C_{\rho_1} & D_{\rho_1} \end{bmatrix} + \cdots + \rho_n \begin{bmatrix} A_{\rho_n} + jE_{\rho_n} & B_{\rho_n} \\ C_{\rho_n} & D_{\rho_n} \end{bmatrix} \tag{33}$$

$$w = \begin{bmatrix} d \\ r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} C_l \\ x_{1_{ref}} \\ x_{3_{ref}} \end{bmatrix} \tag{34}$$

ρ and w are defined as uncertain parameter and external input respectively.

$$B_1(\rho) = B_{1_0} + C_l B_{1_{C_l}}$$
$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \eta_6 & 0 \\ 0 & \eta_{10} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_{1_0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B_{1_{C_l}} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{J} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{35}$$

$$A(p) = A_0 + f_v A_{f_v} + R_s A_{R_s}$$

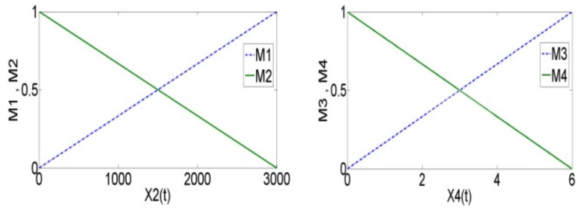
$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_1 x_4 & \eta_2 & 0 & 0 \\ 0 & \eta_5 x_4 & 0 & 0 & 0 & 0 \\ 0 & \eta_7 & \eta_8 x_2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad A_{f_v} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{J} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{R_s}=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&-\frac{1}{L_d}&0&0&0\\0&0&0&-\frac{1}{L_q}&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix} \tag{36}$$

According to , Eq. (36), which emerges from , Eq. (23), and , Eq. (28), just A0 contains nonlinear parameters. Therefore to linearize the system, T-S fuzzy model method is utilized. Because there are two nonlinear elements exist in the A0, four plant rules are assigned.

$$x_2 \in [0 \quad 3000]\,,\ x_4 \in [0 \quad 6]$$
$$v_1(t) = x_2(t)\,,\ v_2(t) = x_4(t) \tag{37}$$

Which v_1 and v_2 are fuzzy variables. According to, Eq. (37), membership functions can be demonstrates as following figures, Fig 4.



(a) (b)

Fig. 4. (a) The membership functions for M1(x2(t)) and M2(x2(t)), (b) the membership functions for M3(x4(t)) and M4(x4(t))

There are four plant rules are considered to cover four local linear systems near specific operation point.

Rule 1:
IF $v_1(t)$ is M_1 and $v_2(t)$ is M_3 THEN
$A(p) = A_{0_1} + f_v A_{f_v} + R_s A_{R_s}$

Rule 2:
IF $v_1(t)$ is M_1 and $v_2(t)$ is M_4 THEN
$A(p) = A_{0_2} + f_v A_{f_v} + R_s A_{R_s}$

Rule 3:
IF $v_1(t)$ is M_2 and $v_2(t)$ is M_3 THEN
$A(p) = A_{0_3} + f_v A_{f_v} + R_s A_{R_s}$

Rule 4:
IF $v_1(t)$ is M_2 and $v_2(t)$ is M_4 THEN
$A(p) = A_{0_4} + f_v A_{f_v} + R_s A_{R_s}$ (38)

According to, Eq. (31), and , Eq. (36), just A_0 varies and A_{f_v}, A_{R_s}, B, C and D are constant in each rule. B, C and D are obtained based on objectives z_2 and z_∞ and external input w.in additional for each rule designed local controller [3, 16, 17]. B is defined as, Eq. (32), and, Eq. (35), and A is represented as, Eq. (36).

$$z_\infty = \begin{bmatrix}x_5\\x_6\end{bmatrix} = \begin{bmatrix}e_1\\e_2\end{bmatrix} \tag{39}$$

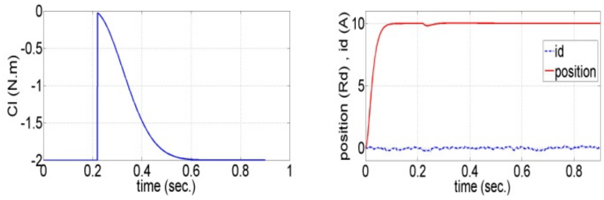
$$z_2 = \begin{bmatrix}x_1\\x_3\\u_1\\u_2\end{bmatrix} = \begin{bmatrix}\theta\\i_d\\v_d\\v_q\end{bmatrix} \tag{40}$$

