Multi-objective economic-statistical design of MEWMA control chart

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Abstract: The various advantages of MEWMA control chart such as the ability to detect small shifts in the process with multiple quality characteristics have motivated users to apply this chart for process monitoring. Considering the high costs of implementing MEWMA control chart, the economic-statistical design of this chart has been increasingly investigated. In most of the previous studies the cost function has been considered as the objective function while the statistical properties have been modelled as constraints in a mathematical programming. According to the dependency of the cost function on statistical properties in the constraints, the results of these methods are not efficient enough. In this paper, two multi-objective approaches, an aggregative and a non-aggregative approach are applied and optimised using a genetic algorithm. The proposed approaches are evaluated through a numerical example from the literature and the efficiency of the multi-objective approaches are verified in comparison with the previous methods.

Keywords: MEWMA control chart; economic-statistical design; Lorenzen and Vance cost function; multi-objective approach; genetic algorithm; GA.

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1 Introduction

Nowadays the competitive market impels companies to develop their production and servicing systems for improving the customers' satisfaction. Hence, establishing and developing quality management systems has turned into a strategic decision of organisations. Shahin (2008) highlighted the fact that improving quality of products through quality management systems plays a fundamental role in increasing the productivity of operations in organisations as well as customers' satisfaction by means of some advanced models.

Although developing quality management systems have confronted some difficulties in different industries (Ashrafi, 2008) but such challenges have led to more customised frameworks and more advanced quality improvement tools. These frameworks have been extensively investigated in real world applications, for instance Shahabuddin (2008) discussed the different aspects and challenges of implementing Six Sigma and suggested true quality control methods which are statistically and practically efficient in real world applications. In a similar research, Manikandan et al. (2008) proposed a method based on Design for Six Sigma (DFSS) road map and presented a plan, identify, design, optimise and validate (PIDOV) approach for problem solving in quality improvement efforts.

In more recent studies researchers have increasingly devoted attention on improving the tools applied in quality management. For instance Xu et al. (2010) investigated recent developments of quality function deployment (QFD) and Sharma (2011) discussed about linking the voice of customer system with QFD. Furthermore several researches have been done to improve the performance of the tools applied in process monitoring such as control charts.

Control charts are one of the most common statistical process control tools which are used to monitor processes and detect assignable causes rapidly. According to their implementation costs, researchers are interested in designing control charts with the least cost. Therefore, some authors such as Duncan (1956) and Lorenzen and Vance (1986) have proposed cost functions which are a function of sample size (n), sampling frequency (h) and control limits (k). What differentiates the Lorenzen and Vance model from Duncan's model is that the Lorenzen and Vance model is based on average run length (ARL) criterion while Duncan's model is based on the probability of type I and II errors.

Afterward, some researches were done based on minimising cost function under different assumption and various control charts, known as economic design which can be seen in Ho and Case (1994). Since the economic approach results in poor statistical performance of control charts (Woodall, 1986) various economic-statistical approaches for designing control charts have been proposed. For instance Saniga (1989) modelled an $\bar{X}$ and R control chart by considering cost function as the objective function and the statistical requirements as the constraints in a mathematical programming. Last review paper in the constrained economic-statistical design of control chart has been done by Celano (2011). In some cases, statistical properties may have the same importance toward the cost, therefore, Evans and Emberton (1991) proposed bi-criterion design of control chart which is based on multi criteria decision making (MCDM) problem. In the multi-objective approach in design of control chart, cost function and statistical properties are considered as objectives and optimised simultaneously.

Two main approaches can be used for multi-objective optimisation problems. In the first approach the objectives are aggregated in an overall function and this function is optimised as the objective of the problem. Although this approach has some advantages

such as simplicity of application, but two drawbacks have been mentioned for this method. Firstly in this approach just one solution is obtained, and secondly Fleming (1985) showed that this approach is unable to deal with concave Pareto front in multi-objective problems.

The second approach is the non-aggregative approach which results in a set of permissible solutions. To the best of our knowledge multi-objective approaches have been used only for univariate control charts, while nowadays in most of cases the quality of a product or a process is represented by two or more quality characteristics which should be monitored by multivariate control charts (Hotelling, 1947).

In this paper, we apply two multi-objective economic-statistical approaches in designing an multivariate exponentially weighted moving average (MEWMA) control chart based on Lorenzen and Vance cost function with considering Taguchi loss approach. The first approach is the aggregative one in which the economic and the statistical objectives are aggregated in a single objective. As the second approach, the normalised normal constraint method is used to generate a set of non-dominated solutions for designing the control chart parameter. For obtaining the optimal design of the control chart genetic algorithm (GA) is used in each approach and the final results are compared with the results reported by Niaki et al. (2010). Note that the in-control and out-of-control run length are calculated by using Markov chain proposed by Runger and Prabhu (1996).

