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The Maximal Backup Covering Tour Problem

Ebrahimi A.D.[1], **Sahraeian R.**[2]

Abstract: In this paper we propose the Maximal Backup Covering Tour Problem (MBCTP) that is a generalization of the Covering Tour Problem (CTP). This problem is defined on an undirected graph $G=(V\bigcup W,E)$, where W is a set of vertices that must be covered. The MBCTP aims to determine a minimum length Hamilton cycle on a subset of V such that every vertex of W is within a pre-specified distance from at least one node in the cycle and also maximizing number of vertices of W set which are covered for second or more times. Mathematical formulation of MBCTP, that is a multi-objective problem, is proposed. We used of Multi-objective Decision Making (MODM) methods for optimizing it. Finally numerical examples are provided to demonstrate the validity of the model.

Keywords: The Maximal Backup Covering Tour Problem, Multi-objective Decision Making, Global Criterion method, Goal-attainment method.

1.1 Introduction

This paper represents an extension of the CTP (that is a combination of Traveling Salesman Problem and Set Covering Problem), namely Maximal Backup Covering Tour Problem. The MBCTP is a multi-objective problem and aims to determine a minimal length tour for a subset of nodes while also maximizing number of vertices of set of W that have been covered for second time, i.e. maximizing backup coverage. The MBCTP can be formally described as follows: let $G=(V\bigcup W,E)$ be an undirected graph, where $V\bigcup W$ is the vertex set. $E=\{(v_i,v_j)|v_i,v_j\in V\bigcup W,i<j\}$ is the edge set. Vertex v_1 is a depot, V is the set of vertices that can be visited, $|V|=n$, $T\subseteq V$ is the set of vertices that must be visited ($v_1\in T$), and W is the set of vertices that must be covered. A distance

[1]Ali Ebrahimi Dorcheh
Department of Industrial Engineering, Shahed University, Tehran, Iran
e-mail: aliebrahimi@shahed.ac.ir
[2] Rashed Sahraeian(✉)
Department of Industrial Engineering, Shahed University, Tehran, Iran
e-mail: sahraeian@shahed.ac.ir

matrix $C=(c_{ij})$, satisfying triangle inequality, is defined for E. The MBCTP consists of defining a tour for a subset of V, which contains all the vertices from T, while at the same time optimizing the following two objectives: (i) the minimization of the tour length and (ii) the maximization of the backup coverage. A feasible solution for a small instance is provided in Fig. 1.1.

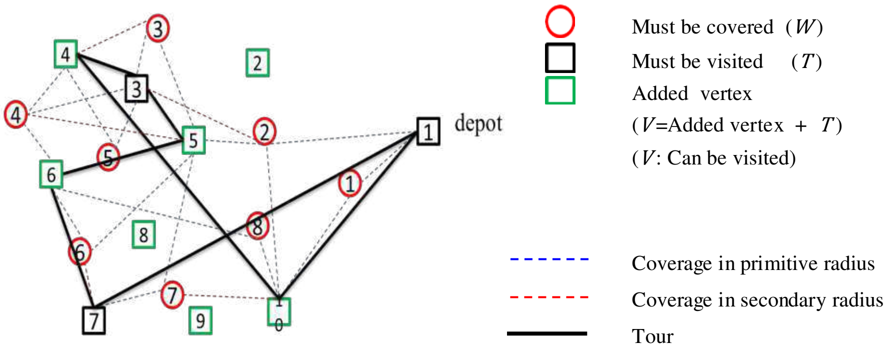

Fig. 1.1. An example of a solution for the MBCTP

An example of the MBCTP is the design of route for mobile health care delivery team in developing countries where services are rendered at a selected number of locations by medical team, and the population living outside these locations must travel on foot to reach them (see, e.g. Foord, 1995 and Hodgson et.al, 1998). Some when some of nature incidents such as flood and earthquake cause that people reaching to selected locations be impossible, also it may be required in high-demand areas to maintain a uniform level of service when vehicles can respond to only one call at a time (Hogan and Revelle, 1986), so it is necessary that backup coverage in selecting of locations for people welfare (set of W) is considered.
As mentioned above the MBCTP is a generalization of the CTP and though the CTP has not received much attention in literature (Gendreau et.al, 1997), it seems that this problem was first introduced by Current, 1981. It is formulated in Current and Schilling, 1989. In this reference and in Current and Schilling, 1994, a two-objective version of the problem is considered: the length of the tour and the number of vertices covered by the tour. Finally this problem proposed by Gendreau and et al. and an exact branch-and-cut algorithm developed for it. To the best our knowledge, backup coverage has not considered for the CTP in the literature. The proposed model inherits its formulation from the previous researches in maximal covering models. The model is formed by using Hogan and ReVelle's maximal backup coverage model (Hogan and Revelle, 1986). They applied backup coverage criterion to Set Covering Location Problem (SCLP) and Maximal Covering Location Problem (MCLP) models.

