A Generalized Linear Test Model to Monitor AR(1) Autocorrelated Polynomial Profiles

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Abstract— **In some statistical process control applications, quality of a process or product can be characterized by a relationship between a response and one or more independent variables, which is typically referred to a profile. In this paper, polynomial profiles are considered where there is a first order autoregressive relation between the error terms in each profile. A remedial measure is first proposed to eliminate the effect of autocorrelation in phase-II monitoring of autocorrelated profiles. Then, a control chart based on the generalized linear test (GLT) is developed to monitor coefficients of polynomial profiles along with an R-chart to monitor the error variance, the combination of which is called *GLT/R* chart. Then, the results obtained from *GLT/R* charts are compared to the prevalent method like multivariate T^2 control chart. Average run length criterion is employed to compare the performances.**

Keywords— **Statistical process control, Polynomial profiles, Phase-II monitoring, Autocorrelation, Average run length, General linear test**

I. Introduction

Sometimes, a relationship between a response and one or more independent variables, referred to a profile, can characterize the quality of a process or a product adequately. Many researchers including Stover and Brill [1], Kang and Albin [2], Mahmoud and Woodall [3], Woodall et al. [4], Wang and Tsung [5], and Woodall [6] discussed practical applications of profiles. Many authors, including Kang and Albin [2], Kim et al. [7], Mahmoud et al. [8], Mahmoud and Woodall [3], Mestek et al. [9], and Stover and Brill [1] studied Phase-I monitoring of simple linear profiles. The purpose of the Phase-I analysis is to evaluate the stability of a process and to estimate process parameters. Some authors including Gupta et al. [10], Kang and Albin [2], Kim et al. [7], Noorossana et al. [11], Zou et al. [12], Niaki et al. [13], and Saghaei et al. [14] investigated Phase-II monitoring of simple linear profiles. In phase-II analysis, one is interested in detecting shifts in the process parameters as soon as possible. Sometimes more complicated models are needed to represent profiles. Kazemzadeh et al. [15] extended three Phase-I methods in polynomial profile monitoring. Zou et al. [16] proposed a multivariate exponentially weighted moving average (*MEWMA*) control chart for monitoring general linear profiles in Phase II. Kazemzadeh et al. [17] transformed polynomial regression to an orthogonal polynomial regression model and proposed a method based on using exponentially weighted moving average (*EWMA*) control charts to monitor the parameters of orthogonal polynomial model in Phase II.

In all previous research works, it is assumed that the error terms in the model are independently and identically distributed normal random variables. However, in some cases these assumptions are violated. Noorossana et al. [18] investigated the effect of non-normality of the error terms on the performances of the *EWMA/R* method proposed by Kang and Albin [2]. Jensen et al. [19] developed a linear mixed model (*LMM*) to account for the autocorrelation within a linear profile. Jensen and Brich [20] showed that use of mixed models have significant advantages when there is autocorrelation within nonlinear regression models. Noorossana et al. [21] considered linear profiles and modeled autocorrelation between profiles as a first order autoregressive AR(1) process. Kazemzadeh et al. [22] considered polynomial profiles in AR(1) process in the presence of between profile autocorrelation. Soleimani et al. [23] investigated the effect of within profile autocorrelation in simple linear profiles and proposed a transformation technique to eliminate the effect of autocorrelation.

In this paper, the research work of Soleimani et al. [23] is first extended to include polynomial profiles. In other words, processes are considered in which the relationship between a response and a single explanatory variable is defined by a k^{th} order polynomial regression, where it is assumed that the error terms within each profile are correlated based on a first order autoregressive model. Moreover, we assume that there is no correlation between polynomial profiles. An application of this problem is discussed by Amiri et al. [24] in which the quality of an automobile engine is characterized by a second order polynomial profile between the torque and speed in rpm with an AR(1) autocorrelation structure between error terms within each profile.

On the other hand, Niaki et al. [13] employed the concept of the linear regression and the generalized linear test to design a control chart for profiles monitoring. Moreover, in order to detect shifts in the error variance an R chart was simultaneously applied. In this paper, the work of Niaki et al. [13] is extended for polynomial profiles. The performance of this method for monitoring polynomial regression profiles is compared with the prevalent multivariate T^2 method through simulation studies via average run length (*ARL*) criterion.