The remainder of the paper is organised as follows: Section 2 discusses about the literature of control charts and the researches proposed for designing them. In the third section MEWMA control chart is illustrated. The Lorenzen and Vance (1986) cost function and the Taguchi loss approach are explained in Section 4. In Section 5, the aggregative and the non-aggregative approaches are discussed. In the sixth section, the MEWMA control chart design using the proposed approaches is discussed and the results are compared with similar studies from the literature. Finally, the conclusion is drawn in the last section.

2 Literature review

2.1 Control charts

Control charts are known as one of the most essential tools for quality engineering efforts. Control charts can distinguish between assignable and common causes of variation and help quality engineers to find the root causes of the problems and improve the quality of processes using the corrective actions. A comprehensive discussion for the most used univariate control charts has been presented by Montgomery (2005), also several researches have discussed process monitoring for multivariate cases (Bersimis et al., 2007) and multi attribute cases (Topalidou and Psarakis, 2009). As another instance, Jarrett and Pan (2009) reviewed some methods in multivariate process control charts and proposed a new multivariate control chart. Meanwhile, some researches have discussed how to implement control charts in different industries, for instance, the application of T^2 control chart has been investigated in the steel making shop by Maiti et al. (2008).

The more recent researches have been proposed to develop and apply control charts under different types of quality characteristics and assumptions. For instance Singh and

Prajapati (2012) discussed the effect of autocorrelation on the performance of exponentially weighted moving average (EWMA) control chart and obtained optimal design of EWMA control chart under different values of autocorrelation coefficients. As another research Busaba et al. (2011) developed EWMA control chart for monitoring processes with multivariate Poisson distributed quality characteristics. Also Bilen and Chen (2009) compared different control charts for processes with auto correlated quality characteristics and proposed a guideline for choosing proper control charts for various conditions based on ARL criterion. Monitoring processes with categorical responses has been also studied by Dasgupta (2009).

According to the importance of the control charts, some researches have been proposed to improve the performance of control charts, for instance, Prajapati (2010) proposed a new X-bar control chart which is as simple as Shewhart X-bar chart in application but more efficient than it. Also Chetouani (2011) applied a CUSUM-based process monitoring using neural adaptive black-box identification. There are also some researches which have investigated the inspection operation in process monitoring, for instance, Azadeh and Sangari (2010), Korytkowski and Wisniewski (2012) studied different strategies for minimising inspection cost and other researches have discussed this operation in fuzzy environment such as Assadi and Cheraghi (2009).

A major concern about control charts is determining the parameters of the charts such that not only the cost of implementing them is minimised but also the statistical properties of the charts are satisfied. As discussed in previous section the economic-statistical design of control chart leads to more appropriate results in comparison with the economic design. Hence, a summary of proposed researches in the field of designing control charts is briefly reviewed in the next sub section.

2.2 Design of control charts

In the field of multivariate control charts, Hotelling (1947) proposed first multivariate control chart (T^2), and the studies about economic design of this control chart was done by Montgomery and Klatt (1972). Chen (1995) extended this research to the economic-statistical design for Hotelling T^2 control chart. Also, Chou (2007) presented an economic design for Hotelling T^2 control chart while the sampling intervals vary and they solved this problem using GA. In the recent studies more attention is concentrated on monitoring the multivariate processes by using control charts such as MEWMA control chart Lowry et al. (1992). This control chart has advantages rather than Shewhart type control chart such as detecting small shift in the mean vector of quality characteristics and robustness to the normality assumption in distribution of quality characteristics. Meanwhile, in designing this control chart, smoothing parameter (λ) should be determined as well. Linderman and Love (2000) presented an economic-statistical design of MEWMA control chart and used Hooke and Jeeves (1961) algorithm to solve their model but they used simulation method in obtaining ARL. Same approach has been applied by Molnau et al. (2001), while they used Markov chain in determining ARL. Testik and Borror (2004) have reviewed the strategies in design of MEWMA control chart. Recently, Niaki et al. (2010) proposed an economic-statistical design of MEWMA control charts using Taguchi loss approach. They extended the Lorenzen and Vance model to include intangible external costs. Celano (2010) proposed a new approach for economic-statistical design of the EWMA control chart where the

availability of inspection resources is considered as a constraint in the mathematical modelling of the problem.

In the research area of multi-objective economic-statistical design of control charts, Celano and Fichera (1999) proposed an aggregative multi-objective design of $\bar{X}$ control chart, where the objectives are aggregated in an overall function and this function is optimised it by using GA. Zarandi et al. (2007) used aggregative approach considering fuzzy cost parameters and used adaptive neuro-fuzzy interface system (ANFIS) to transform the complex model to a fuzzy rule-based one, and then obtained optimal design of $\bar{X}$ control chart by using GA. In the non-aggregative approach, Chen and Liao (2004) and Asadzadeh and Khoshalhan (2009) used non-aggregative discrete optimisation methods for designing $\bar{X}$ control chart.