$$w = \begin{bmatrix}d\\r_1\\r_2\end{bmatrix} = \begin{bmatrix}C_l\\x_{1_{ref}}\\x_{3_{ref}}\end{bmatrix} \tag{41}$$

$$A_{0_1}=\begin{bmatrix}0&1&0&0&0&0\\0&0&-335.67&2764.86&0&0\\0&22.59&0&0&0&0\\0&-30.17&-7167&0&0&0\\-1&0&0&0&0&0\\0&0&-1&0&0&0\end{bmatrix}$$
$$A_{0_2}=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&2764.86&0&0\\0&0&0&0&0&0\\0&-30.17&-7167&0&0&0\\-1&0&0&0&0&0\\0&0&-1&0&0&0\end{bmatrix}$$
$$A_{0_3}=\begin{bmatrix}0&1&0&0&0&0\\0&0&-335.67&2764.86&0&0\\0&22.59&0&0&0&0\\0&-30.17&0&0&0&0\\-1&0&0&0&0&0\\0&0&-1&0&0&0\end{bmatrix}$$
$$A_{0_4}=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&2764.86&0&0\\0&0&0&0&0&0\\0&-30.17&0&0&0&0\\-1&0&0&0&0&0\\0&0&-1&0&0&0\end{bmatrix} \tag{42}$$

$$C=\begin{bmatrix}0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\quad D=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix} \tag{43}$$

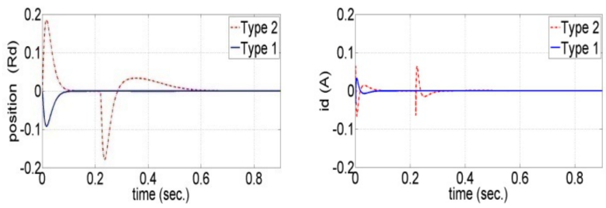
Figure 5 shows real angular position and real direct current that track 10 Rd and 0 A, respectively in presence of load torque disturbance.



(a) (b)

Fig. 5. (a) Load torque (b) Real angular position (rad versus time) and Real direct current id (A versus time) in presence of load torque disturbance

Figure 6 demonstrates disturbance rejection on angular position and direct current id for tow type of disturbances.



(a) (b)

Fig. 6. (a) Disturbance rejection on angular position (b) Disturbance rejection on direct current id (Type 1: step disturbance, Type 2: disturbance based on Fig. 5. (a))

Figure 7 and 8 compare tracking of angular position of motor between our method and classical method (linearization of system in one operation point). However, figure 7 shows the tracking for 10 Rd but figure 8 compare tracking for arbitrary reference. As it can be seen from two figures, proposed method presents good performance in quick response and accurate tracking.

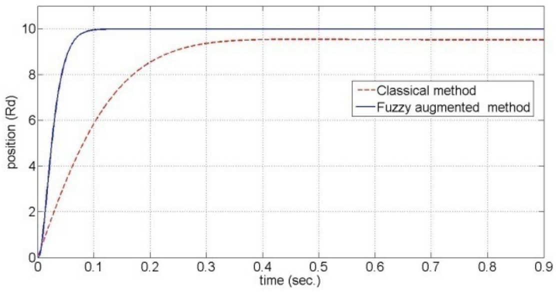


Fig. 7. Comparison between desired response of our method and classical method.

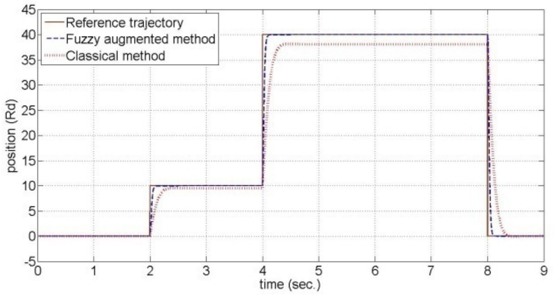


Fig. 8. Comparison between desired trajectory response of our method and classical method.

Figure 9 illustrates the robustness of the method in front of uncertainties. As it can be seen, despite of this fact that uncertainties vary in its range but the performance of the system is appropriate.