The structure of the remainder of the paper is as follows: In Section II, the problem formulation as well as assumptions are given. The transformation technique and application of the general linear test to monitor polynomial profiles are presented in Section III. Traditional multivariate T^2 control chart adopted for monitoring polynomial profiles is presented in Section IV. In Section V, the effect of autocorrelation on the performance of *GLT/R* control chart is shown. Then the performances of the GLT/R and T^2 methods are compared in terms of ARL criterion. Concluding remarks are given in Section VI.

II. Modeling and assumptions

Having a single explanatory variable x and assuming *j*th sample is being collected over time, the observations are $(x_i, x_i^2, ..., x_i^k, y_{ij})\ ;\ i = 1, 2, ...n$. In the other words, the subscript *i* shows the *i*th observation within each profile, and subscript *j* shows the *j*th profile collected over time. When the process is in-control, the autocorrelated polynomial profile is modeled as:

$$\begin{aligned} y_{ij} &= A_0 + A_1 x_i + A_2 x_i^2 + ... + A_k x_i^k + \varepsilon_{ij} \\ \varepsilon_{ij} &= \Phi \varepsilon_{(i-1)j} + a_{ij} \end{aligned} , \tag{1}$$

where y_{ij} is the response variable of *i*th observation in *j*th profile and ε_{ij}'s are the correlated error terms, a_{ij}'s are independent and identically distributed normal random variables with mean zero and variance σ^2, $A_0, A_1, ..., A_k$ are model parameters, and $-1 < \Phi < 1$ is the autocorrelation coefficient. Moreover, it is assumed x-values are fixed and constant from profile to profile. In this paper, we consider a Phase-II monitoring case, in which the in-control values of the parameters $A_0, A_1, ..., A_k$ and σ^2 are assumed known.

In the next section, the transformation technique proposed by Soleimani et al. [23] is first employed to eliminate the autocorrelation effect. Then, the GLT/R method by Niaki et al. [13] is utilized to monitor polynomial profiles.

III. Proposed method

A. Extension of Transformation Technique

In order to eliminate the existing within-profile autocorrelation of polynomial profiles the transformation technique proposed by Soleimani et al. [23] is used in the first step of the proposed methods. In this technique, all observations on the response variable are transformed via the following equation:

$$Y'_{ij} = Y_{ij} - \Phi Y_{(i-1)j} . \tag{2}$$

If observations Y_{ij} and $Y_{(i-1)j}$ in Eq. (2) are replaced by their equivalents on regression model (1), a polynomial regression model with independent error terms is obtained as

$$\begin{aligned} Y'_{ij} &= A_0(1-\Phi) + A_1(X_i - \Phi X_{i-1}) + \\ & A_2(X_i^2 - \Phi X_{i-1}^2) + ... + A_k(X_i^k - \Phi X_{i-1}^k) + \\ & (\varepsilon_{ij} - \Phi \varepsilon_{(i-1)j});\ i = 1, 2, ... n \end{aligned} \tag{3}$$

That results in

$$Y'_{ij} = A'_0 + A'_1 X'_i + A'_2 X'^2_i + ... + A'_k X'^k_i + a_{ij} \tag{4}$$

where $Y'_{ij} = Y_{ij} - \Phi Y_{(i-1)j}$,
$X'_i = X_i - \Phi X_{i-1},\ X'^2_i = X_i^2 - \Phi X_{i-1}^2, ...,\ X'^k_i = X_i^k - \Phi X_{i-1}^k$
, and $A'_0 = A_0(1-\Phi), A'_1 = A_1, A'_2 = A_2, ..., A'_k = A_k$.

In Eq. (4), a_{ij} s are independent random variables with mean zero and variance σ^2. In this paper, we consider Phase-II monitoring of polynomial profiles where Φ is assumed a known parameter.

B. GLT/R

After eliminating the effect of autocorrelation within polynomial profiles, GLT/R method proposed by Niaki et al. [13] in simple linear profile monitoring is utilized here for monitoring polynomial profiles.

