

# Compact Filtering Unequal Wilkinson Power Divider with 1:15 Power Dividing Ratio Using Composite Right/Left Handed-Coupled Lines

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**Abstract** – A novel design approach for a compact unequal Wilkinson power divider (WPD) with arbitrary large power dividing ratio, 1:15, and filtering response based on composite right/left-handed coupled-line (CRLH-CL) circuit structure is proposed and investigated. The presented structure is stemmed from a typical unequal WPD by utilizing asymmetric CRLH-CL section as a  $\lambda/4$  transformer between input and output ports and integrating two CRLH-CL band-pass filter transformers near the output ports. The analytical closed-form design equations for the proposed structure are derived and a design procedure for physical implementation is presented. To theoretically verify the proposed design approach, a 1:15 filtering WPD operating at center frequency of 1.4 GHz, is designed and simulated.

## 1 INTRODUCTION

Among all kinds of power dividers, the Wilkinson power divider (WPD) is one of those commonly used components in RF and microwave modules and subsystems due to its planar and simple structure, good input and output port matching and ideal isolation characteristics [1]. Based on the power division ratio, there are two typical categories of WPDs: the equal WPDs which are simple to design and realize and the unequal ones which are difficult to implement and have limitations in design and fabrication, especially in a case of large power dividing ratio where the too high- or too low-impedance transmission lines with very narrow or wide conductor width are needed. To solve this issue, several techniques have been proposed such as adopting defected ground structures (DGS) under the microstrip line [2], using coplanar waveguide (CPW) with electromagnetic band gap structures (EBGs) [3], grooving substrate under the strip lines [4], and so on. However, all of these techniques suffer from the restricted power division ratio as the highest realized splitting ratio using these techniques is 1:11 [5] and there is no general solution for simplified design process. Apart from unequal WPDs with arbitrary large power dividing ratio, band-pass filters (BPFs) play an important role in many RF/microwave systems such as beam-forming networks and mostly need to be coexist with power dividers in the same front-end to divide and filter signals simultaneously. To further reduce the circuit size, integrating two functions into one device i.e., filtering WPD, is necessary. In the past, a few integrated designs based on conventional microstrip TLs were proposed such as [6]. However, these designs exhibit degrees of limitation such as large occupied area, restricted

power dividing ratio as the highest implemented power ratio does not exceed 1:4 or non-exact design equations. Nevertheless, there is still scope for increasing the power dividing ratio as well as selectivity of each transmission path by using composite right/left-handed coupled-line, CRLH-CL, which is realized by loading host coupled transmission line medium with series capacitors and shunt inductors to the ground.

In this paper, a novel structure of the WPD with the ability to provide high power-dividing ratio as well as filtering response with reduced occupied area based on CRLH-CL is proposed, analyzed and designed. The proposed WPD is composed of an asymmetric CRLH-CL section as a  $\lambda/4$  impedance transformer to realize arbitrary power dividing ratio with reduced arm lengths and two CRLH-CL filter transformers near the output ports to satisfy the matching requirement for the system impedance of  $50\Omega$  as well as to achieve the desired selectivity in each transmission path and a lumped resistor for output port isolation. To show the capability of the proposed design, a typical example with arbitrary high power dividing ratio of 1:15 operating on center frequency of 1.4 GHz is presented with simulated results.

## 2 THEORY AND DESIGN

### 2.1 Configuration

Figure 1(a) shows the configuration of the proposed unequal WPD with arbitrary large power dividing ratio and band-pass response. It consists of an asymmetric CRLH-CL structure as a  $\lambda/4$  transformer between input and output ports to realize arbitrary power dividing ratio and two symmetric CRLH-CL filter transformers near the output ports to satisfy the matching requirement for the system impedance of  $50\Omega$  as well as to achieve the desired selectivity in each transmission path. Figure 1(b) shows the CRLH-CL structure used in this design to replace the  $\lambda/4$  transmission line transformers in conventional unequal WPDs. To derive the simplified design procedure as well as closed-form design equations, in-and anti-phase analysis, [7], based on metamaterial theory is carried out. First of all, the CRLH-CL is investigated using coupled-mode formulation in Section 2.2 to derive closed-form expressions for loading element parameters in terms of modal