3 MEWMA control chart

The univariate statistic of EWMA control chart first proposed by Robert (1959) is presented in equation (1).

$$z_t = \lambda\left(x_t - \mu\right) + (1-\lambda)z_{t-1}, \tag{1}$$

where, $z_0 = 0$, and $0 < \lambda \leq 1$ is the smoothing parameter. Because of two main advantages of this control chart, more attentions are being attracted for using it in many processes by engineers. The first advantage is that the statistic of this chart considers the effect of the previous sampled data and the second advantage is the robustness of this chart toward non-normality of quality characteristics. These advantages motivated researchers to propose a similar control chart for processes with multiple quality characteristics as well.

The extension of the EWMA statistic to the multivariate case was proposed by Lowry et al. (1992).

Consider a process with p quality characteristics monitored over time t, where $x_1, x_2, \ldots, x_p$ represent a $p \times 1$ vector of independent normally distributed quality characteristics, $x_t \sim N_p(\mu, \Sigma)$. μ is the mean vector and Σ is the known covariance matrix of quality characteristics. If the process is in control, without loss of generality, it can be assumed that the process mean is the zero vector. If it is required to consider the effects of previous sampled statistics, one can use discounted weighting of vectors x_t using EWMA vectors represented as z_t recursively by equation (2).

$$z_t = R\left(x_t - \mu\right) + (I - R)z_{t-1}, \tag{2}$$

where R is a diagonal matrix with the elements of λ ($0 < \lambda \leq 1$) and z_0 is a zero vector. The MEWMA statistic proposed by Lowry et al. (1992) is plotted using the statistic calculated by in equation (3).

$$Q_t^2 = z_t' \Sigma_z^{-1} z_t \tag{3}$$

In this equation z_t's are obtained from equation (1) and also Σ_z is the variance-covariance matrix of z_t's and computed as follows:

$$\Sigma_z = \left(\frac{\lambda}{2-\lambda}\right)\Sigma_x. \tag{4}$$

An out-of-control signal is alarmed when $Q_t^2 \geq L$, where L is a predefined control limit determined by the user.

A certain advantage of MEWMA control chart is its power for detecting small shifts in the process mean vector, meanwhile the smaller values for the elements of λ increase this power. Designing an MEWMA control chart includes determination of sample size (n), sampling interval (h), the upper control limit (L) and the smoothing parameter (λ) which is discussed in the further sections.

4 Cost function

The conventional researches of designing charts assumed the Duncan (1956) or Lorenzen and Vance (1986) model for the cost function, while more recent researches are trying to develop the models to incorporate more economic aspects of a process. For instance, the research proposed by Niaki et al. (2010) incorporates the costs due to deviation of the process mean from target value of the process, or the research proposed by Celano (2010) which considers the inspection resource availability as constraints in model of the problem respectively.

In this paper we use the Lorenzen and Vance model as the base cost function and incorporate it with the Taguch loss function. Hence, the Lorenzen and Vance model as well as the Taguchi loss approach are explained in details in the following subsections:

4.1 Lorenzen and Vance model

The cost model which is used in this paper is the model presented by Lorenzen and Vance (1986). This model can be decomposed to three main components. The first component represents the costs when the process is in-control, the second component includes the costs when an assignable cause has occurred and the process parameters are changed to the out-of-control state. Since the expected total costs are defined as the total expected cost per unit of time, the expected total cost in a production cycle should be divided by the expected cycle length. The assumption of this model is that the process has been started from an in-control state and the time to the occurrence of an assignable cause follows an exponential distribution by the mean of $\frac{1}{\theta}$. The cost function is illustrated in equation (5) as follows:

$$C = \frac{\frac{C_0}{\theta} + C_1\left(-\tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right) + \frac{SF}{ARL_0} + W}{\frac{1}{\theta} + \left(1-\gamma_1\right)\frac{ST_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2} + \left(\frac{a+bn}{h}\right)\frac{\frac{1}{\theta} - \tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2}{\frac{1}{\theta} + \left(1-\gamma_1\right)\frac{ST_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2}, \tag{5}$$

where

h	sampling intervals (the time between alternate samplings)
C_0	hourly costs of non-conforming items produced in an in control process
C_1	hourly costs of non-conforming items produced in an out of control process
τ	expected time between the occurrence of the assignable cause and the time of the last sample taken before the assignable cause = $1 - (1 + \theta h)e^{(-\theta h)} / \theta(1 - e^{(-\theta h)})$
E	one unit of sampling time
ARL_0	average run length of an in control process
ARL_1	average run length of an out of control process
T_0	mean search time to assure the signal is a false alarm
T_1	mean time to detect the assignable cause
T_2	mean time to perform the corrective actions
γ_1	1 if production is not stopped during investigation for assignable cause, 0 if production is stopped during investigation for assignable cause
γ_2	1 production is not stopped during repairing 0 if production is stopped during
S	the mean number of samples taken when the process is in-control state, equal to: $e^{(-\theta h)} / 1 - e^{(-\theta h)}$
F	costs incurred from false alarm
W	cost of corrective actions to repair the assignable cause
a	fixed cost per sample
b	cost per unit sampled.