This method is the general linear test to monitor the coefficients of polynomial regression model when applied to profile monitoring. In Phase II of the proposed procedure, a sample size *n* is collected from process periodically at time *j* and the regression parameters $(A_0, A_1, A_2, ..., A_k)$ are estimated by the least square method. In order to monitor polynomial profile coefficients, *F*-statistic is employed by following equation:

$$F_j^* = \frac{\sum_{i=1}^{n-1}\left(Y'_{ij} - A'_0 - A'_1X'_i - A'_2X'^2_i - ... - A'_kX'^k_i\right)^2 - \sum_{i=1}^{n-1}\left(Y'_{ij} - \hat{A}'_0 - \hat{A}'_1X'_i - \hat{A}'_2X'^2_i - ... - \hat{A}'_kX'^k_i\right)}{k+1} \div$$

$$\frac{\sum_{i=1}^{n-1}\left(Y'_{ij} - \hat{A}'_0 - \hat{A}'_1X'_i - \hat{A}'_2X'^2_i - ... - \hat{A}'_kX'^k_i\right)}{n-k-1} \tag{5}$$

The coefficients of polynomial profiles are in-control when $F_j^* < F_{\left(1-\alpha; df_{R_j} - df_{F_j}; df_{F_j}\right)}$. In other words, when $F_j^* < F_{\left(1-\alpha; df_{R_j} - df_{F_j}; df_{F_j}\right)}$, all of the polynomial coefficients simultaneously are in-control.

Note that by the aforementioned approach only the mean of process is monitored. In order to detect a shift in the error variance, an R chart may be used simultaneously. The *R* control chart statistic denoted by R_j is calculated by $R_j = max(e_{ij}) - min(e_{ij})$, where the residuals of the transformed model (e_{ij}) is obtained as

$$e_{ij} = y'_{ij} - (A'_0 + A'_1 X'_i + A'_2 X'^2_i + \ldots + A'_k X'^k_i) \;\; ; \;\; i = 1, 2, \ldots, n \tag{6}$$

The lower and upper control limits for R control chart is

$$LCL = \sigma\left(d_2 - Ld_3\right) \quad \text{and} \quad UCL = \sigma\left(d_2 + Ld_3\right) \tag{7}$$

respectively, where $L(>0)$ is a constant chosen to give a specified in-control ARL. The values of d_2 and d_3 are constants that depend on the sample size *n*.

IV. Traditional Multivariate T² Method

In this Section, the proposed transformation technique is applied to the well-known multivariate T^2 method.

This method is a modified version of the T^2 control chart proposed by Kang and Albin [2]. To reduce the effect of autocorrelation that exists between error terms within profiles, all the parameters, $(A_0, A_1, A_2, \ldots, A_k)$, of the original model are replaced by their transformed ones. This method is used when the number of parameters (k) is not very large. The modified T^2 statistic is obtained by Eq. (8) as follows:

$$T_j^2 = \left[\left(\hat{A}'_{oj}, \hat{A}'_{1j}, \hat{A}'_{2j}, \ldots, \hat{A}'_{kj}\right) - \left(A'_{oj}, A'_{1j}, A'_{2j}, \ldots, A'_{kj}\right)\right]^T \Sigma^{-1} \left[\left(\hat{A}'_{oj}, \hat{A}'_{1j}, \hat{A}'_{2j}, \ldots, \hat{A}'_{kj}\right) - \left(A'_{oj}, A'_{1j}, A'_{2j}, \ldots, A'_{kj}\right)\right] \tag{8}$$

where

$$\Sigma = [\,\sigma^2 (X^T X)^{-1}] \tag{9}$$

TABLE I
The effect of autocorrelation coefficient on in-control ARL performance of GLT/R control chart under different shifts in intercept, second parameter, third parameter and error standard deviation without utilizing the proposed transformation method