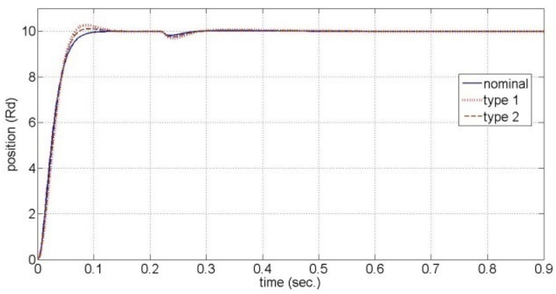


Fig. 9. Robustness of Real angular position response in presence changes uncertain parameters

6. CONCLUSION

In this paper, robust H_2/H_∞ multi-objective state feedback controller for nonlinear uncertain systems is purposed. Firstly, uncertain parameters and Quantification of uncertainty on physical parameters is defined. Then, to approximate uncertain nonlinear systems, the T-S fuzzy linear model is employed. Afterward, in order to improve tracking control, some states (error of tracking) are augmented to the system. Finally, based on fuzzy linear model with augmented state, a H_2/H_∞ multi-objective state feedback controller is developed to achieve the robustness design of nonlinear uncertain systems. The simulation results on the 3-phase permanent magnet synchronous motor (PMSM) shows that the method can efficiently reached to desire performance requirements like robustness, disturbance rejection and tracking .

REFERENCES

[1] Wudhichai Assawinchaichote, S.K.N., Peng Shi, Fuzzy Control and Filter Design for Uncertain Fuzzy Systems. 2006: Springer-Verlag.

[2] Wu, S.M., et al., Discrete H_2/H_∞ Nonlinear Controller Design Based on Fuzzy Region Concept and Takagi–Sugeno Fuzzy Framework. Circuits and Systems I: Regular Papers, IEEE Transactions on, 2006. 53(12): p. 2838-2848.

[3] Pascal Gahinet, A.N., Alan J. Laub,Mahmoud Chilali, LMI Control Toolbox User’s Guide. 1995: MathWorks.

[4] Qing, H., et al. Robust multiobjective control of high-rise roped elevator system based on T-S fuzzy model. in Control and Decision Conference, 2009. CCDC '09. Chinese. 2009.

[5] Sofianos, N.A. and O.I. Kosmidou. Guaranteed cost LMI-based fuzzy controller design for discrete-time nonlinear systems with polytopic uncertainties. in Control & Automation (MED), 2010 18th Mediterranean Conference on.

[6] Wen-Jer, C., H. Wei-Han, and C. Wei. Fuzzy Control of Inverter Pendulum Robot via Perturbed Time-Delay Affine T-S Fuzzy Models. in Robotics, Automation and Mechatronics, 2008 IEEE Conference on. 2008.

[7] El Messoussi, W., O. Pages, and A. El Hajjaji. Observer-based robust control of uncertain fuzzy dynamic systems with pole placement constraints: an LMI approach. in American Control Conference, 2006. 2006.

[8] Ho Jae, L., P. Jin Bae, and C. Guanrong, Robust fuzzy control of nonlinear systems with parametric uncertainties. Fuzzy Systems, IEEE Transactions on, 2001. 9(2): p. 369-379.

[9] Hoseini, S.A. and B. Labibi. Robust fuzzy controller design with bounded control effort for nonlinear systems with parametric uncertainties. in Networking, Sensing and Control, 2009. ICNSC '09. International Conference on. 2009.

[10] Zhaona, C., et al. Feedback controller design for T-S fuzzy control systems based on Lyapunov approach. in Intelligent Control and Automation, 2008. WCICA 2008. 7th World Congress on. 2008.

[11] Sofianos, N.A. and O.I. Kosmidou. Robust fuzzy control of nonlinear discrete-time systems with polytopic uncertainties. in Control and Automation, 2009. MED '09. 17th Mediterranean Conference on. 2009.

[12] Chiu, C.S., Robust adaptive control of uncertain MIMO non-linear systems - feedforward Takagi-Sugeno fuzzy approximation based approach. Control Theory and Applications, IEE Proceedings -, 2005. 152(2): p. 157-164.

[13] Jie, W., et al. H_∞ fuzzy tracking control for boiler-turbine systems. in Control and Automation, 2009. ICCA 2009. IEEE International Conference on. 2009.

[14] Laghrouche, S., F. Plestan, and A. Glumineau. A higher order sliding mode controller for a class of MIMO nonlinear systems: application to PM synchronous motor control. in American Control Conference, 2004. Proceedings of the 2004. 2004.

[15] Laghrouche, S., et al. Robust second order sliding mode control for a permanent magnet synchronous motor. in American Control Conference, 2003. Proceedings of the 2003. 2003.

[16] Haiping, D., et al., Robust Fuzzy Control of an Active Magnetic Bearing Subject to Voltage Saturation. Control Systems Technology, IEEE Transactions on. 18(1): p. 164-169.

[17] Li, T.H.S. and T. Shun-Hung, T–S Fuzzy Bilinear Model and Fuzzy Controller Design for a Class of Nonlinear Systems. Fuzzy Systems, IEEE Transactions on, 2007. 15(3): p. 494-506.