The cycle includes an in-control phase and an out-of-control phase alternately. After performing a corrective action to remove the assignable cause, the process restores to the in-control state and the cycle restarts accordingly. The total cost in each cycle includes the costs of sampling, investigation, and repairing costs as well as the costs due to non-conformities. ARL is a measure to evaluate a designed control chart. When the process is in-control, fewer false alarms are preferred to be signalled; it means that higher value of ARL_0 is more preferable. On the other hand, for an out-of-control process the shorter ARL is preferred since it shows that the chart can sooner detect the out-of-control state, so a lower value of ARL_1 is more preferable. In this paper, to compute the ARL, the Markov chain approach which is proposed by Runger and Prabhu (1996) is used.

4.2 Taguchi loss approach

For incorporating external costs Taguchi loss approach is used in this paper. Using Taguchi loss approach, the costs for deviation from target values of responses is

considered in the cost function. In this paper we used the multivariate loss function presented by Kapur and Cho (1996) shown in equation (6).

$$L(y_1, y_2, \ldots, y_p) = \sum_{i=1}^{p} \sum_{j=1}^{i} k_{ij} (y_i - t_i)(y_j - t_j), \tag{6}$$

where k_{ij} represents the interaction cost for deviation of the ith and jth response from their target values, t_i and t_j. The expected external cost for the in control process J_0 and the expected external cost for the out of control process J_1 are shown in equations (7) and (8), respectively.

$$J_0 = \sum_{i=1}^{p} \sum_{j=1}^{i} k_{ii} \left[(\mu_{0i} - t_i) + \sigma_i^2 \right] + \sum_{i=2}^{p} \sum_{j=1}^{i-1} k_{ij} \left[(\mu_{0i} - t_i)(\mu_{0j} - t_j) + \sigma_{ij} \right], \tag{7}$$

$$J_1 = \sum_{i=1}^{p} \sum_{j=1}^{i} k_{ii} \left[(\mu_{1i} - t_i) + \sigma_i^2 \right] + \sum_{i=2}^{p} \sum_{j=1}^{i-1} k_{ij} \left[(\mu_{1i} - t_i)(\mu_{1j} - t_j) + \sigma_{ij} \right]. \tag{8}$$

After computing J_0 and J_1 the external costs are imported to C_0 and C_1 using equation (9).

$$\begin{aligned} C_0 &= J_0 P + C_0', \\ C_1 &= J_1 P + C_1', \end{aligned} \tag{9}$$

where C_0 and C_1 represent the total costs for in-control and out-of-control process, respectively. Then by replacing these parameters to the Lorenzen and Vance cost function, the optimal design of MEWMA control chart are obtained by minimising the new function which is named as ATL.

5 Multi-objective approaches for economic-statistical model

The multi-objective economic-statistical design of MEWMA control chart discusses about determination of sample size (n), sampling interval (h) and control limit (L) and the smoothing vector (λ) such that the cost function (ATL) and the in-control (ARL_0) and out-of-control (ARL_1) ARL are optimised, simultaneously. The ATL function and ARL_1 are minimisation objectives while ARL_0 is a maximisation one. Note that the optimisation of such objectives imposes some conflicts to the model. For instance, to increase ARL_0, the control limit (L) should be widened while such an action deteriorates ARL_1. To avoid this, the sample size (n) should be increased which might increase the ATL value. Besides, according to the non-linear relation of sample size (n) and the control limit (L) with the ATL, there is a complex structure of correlation among these objectives. All these issues impel us to use powerful multi-objective approaches to determine sample size (n), control limit (L) and sampling frequency (h) such that all objectives are optimised simultaneously.

In this section two different approaches of multi-objective economic-statistical designs are discussed. The first approach is the aggregative and the second one is the non-aggregative approach. The details of the approaches are mentioned as follows:

5.1 Aggregative approach

In some cases statistical properties are the same important as cost, therefore, statistical properties should be considered as objectives. To extend the single objective economic-statistical design of MEWMA control chart to a multi-objective model the cost function and the statistical properties are considered as the objectives in the proposed model which is illustrated in equation (10).

$$\text{Minimise ATL } \left(1+W_1\left(\left(W_2 / ARL_0\right)-\left(W_3 / ARL_1\right)\right)\right), \tag{10}$$

In which $W_1 = 0.01$, $W_2 = 200$, and $W_3 = 10$ are the coefficients which enable the model to include the statistical properties in the objective function. To obtain an optimal design of MEWMA control chart based on multi-objective approach, GA has been applied to optimise this objective function.

The main advantage of this approach is its simplicity because no multi-objective optimisation framework is needed. The only challenge in this approach is the way that objectives are aggregated mostly determined by interchanging with the quality manager.