λ (Shift in the intercept)												
Φ Autocorrelation coefficients		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	200.1	197.6	185.2	159.2	124.7	95.6	69.2	48.5	34.7	24.3	18.0
	0.1	181.3	178.8	150.6	123.9	95.9	72.5	51.2	36.5	26.3	19.1	14.1
	0.3	81.3	75.4	66.7	53.1	41.3	32.9	23.8	18.3	13.7	10.7	8.6
	0.5	24.5	23.9	21.9	18.6	15.7	13.3	11.1	9.1	7.4	6.2	5.1
	0.7	7.2	7.1	6.7	6.4	5.8	5.5	4.9	4.5	4.1	3.6	3.3
	0.9	2.6	2.5	2.6	2.5	2.5	2.3	2.3	2.2	2.1	2.1	2.0
β (Shift in the second parameter)												
Φ Autocorrelation coefficients		0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
	0	200	195.4	191.8	190.9	170.2	149.3	130.4	107.6	89.3	71.5	57.9
	0.1	182.3	178.2	171.2	161.1	140.4	121.1	101.2	83.7	67.0	53.8	43.6
	0.3	79.8	80.8	74.2	68.2	60.1	52.4	45.1	38.6	31.9	26.9	22.1
	0.5	24.6	24.3	23.6	22.4	20.6	18.6	16.8	14.9	13.1	11.7	10.2
	0.7	7.1	7.0	7.0	6.7	6.5	6.3	5.8	5.5	5.2	4.9	4.5
	0.9	2.6	2.6	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1
δ (shift in the third parameter)												
Φ Autocorrelation coefficients		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
	0	200.5	199.2	188.4	166.1	135.7	108.3	79.9	58.1	41.5	30.5	21.9
	0.1	185.1	177.2	156.8	136.3	111.1	83.2	60.9	44.1	32.2	23.4	17.3
	0.3	81.5	78.6	69.1	58.7	48.1	37.4	29.3	22.7	17.1	13.2	10.5
	0.5	24.6	24.4	22.5	20.1	17.3	14.8	12.7	10.5	8.7	7.2	5.9
	0.7	7.2	7.1	6.9	6.5	6.2	5.6	5.2	4.6	4.2	3.7	3.3
	0.9	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.1	2.1	1.9
γ (shift in the standard deviation)												
Φ Autocorrelation coefficients		1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
	0	200.7	95.3	40.0	19.5	10.7	4.7	3.5	2.8	2.3	2.3	2.1
	0.1	186.7	108.1	50.2	24.3	13.2	8.1	5.6	4.1	3.2	2.5	2.1
	0.3	80.1	71.4	47.8	28.2	16.6	10.4	6.9	5.1	3.8	3.1	2.5
	0.5	24.0	23.7	22.2	18.1	13.8	9.8	7.4	5.4	4.3	3.4	2.8
	0.7	7.1	7.1	6.9	6.7	6.2	5.5	4.8	4.1	3.5	3.1	2.6
	0.9	2.7	2.6	2.6	2.6	2.5	2.5	2.3	2.3	2.1	2.1	1.9

V. Simulation Experiments

In this section, we first evaluate the performance of the *GLT/R* control chart for monitoring polynomial profiles when within-profile autocorrelation presents and the proposed transformation method is not utilized. The following example is used to study the performance:

$$\begin{aligned} y_{ij} &= 3 + 2x_i + x_i^2 + \varepsilon_{ij} \\ \varepsilon_{ij} &= \Phi \varepsilon_{(i-1)j} + a_{ij} \end{aligned} \tag{10}$$

Where a_{ij} follows a normal distribution with mean zero and variance one and x-values are $1, 2, 3, 4, 5, 6, 7, 8, 9, \text{and} 10$. In the simulation experiments, the effect of different autocorrelation coefficients Φ on the performance of *GLT/R* control chart

TABLE II

Out-of-control ARL comparisons under shifts from A_0 to $A_0 + \lambda\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	λ										
Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
T^2	200	173.7	122.3	76.2	44.6	26.9	16.6	10.4	7.8	4.7	3.5
GLT/R	199.4	162.3	102.4	56.8	29.7	16.2	9.5	6.1	3.8	2.8	2.2
$\Phi = 0.9$	λ										
Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
T^2	200.1	198.4	196.7	196.2	195.2	193.0	187.3	183.0	180.6	172.2	169.8
GLT/R	200.5	192.2	199.2	200.5	191.4	191.7	186.8	173.7	163.6	160.8	157.6