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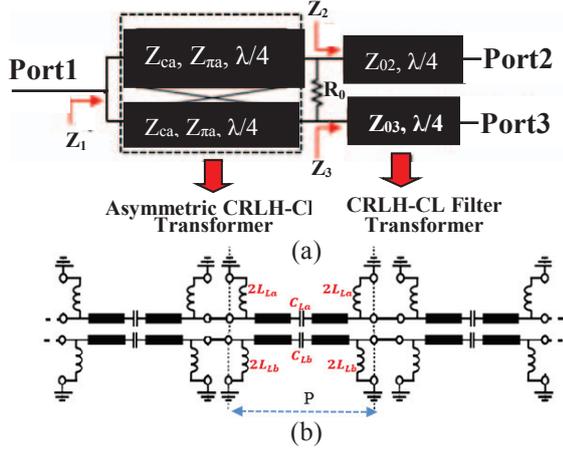


Figure 1: (a) structure of the proposed WPD, (b) a typical structure of the asymmetric CRLH-CL.

characteristic impedances. Then, the overall power splitting circuitry is analyzed in Section 2.3, based on the desired power division ratios and the filtering function.

## 2.2 Analysis of CRLH-CL

Figure 1(b) shows the structure of asymmetric CRLH-CL as the most general case of CRLH-CL which consists of two different CRLH transmission lines (TLs). It can be assembled from  $N$  unit cells that are composed of left-handed (LH) and right-handed (RH) sections. The LH section is based on lumped loading shunt inductors ( $L_{La}$  and  $L_{Lb}$ ) and series capacitors ( $C_{La}$  and  $C_{Lb}$ ) and RH section is realized with conventional microstrip coupled lines. The length of the unit cell,  $p$ , is much smaller than wavelength at the design frequency range; hence the asymmetric CRLH-CL implemented by periodically chaining  $N$  number of unit cells can be regarded effectively homogeneous and characterized by the general coupled-mode differential equations [7]. Its solution as a superposition of two normal modes of propagation,  $c$  and  $\pi$  modes, are characterized by four parameters including  $R_{c/\pi}$ ,  $\gamma_{c/\pi}$ ,  $Z_{c/\pi a}$  and  $Z_{c/\pi b}$  which are defined as the modal voltage ratio between two lines,  $(v_b/v_a)_{c/\pi a}$ , modal propagation constants and modal characteristic impedances of lines 'a' and 'b', respectively and can be expressed as following:

$$Z_{c/\pi a} = \frac{\gamma_{c/\pi}}{Y_a + Y_m R_{c/\pi}}, \quad Z_{c/\pi b} = \frac{\gamma_{c/\pi}}{Y_b + Y_m / R_{c/\pi}} \quad (1)$$

$$\gamma_{c/\pi} = j\beta_{c/\pi} = \sqrt{Z_{a/b} Y_{a/b}} \quad (2)$$

where,  $Z_a/Y_a$  and  $Z_b/Y_b$  are the per unit length series impedance and shunt admittance of isolated lines 'a' and 'b', respectively, and  $Y_m$  is the per unit length mutual admittance between two lines which are determined as:

$$Y_{a/b} = \frac{1 - \omega^2 C_R L_{La/b}}{j\omega p L_{La/b}}, \quad Z_{a/b} = \frac{1 - \omega^2 C_{La/b} L_R}{j\omega p C_{La/b}}, \quad Y_m = j\omega \frac{C_m}{p} \quad (3)$$

where parameters  $L_R$ ,  $C_R$  and  $C_m$  are the per unit cell self-inductance, self-capacitance, and mutual capacitance, respectively, that model the edge-coupled interconnecting transmission line segments. For a lossless quasi-TEM-mode coupled lines and the fact that  $\pi/2$  phase shift should be provided through asymmetric CRLH-CL,  $\gamma_{c/\pi} = j\beta_{c/\pi} = -j\pi / 2Np$  may be assumed [7]. Moreover, from the given assumption, the following relation between the ratio parameters, i.e.,  $R_c$  and  $R_\pi$ , can be obtained [7]:

$$\gamma_{c/\pi} = j\beta_{c/\pi} = -j \frac{\pi}{2Np}, \quad R_c = -R_\pi = \sqrt{\frac{Z_{cb}}{Z_{ca}}} = k \quad (4)$$

Now, by putting  $\gamma_{c/\pi}$  and  $R_{c/\pi}$  from (4) into (1), the modal characteristic impedances become:

$$Z_{c/\pi a} = \frac{-j\pi}{2Np} \left( \frac{1}{Y_a \pm Y_m k} \right), \quad Z_{c/\pi b} = \frac{-j\pi}{2Np} \left( \frac{1}{Y_b \pm Y_m / k} \right) \quad (5)$$