5.2 Non-aggregative approach

In the aggregative approach discussed in Section 5.1, the objective functions are summarised in single objective using an additive function, then by optimising this function a suitable solution is found. Although this solution might satisfy the required costs and statistical properties, but most managers are more interested to have a set of possible solutions called as Pareto optimal solutions to choose among from.

A relevant term used in the literature of multi-objective optimisation is the Pareto solution, which is a solution with permissible values for all objectives, while no other solution can be found by which some of the objectives improve without exasperating the other objectives.

In normalised normal constraint method proposed by Messac et al. (2003) a hyper plane of Pareto solutions is achieved in multi-objective problems. Note that the objective functions should be minimisation type ones in normalised normal method, so the maximisation objectives should be initially transformed to minimisation ones before being used in this method. Note that the ARL_0 is a maximisation type objective, so it should be transformed to a minimisation one before being used in normalised normal constraint method. The ARL_1 and cost function are minimisation objectives and they do not need any manipulation. The steps of this method are explained as follows:

Step 1 Optimise each objective separately and obtain $\mu_i\,(x_i^*)$ for $i = 1, 2, 3$, here in for the three objectives of ARL_0, ARL_1 and cost, respectively.

In the notations presented in this section, consider that each decision variable of x_i is a vector of the control chart designing variables including n, h, L, λ and $\mu_i\,(x_i^*)$ represents the value of the ith objective function based on x_i.

By conducting an optimisation for each objective function, the solution which results the best value for the ith objective (x_i^*) is found, however this solution might result in impermissible values for other objectives. The result of each optimisation is a three dimensional point that each dimension represents the

objective values of the corresponding function for a certain solution. The mentioned points are shown in equations (11), (12) and (13) as follow:

$$\left[\mu^*_{ARL_0}, ARL_1\left(x_1^*\right), \text{cost}\left(x_1^*\right)\right], \tag{11}$$

$$\left[ARL_0\left(x_2^*\right), \mu^*_{ARL_1}, \text{cost}\left(x_2^*\right)\right], \tag{12}$$

(13)

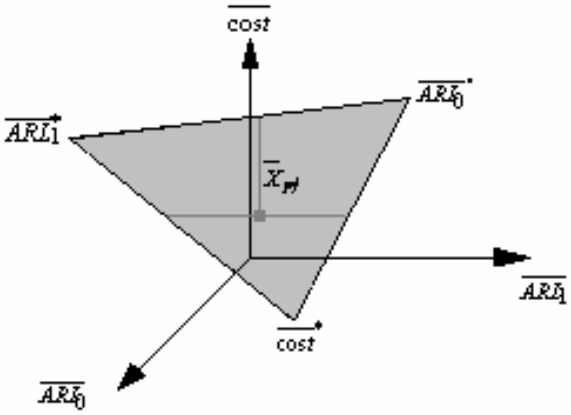
$$\left[ARL_0\left(x_3^*\right), ARL_1\left(x_3^*\right), \mu^*_{\text{cos}t}\right] \tag{13}$$

The hyper plane which comprises the best values of all the objectives, is called the *Utopia* hyper plane given in equation (14),

$$\mu^u = \left[\mu^*_{ARL_0}, \mu^*_{ARL_1}, \mu^*_{\text{cos}t}\right], \tag{14}$$

where μ^u the *Utopia* set of objectives. Figure 1 illustrates the hyper plane for a three objective case.

Figure 1 The *Utopia* hyper plane of Pareto solutions



Step 2 Since the scales of objectives are different from each other, a normalisation procedure should be performed on the objectives. For this purpose the distance of each objective function from its *Utopia* value and also the distance of the *Utopia* point of each objective from its Nadir point should be measured. Nadir point is the point where the objective function has its worst value in comparison with the other solutions. To obtain the Nadir point of the ith objective function use equations (15), (16) and (17). The Nadir vector is also represented in equation (18).

$$\mu^N_{ARL_0} = \max\left[\mu^*_{ARL_0}\left(x_1^*\right), ARL_0\left(x_2^*\right), ARL_0\left(x_3^*\right)\right], \tag{15}$$

$$\mu^N_{ARL_1} = \max\left[ARL_1\left(x_1^*\right), \mu^*_{ARL_2}\left(x_1^*\right), ARL_1\left(x_1^*\right)\right], \tag{16}$$

$$\mu^N_{\text{cos}t} = \max\left[\text{cost}\left(x_1^*\right), \text{cost}\left(x_2^*\right), \mu^*_{\text{cost}}\left(x_3^*\right)\right]. \tag{17}$$

$$\mu^N = \left[\mu^N_{ARL_0}, \mu^N_{ARL_1}, \mu^N_{\text{cost}}\right]. \tag{18}$$