TABLE III

Out-of-control ARL comparisons under shifts from A_2 to $A_2 + \delta\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	δ										
Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
T^2	199.2	174.6	127.5	80.1	49.7	29.4	18.5	11.9	7.7	5.4	3.9
GLT/R	198.9	28.1	3.7	1.3	1.1	1.0	1.0	1.0	1.0	1.0	1.0
$\Phi = 0.9$	δ										
Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
T^2	200.4	191.6	167.8	138.4	109.7	84.2	63.6	47.6	35.7	27.4	20.3
GLT/R	197.4	155.8	71.9	33.5	16.5	8.8	4.9	3.1	2.1	1.6	1.3

TABLE IV

Out-of-control ARL comparisons under standard deviation shifts from σ to $\gamma\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	γ										
Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
T^2	197.4	72.1	32.4	17.9	11.5	8.0	5.9	4.6	3.7	3.2	2.7
GLT/R	196.6	175.2	125.2	76.1	41.1	23.7	14.2	9.3	6.7	4.9	3.9
$\Phi = 0.9$	γ										
Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
T^2	200.6	70.4	32.8	18.1	11.3	7.9	5.8	4.6	3.7	3.2	2.7
GLT/R	198.3	174.2	125.5	75.3	41.2	23.6	14.4	9.4	6.7	5.1	3.9

under different shifts in the intercept λ, the second parameter β, the third parameter δ, and error standard deviation γ using in-control average run length criterion is studied in 10,000 simulation runs. The results are summarized in Table I. In this table, λ, β, δ and γ are measured in multiples of σ and the in-control average run length is considered 200. As shown in Table I, when the transformation technique is not used, the in-control *ARL*s of *GLT/R* control chart decrease in the presence of autocorrelation within profiles, leading to its poor performance. Moreover, this effect is more considerable when the autocorrelation coefficient gets bigger. When the proposed transformation method is used, the performances of T^2 and *GLT/R* are compared employing the same example introduced earlier in Eq. (10). Two autocorrelation coefficients $\Phi = 0.1$ (weak autocorrelation) and $\Phi = 0.9$ (strong autocorrelation) are considered where all control-charting methods are designed to have an overall in-control ARL of 200. To achieve this, in the *GLT/R* control chart, we set the value of L equal to 3.8 for both $\Phi = 0.1$ and

$\Phi = 0.9$ autocorrelation coefficients. For T^2 chart *UCL* is set 12.84. Finally, For *GLT/R* chart *UCL* is set 12.916. We used 10,000 simulation runs to study out-of-control *ARL* under different shifts in the intercept, the second parameter, the third parameter, and the error standard deviation. The results are summarized in Tables II through IV. The results in Table II show that under the intercept shift from A_0 to $A_0 + \lambda\sigma$ in both weak and strong autocorrelations ($\Phi = 0.1$ and $\Phi = 0.9$), the *GLT/R* chart uniformly performs better than the other one. Further, it can be seen that the out-of-control *ARL*s for the strong autocorrelation case are larger than the ones in the weak autocorrelation situation. Similar results are obtained under shifts in the second parameters. So we avoid reporting the results in the paper. Based on the results of Table III under shifts in the third parameter δ, in both weak and strong autocorrelation coefficients, *GLT/R* method outperforms the T^2 control chart. Finally, the results in Table 4 show that under the standard deviation shift from σ to $\gamma\sigma$ in both weak and strong autocorrelation situations, the T^2 control chart performs uniformly better than the GLT/R method.

VI. Conclusions

In this paper, the effect of within-profile autocorrelation on the performance of a *GLT/R* chart designed to monitor polynomial profiles under independency of the error terms was first investigated. The results showed that autocorrelation leads to poor performance of the *GLT/R* control chart. Then, the transformation technique of Soleimani et al. (2009) that was originally proposed for simple linear profile was extended and employed for the polynomial profile. Finally, the performances of *GLT/R* and T^2 control charts are compared in terms of average run lengths criterion using 10,000 simulation runs. The results showed that the *GLT/R* scheme performs better than the T^2 charts under the different shifts in the regression parameters. However, the T^2 method was better than the *GLT/R* method under the shifts in the standard deviation.

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