After some algebra, we obtain the following for  $Y_{a/b}$ ,  $Z_{a/b}$  and  $Y_m$  from (6)-(7).

$$Y_{a/b} = -j \frac{\pi}{4Np} \left( \frac{1}{Z_{ca/b}} + \frac{1}{Z_{\pi a/b}} \right), \quad Z_{a/b} = -\frac{\pi^2}{Y_{a/b} (2Np)^2} \quad (6)$$

$$Y_m = j \frac{\pi}{4Npk} \left( \frac{1}{Z_{\pi a}} - \frac{1}{Z_{ca}} \right) = j \frac{\pi k}{4Np} \left( \frac{1}{Z_{\pi b}} - \frac{1}{Z_{cb}} \right) \quad (7)$$

Regarding (3) and (6)-(7), the design formulas for the unknown values of loading elements  $L_{La/b}$ ,  $C_{La/b}$  and  $C_m$  can be found as the following for the required modal characteristic impedances to produce desired power dividing ratio of  $k^2$ , at a given frequency  $\omega$ .

$$C_{La/b} = \frac{1}{j\omega p Z_{a/b} + \omega^2 L_R}, \quad L_{La/b} = \frac{1}{j\omega p Y_{a/b} + \omega^2 C_R}, \quad C_m = \frac{p Y_m}{j\omega} \quad (8)$$

A symmetrical CRLH-CL is a special case of asymmetric CRLH-CL which consists of two identical CRLH transmission lines having the same per unit length shunt admittance ( $Y_a=Y_b=Y$ ) and series impedance ( $Z_a=Z_b=Z$ ) values. In this case, the  $c$ -mode and  $\pi$ -mode are normally referred to as the even-mode and the odd-mode, respectively.

## 2.3 Power-dividing ratio and filtering response analysis

Based on the analytical results of the asymmetric and symmetric CRLH-CLs, the overall structure of the WPD depicted in Figure 1(a) is investigated. For the WPD, all of the input power injected into port 1 should be asymmetrically divided into two output ports with desired power-dividing ratio of  $k^2$ . Moreover, to have lossless power division, no currents should follow through the isolation resistor  $R_0$  and perfect input port matching should be fulfilled at input and output ports. With these conditions, the equivalent  $c$ -mode characteristic impedances of the asymmetric CRLH-CL section,  $Z_{ca}$  and  $Z_{cb}$ , and isolation resistor,  $R_0$ , should satisfy the following equations and the output impedances  $Z_2$  and  $Z_3$  should be chosen  $Z_3=k^2 Z_2$ .

$$Z_{ca} = \sqrt{\frac{1+k^2}{k^2}} Z_1 Z_2, \quad Z_{cb} = \sqrt{(1+k^2)k^2} Z_1 Z_2 = k^2 Z_{ca} \quad (9)$$

$$R_0 = (1+k^2)Z_2 \quad (10)$$

From (9), the values of  $Z_{ca}$  and  $Z_{cb}$  can be evaluated based on desired power-dividing ratio of  $k^2$ . As analyzed in the previous section, the required modal characteristic impedances,  $Z_{ca}$  and  $Z_{cb}$ , can be easily adjusted with a wide range by appropriate choosing the loading-element values,  $L_{La/b}$  and  $C_{La/b}$ , as well as coupling strength value,  $C_m$ , without manufacturing difficulty. Particularly, large power-dividing ratio can be achieved, while it may not be realized by employing conventional coupled-line structures. For example, for  $k^2=15$  in this design, the equivalent c-mode characteristic impedances can be calculated as  $Z_{ca} = 26.23$  and  $Z_{cb} = 393.5$ . Realizing such c-mode characteristic impedances with conventional coupled lines, requires tightly edge coupled microstrip lines with line widths and spacing far below the fabrication limitation. In contrast, with the asymmetric CRLH-CL structure, this power-dividing ratio is feasible as exhibited in the next Section. As well as the large power-dividing ratio, the proposed WPD enables the filtering response by using two symmetric CRLH-CL filter transformers near the output ports. In ideal WPD, the electrical lengths of two output transformers should be equal to  $90^\circ$ , however, in general application, their practical length can be determined as an odd integer multiple of  $90^\circ$ . As in the proposed power dividing circuitry, such transformers are replaced with the CRLH-CL filter transformers, hence, their transmission matrixes should be approximately equal to each other. This implies that the even-order CRLH-CL filter with system impedance of  $Z_0$  equal to the characteristic impedance of the ideal  $\lambda/4$  transformers,  $Z_{o2}$  and  $Z_{o3}$  (see Figure 1(a)), should be employed instead [8]. To derive the design equations for the given CRLH-CL filter specifications, first, a symmetric CRLH-CL section with  $\pi/2$  electrical length around the operating frequency, is modeled by an admittance inverter,  $J$ , with a system impedance of  $Z_0$ . By equating their image impedances and propagation constants, the following equation are obtained for even- and odd-mode characteristic impedances of symmetric CRLH-CL [8]:

$$Z_{e/o} = Z_0(1 \pm JZ_0 + (JZ_0)^2) \quad (11)$$

The abovementioned equation shows that the even-mode and odd-mode characteristic impedances of a CRLH-CL section can be characterized by the system impedance of  $Z_0$  and the admittance inverter of  $J$ , which has the same representation of those of a conventional coupled line sections[8]. Accordingly, the design equations of the conventional couple-line filters can be adopted to design the CRLH-CL filter transformers. The admittance inverter values of a

typical band pass filter are given by the following equations [8]:

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{2g_1}}, Z_0 J_i = \frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}}, Z_0 J_{N+1} = \sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}} \quad (12)$$

Where  $g_0, g_1, \dots, g_{n+1}$  are the elements of an equal-ripple low-pass prototype with a normalized cutoff  $\Omega_c=1$ ,  $\Delta$  is the fractional bandwidth of the band-pass filter,  $j_1, j_2, \dots, j_{n+1}$  are the characteristic admittances of  $J$  inverters, and  $Z_0$  is the system impedance which should be equal to the characteristic impedances of each output transformers. Substituting the admittance inverter values from (12) in (11), the even- and odd-mode characteristic impedances of CRLH-CL sections can be obtained straightforwardly. Subsequently, the proposed CRLH-CL filter transformers can be properly designed for satisfying matching conditions and improving the selectivity in each transmission path.

### 3 DESIGN EXAMPLE WITH SIMULATION RESULTS

For more demonstration, a filtering unequal WPD with high power dividing ratio of 1:15 is designed and simulated at 1.4 GHz. Figure 2 shows the layout of the proposed WPD with an input asymmetric CRLH-CL impedance transformer to realize 1:15 power dividing ratio and two output symmetric CRLH-CL filter transformers to satisfy the matching requirement for the system impedance of  $50\Omega$  as well as to achieve the desired selectivity in each transmission path. Each output CRLH-CL impedance transformer is second-order band-pass filter which is designed for center frequency of 1.4 GHz and fractional bandwidth of 15% and is composed of three CRLH-CL sections with equivalent even- and odd-mode characteristic impedances derived from (11) and (12) as following (see Figure 2):  $Z_{e1,a} = Z_{e3,a} = 40.01, Z_{o1,a} = Z_{o3,a} = 19.27, Z_{e2,a} = 30.34, Z_{o2,a} = 21.88, Z_{e1,b} = Z_{e3,b} = 154.96, Z_{o1,b} = Z_{o3,b} = 74.63, Z_{e2,b} = 117.52, Z_{o2,b} = 84.74$ . To realize such characteristic impedances, each CRLH-CL section is designed to have only one unit-cell formed of a coupled microstrip line loaded by two shunt shorted stub inductors and series interdigital capacitors (IDCs) with inductance and capacitance values extracted from (8) which can be directly mapped to the geometrical parameters as indicated in the capture of Figure 2. IDCs and stub inductors are chosen due to the fact that they suggest higher degree of design flexibility, due to the more available design parameters. The asymmetrical CRLH-CL section between input and output ports compromises two 6-mm long coupled lines with chosen strip width and spacing of 1.8mm and 0.9 mm, respectively loaded by surface mount technology (SMT) components with inductance and capacitance values derived from (8) as following:

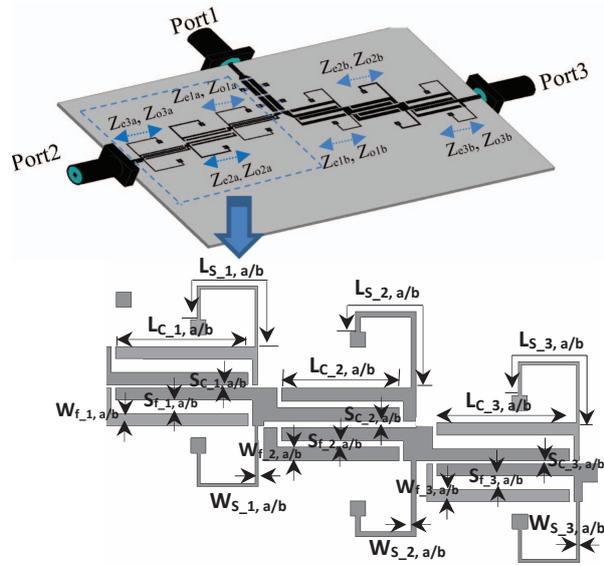


Figure 2: Layout of the proposed filtering 1:15 unequal WPD; where:  $W_{f1,a}=W_{f3,a}=0.3\text{mm}$ ,  $W_{f2,a}=0.4\text{mm}$ ,  $S_{f1,a}=S_{f3,a}=0.2\text{mm}$ ,  $S_{f2,a}=0.4\text{mm}$ ,  $LC_{1,a}=LC_{2,a}=LC_{3,a}=8.7\text{mm}$ ,  $SC_{1,a}=SC_{3,a}=0.1\text{mm}$ ,  $SC_{2,a}=0.6\text{mm}$ ,  $W_{f1,b}=W_{f3,b}=0.8\text{mm}$ ,  $W_{f2,b}=0.9\text{mm}$ ,  $S_{f1,b}=S_{f3,b}=0.9\text{mm}$ ,  $S_{f2,b}=0.4\text{mm}$ ,  $LC_{1,b}=LC_{3,b}=8.7\text{mm}$ ,  $LC_{2,b}=7.7\text{mm}$ ,  $SC_{1,b}=SC_{3,b}=0.2\text{mm}$ ,  $SC_{2,b}=0.4\text{mm}$ .

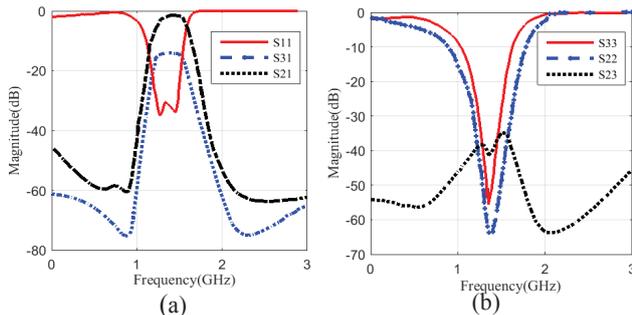


Figure 3: Simulated (a) insertion loss ( $S_{21}$  and  $S_{31}$ ) and input return loss ( $S_{11}$ ) parameters, (b) isolation ( $S_{23}$ ) and output return loss ( $S_{22}$  and  $S_{33}$ ) parameters.

$C_{L,a}=3.2\text{pF}$ ,  $L_{L,a}=3.2\text{nH}$ ,  $C_{L,b}=0.3\text{pF}$ ,  $L_{L,b}=16\text{nH}$ . Due to the relatively large values of the LC reactive loading elements, SMT components are preferred to IDCs and stub inductors in this section. It should be also noted that, in this design, the Rogers R04003 substrate with relative permittivity of 3.55 and a thickness of 0.813 mm is employed. The proposed structure is simulated with Full-wave High Frequency Simulator (HFSS) and the obtained results are depicted in Figure 3. As it can be inferred from these figures, the desired power division ratio with relatively sharp band-pass responses are observed. The center frequency and 3-dB fractional bandwidth of each CRLH-CL filter transformer are approximately 1.4 GHz and 14.8%. The simulated insertion losses,  $|S_{21}|$  and  $|S_{31}|$ , are about 0.45 and 12.8 dB, respectively; the output return losses,  $|S_{22}|$  and  $|S_{33}|$ , are better than 32 dB and the input return loss  $|S_{11}|$  is greater than 20 dB around the desired

frequency band. The simulated isolation,  $|S_{23}|$ , is around 42 dB at the center frequency and better than 34 dB within the whole band. It is worth mentioning that the proposed power divider has the most compact size and the highest power division ratio among the available unequal filtering WPDs, since its circuit size is reduced to 40% of the conventional counterpart of exactly the same electrical parameters and the corresponding power splitting ratio is enhanced to 1:15.

#### 4 CONCLUSION

This paper presents a novel miniaturized filtering unequal WPD with high power-dividing ratio application using CRLH-CLs. By using circuit model, a design procedure has been derived for the proposed power divider. To validate the theoretical design procedure, an unequal compact WPD with a high power division ratio of 1:15 is simulated at 1.4 GHz. The simulation results verify that the proposed WPD not only exhibits a sharp frequency selectivity but also a large power splitting ratio with desired performance.

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