By having the μ^u and the μ^N, D matrix is defined as equation (19) as follows:

$$D = \begin{Bmatrix} l_1 \\ l_2 \\ l_3 \end{Bmatrix} = \begin{Bmatrix} \mu^N_{ARL_0} - \mu^u_{ARL_0} \\ \mu^N_{ARL_1} - \mu^u_{ARL_1} \\ \mu^N_{\text{cost}} - \mu^u_{\text{cost}} \end{Bmatrix}, \tag{19}$$

and the objectives are normalised using equation (20), (21) and (22);

$$\bar{\mu}_{ARL_0} = \frac{ARL_0 - \mu^*_{ARL_0}}{\mu^N_{ARL_0} - \mu^u_{ARL_0}}, \tag{20}$$

$$\bar{\mu}_{ARL_1} = \frac{ARL_1 - \mu^*_{ARL_1}}{\mu^N_{ARL_1} - \mu^u_{ARL_1}}, \tag{21}$$

$$\bar{\mu}_{\text{cost}} = \frac{\text{cost} - \mu^*_{\text{cost}}}{\mu^N_{\text{cost}} - \mu^u_{\text{cost}}}. \tag{22}$$

Step 3 Define the direction $\overline{N}_k$ from $\bar{\mu}^*_k$ to $\bar{\mu}^*_{\text{cost}}$ for $k \in \{1, 2\}$ using equations (23) and (24).

$$\bar{N}_1 = \bar{\mu}^*_{\text{cost}} - \bar{\mu}^*_{ARL_0}, \tag{23}$$

$$\bar{N}_2 = \bar{\mu}^*_{\text{cost}} - \bar{\mu}^*_{ARL_1}. \tag{24}$$

Step 4 In this step we generate new points on the *Utopia* hyper plane by using equation (25).

$$\bar{X}_{pj} = \sum_{i=1}^{3} \alpha_{kj} \bar{\mu}^*_k, \quad \forall j = 1, 2, \ldots, s \tag{25}$$

where $0 \le \alpha_{kj} \le 1$ and defined by the user, conditioned that $\sum_{k=1}^{3} \alpha_{kj} = 1$. s is the number of Pareto solutions that should be generated, and is determined by the user.

Step 5 Generate Pareto solutions for each value of $\bar{X}_{pj}$ by solving the model presented in equation (26).

$$\begin{aligned} &\min_x \bar{\mu}_{\text{cost}} \\ &\text{subject to:} \\ &\bar{N}_k \left(\bar{\mu} - \bar{X}_{pj}\right)^T \le 0, \quad K = 1, 2, \\ &\bar{\mu} = \left\{\bar{\mu}_{ARL_0}, \bar{\mu}_{ARL_1}, \bar{\mu}_{\text{cost}}\right\}. \end{aligned} \tag{26}$$

Regarding to the defined value for s, the optimisation model (26) is iterated s times and s Pareto solutions are achieved respectively. For the optimisation of this model GA is used, as explained in the next section.

5.3 Genetic algorithm

According to the non-linearity form of this objective function, GA can achieve better results in comparison with the classical optimisation techniques. The steps of GA used in this paper are explained as follows:

- *Initial population generations*: Each combination of MEWMA parameters, n, h, L, and λ composes a chromosome including four genes in decimal coding. In the first step, 20 chromosomes are generated randomly and the objective function of each chromosome is computed. Then using elitist rule, five pairs of chromosomes are selected in which chromosomes with the best value of fitness are selected. The selected population is the input population to generate the offspring's. Parents and offspring's are preserved after cross-over and mutation operation. Then new population is selected based on elitist rule.
- *Cross-over*: In this step, five pairs are crossed over together to produce new answers. In each pair, two similar genes are randomly replaced with each other by the probability of $P_c = 0.68$.
- *Mutation*: The mutation step in each loop is performed with probability of $P_m = 0.2$. In this step, each gen in chromosomes is mutated by the size of d, which is calculated by multiplying three factors. The first factor is a standard normal random number. The second factor is the length of the feasible range within which the gene generation is performed. The length of n, h, L and λ are 30, 10, 30 and 1, respectively. The third factor is a fix number denoted by $C_m = 0.00106$. These basic parameters are extracted from Niaki et al. (2010) which are tuned based on the response surface methodology (RSM).
- *Evaluation*: In this step, the objective value of the chromosomes includes parents, offspring's and muted offspring's are compared and sorted. At the end of this step, the ten chromosomes with the lower objective value are exported to the first step to repeat the procedure.
- *Stopping rule*: Two stopping rules are applied in the proposed GA. The first criterion is the number of iterations which is set at ten and the second criterion is the difference between the minimum objective value of three alternate steps which is 0.01. The chromosome with the minimum objective value in the last iteration is selected as the optimal solution.

5.4 GA optimisation procedure

Since GA method is a search-based algorithm, the quality of the input data seriously impacts the quality of the final out puts of the model. Hence, optimisation of the model presented in equation (18), a two stage GA model is used.

In the first stage a population size of 20 gens are produced and as explained in Section 5.3 using the elitist rules and the mutation operator, the populations with better fitness functions are generated iteratively. When the stopping rule rules are realised, among them, 10 gens with the best values of fitness functions are selected as the initial population for the second stage optimisation.

The second stage optimisation is a similar optimisation model in which the initial populations are the solutions which have approximately suitable fitness value. The stopping rules of this stage are the same as the first stage rules.

6 Numerical example

The proposed approach is evaluated through a numerical example extracted from Niaki et al. (2010). In this numerical example, y_1 is the radius and y_2 is the weight of automobile piston. The specification limits of y_1 and y_2 are 20 ± 0.1 in 4 ± 0.05 lb, respectively, and the correlation matrix is given in equation (27).

$$\sum = \begin{bmatrix} 0.002549 & 0.000731 \\ 0.000731 & 0.000798 \end{bmatrix} \tag{27}$$

Considering the correlation of the quality characteristics for monitoring the process, an MEWMA control chart should be designed. The required input parameters of the example are as follows:

$$\frac{1}{\theta} = 100,\ E = 0.05,\ T_0 = 0,\ T_1 = 2,\ T_2 = 2,\ \gamma_1 = \gamma_2 = 1,$$
$$C_0 = 10,\ C_1 = 100,\ F = 50,\ W = 25,\ a = 0.5,\ b = 0.1,$$
$$\mu = [0.0505 \quad 0.0282],\ K = \begin{bmatrix} 1500 & -1{,}000 \\ -1{,}000 & 8{,}000 \end{bmatrix}$$

The ARL computation is done using the Markov chain method proposed by Runger and Prabhu (1997). In most practical applications, m = 25 for the in-control state and $m_1 = m_2 = 5$ for the out-of-control state are suggested. The parameters of m, m_1, m_2 are the input parameters in Markov chain algorithm for ARL computation. Furthermore, when the process mean shifts from the initial vector of zero to the new vector of μ, the value of non-centrality parameter is obtained using equation (28) as follows:

$$\delta = \left(\mu' \Sigma^{-1} \mu\right)^{0.5} = 1.320 \tag{28}$$

The aggregated function presented in equation (10) is used in multi-objective economic statistical design of MEWMA control chart and the obtained results are compared with the results reported by Niaki et al. (2010), shown in Table 1. It can be concluded that the proposed aggregative method performs better than the traditional economic-statistical model from the view point of the statistical properties. It can be seen that the ARL_0 value is larger than Niaki et al. (2010) and also the value of ARL_1 is smaller. The results show significant improvement in statistical properties toward less than 1% increase in cost.

Table 1 Optimal design of MEWMA control chart using aggregative approach

Model	*n*	*h*	*L*	λ	*Cost*	ARL_0	ARL_1
Economic-statistical Niaki et al. (2010)	11	0.484	11	0.802	1,007.382	246.170	1.356
Multi-objective ES (The proposed aggregative approach)	23	0.7851	13.62	0.853	1012.6	908.044	1.035
Improvement (%)	-	-	-	-	–0.52	268.87	23.67

For the non-aggregative approach, ARL_0 and ARL_1 and cost function are optimised separately and the corresponding optimum solution, x_i^* are obtained. Also the vectors of μ^u and μ^N are reported in Table 2.

Table 2 The initial computations for normalised normal constraint method

i	x_i^*	ARL_0	ARL_1	*Cost*
1	$n = 28$, $h = 1.04$, $l = 11$, $\lambda = 0.736$	6,993	1.12	1,016.3
2	$n = 21$, $h = 1.835$, $l = 11.6$, $\lambda = 0.626$	255	1.0457	1,014.7
3	$n = 11$, $h = 0.484$, $l = 11$, $\lambda = 0.80$	246	1.356	1,007.38
μ^u	[6,993, 1.0457, 1,007.8]			
μ^N	[246, 1.356, 1,016.3]			

Using the computations reported in Table 2, by assuming $s = 10$, the following Pareto solutions are obtained and reported in Table 3. In this table, for each possible combination of a_{kg} a permissible solution for the problem is obtained and a variety of solutions are in hand for the manager to choose among from.

For comparing the obtained results of the non-aggregative approach with the results reported by Niaki et al. (2010), the improvement percentages for ARL_0, ARL_1 and the cost are reported in Table 3, as well.

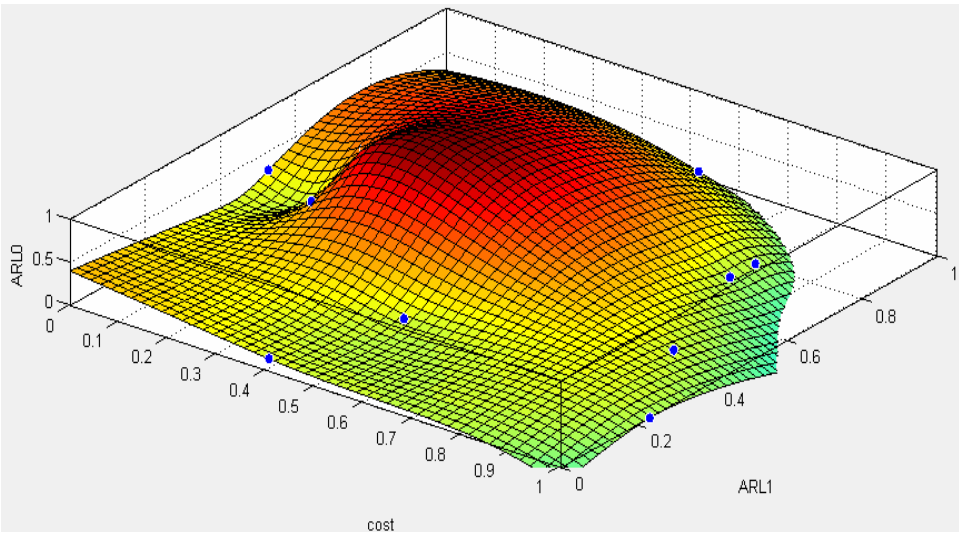
As shown in Table 3, although the cost values have increased less than 1% in comparison with the cost reported by Niaki et al. (2010), but the values of ARL_0, ARL_1 have improved significantly. Note that improvement of ARL_0 would result in less false alarms as well as wrong process interruptions. Moreover, improving ARL_1 results in decreasing probability of type II error and less non-conforming items sent to customers consequently. According to these two notions, it can be claimed that the more improvement of obtained solutions in statistical properties of the chart would redress the very few increase in the cost values. Meanwhile, the obtained Pareto solutions enable managers to have more choices for investigating the tradeoffs between economic and statistical objectives. A schematic image of the obtained Pareto solutions is illustrated in Figure 2.

Since the scales of objective values are different from each other, the corresponding data have been normalised before plotting the Pareto surface.

Table 3 The Pareto solutions for optimal design of MEWMA control chart using non-aggregative approach

NO.	$(\alpha_1, \alpha_2, \alpha_3)$	*n*	*h*	*L*	*r*	ARL_0	ARL_1	*cost*	*Improvement percent*		
									ARL_0	ARL_1	*cost*
1	1/5 2/5 2/5	17	0.666	11.753	0.922	356.819	1.0936	1,009.7	44.72%	18.88%	–0.23%
2	1/3 1/3 1/3	17	1.221	12.070	0.803	419.705	1.1106	1,011.2	69.92%	17.40%	–0.38%
3	2/5 2/5 1/5	18	1.528	12.245	0.827	457.490	1.0881	1,012.6	76.65%	18.20%	–0.58%
4	2/5 1/5 2/5	20	0.76	11.69	0.847	346.491	1.0429	1,011.1	35.31%	23.3%	–0.37%
5	1/5 3/5 1/5	20	1.172	14.318	0.980	1285.77	1.0883	1,012.2	422.64%	19.33%	–0.48%
6	2/9 3/9 4/9	18	0.699	11.318	0.821	288.138	1.0673	1,010.1	16.81%	20.90%	–0.27%
7	4/11 3/11 4/11	16	1.357	12.084	0.910	421.029	1.1335	1,011.5	71.02%	15.67%	–0.41%
8	1/5 1/5 3/5	16	0.653	11.211	0.803	273.465	1.1121	1,009.3	10.57%	17.40%	–0.19%
9	3/13 4/13 6/13	18	0.707	11.249	0.922	277.302	1.0612	1,010.1	12.67%	21.37%	–0.27%
10	3/6 2/6 1/6	19	1.502	12.639	0.746	559.129	1.0858	1,013	125.5%	19.3%	–0.6%

Figure 2 The fitted Pareto solution surface for the normalised objective values (see online version for colours)



7 Conclusions

In this paper two multi-objective approaches were applied and compared for economic-statistical design of an MEWMA control chart. Based on the proposed aggregative multi-objective economic-statistical model the statistical properties including the in-control and out-of-control ARL are considered as objectives in addition to the cost function. The Lorenzen and Vance cost model by considering Taguchi loss was used as the cost function.

Genetic algorithm was used to obtain optimal parameters of MEWMA control chart. The performances of the proposed approaches were evaluated through a numerical example and the results were compared with the traditional economic-statistical model. The results showed that by less than 1% increasing in overall cost, a significant improvement is obtained in statistical properties, both in aggregative and non-aggregative approaches. It was shown that the proposed method can result in appropriate economical statistical design of MEWMA control charts and using the non-aggregative approach a variety of solutions can be obtained for decision makers and managers.

On the other hand, one of the limitations of the aggregative approach is difficulty in determining the appropriate weight for the objectives.

The main contribution of this paper was proposing different multi-objectives approaches for designing the MEWMA control chart. Applying these approaches would result in more efficient process monitoring, cost reduction and consequently more satisfaction of the management.

Multi-objective design and optimisation of other multivariate control charts such as T^2 and MCUSUM charts can be a fruitful area for future research. In addition, incorporating the preventive maintenance in to the cost model would increase the applicability of the model.

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