First coverage was traded off against backup coverage in their models. It was shown that backup coverage can be provided without substantial first coverage loss, (Araz et.al, 2007).
In this paper two powerful MODM methods is provided for solving and demonstrating validity of the model, Global Criteria (GC) and Goal-attainment (GA) methods which are describe in the next sections. In this paper three small instances which are generated randomly is considered to demonstrate validity of the model.

1.2 Model Definition

The MBCTP can be formulated as a multi-objective integer linear program as follows: for $v_k \in V$, let y_k be a binary variable equal to 1 if and only if vertex v_k belongs to the tour. If $v_k \in T$, then y_k is necessarily equal to 1. For $v_i, v_j \in V$ and $i < j$, let x_{ij} be a binary variable equal to 1 if and only if edge (v_i, v_j) belongs to the tour. Define binary coefficients δ_{lk} equal to 1 if and only if $v_l \in W$ can be covered by $v_k \in V$ in primitive radius (i.e. $c_{lk} \le c$,where *c* is primitive coverage radius), and let $S_l = \{v_k \in V | \delta_{lk} = 1\}$ for every $v_l \in W$. Hence assume that $|S_l| \ge 2$ for all $v_l \in W$ and that the degenerate tour (v_1) is infeasible. Also let binary coefficients δ'_{lk} equal to 1 if and only if $v_l \in W$ can be covered by $v_k \in V$ in secondary radius (i.e. $c_{lk} \le d$,where *d* is secondary coverage radius and $d > c$), and let $S'_l = \{v_k \in V | \delta'_{lk} = 1\}$ for every $v_l \in W$. U_l is a binary variable that is associated with backup coverage and equal 1 if and only if $v_l \in W$ in primitive or secondary radius covered by a vertex of $v_k \in V$ for second or more times and there are a benefit coefficients for every $v_l \in W$ as a_l that indicates importance of vertices of *W* set (or demand points). Then the MBCTP can be stated as:

$$Min \quad \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} c_{ij} x_{ij} \tag{1}$$

$$Max \quad \sum_{l \in W} a_l U_l \tag{2}$$

St:

$$\sum_{v_k \in S_l} y_k \ge 1 \qquad \forall l \in W \tag{3}$$

$$\sum_{v_k \in S_l} y_k - U_l \geq 1 \qquad \forall l \in W \tag{4}$$

$$\sum_{i=1}^{k} x_{ik} + \sum_{j=k+1}^{n} x_{kj} = 2y_k \qquad \forall v_k \in V \tag{5}$$

$$\sum_{\substack{v_i \in S, v_j \in V \setminus S \\ or \\ v_j \in S, v_i \in V \setminus S}} x_{ij} \geq 2y_k \qquad \begin{array}{l} \forall S \subset V, 2 \leq |S| \leq n-2 \\ T \setminus S \neq \phi, v_k \in S \end{array} \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad i, j \in V, i < j \tag{7}$$

$$y_k \in \{0,1\} \qquad \forall v_k \in V \setminus T \tag{8}$$

$$y_k = 1 \qquad \forall v_k \in T \tag{9}$$

$$U_l \in \{0,1\} \qquad \forall l \in W \tag{10}$$

In this formulation, objective (1) minimize the tour length and objective (2) provide maximum backup coverage, constraints (3) ensure that every vertex of W is covered by the tour, while constraints (4) are associated with backup coverage. Constraints (5) are degree constraints and constraints (6) are connectivity constraints and they force the presence of at least two edges between any set S and $V \setminus S$, for every proper subset S of V such that $T \setminus S \neq \phi$ and S contains a vertex v_k belonging to the tour. Finally constraints (7), (8), (9) and (10) set the integrality requirements. Note that in this formulation, we do not allow the case where $x_{1k} = 2$ for some k since this can only happen if the tour (v_1, v_k, v_1) is feasible.

1.3 Multi-objective Optimization

Multi-objective programming is a mathematical programming method that can investigate multiple definite objectives. It aims to help the decision-makers obtain a better policy under the constraints on finite resources and conflicting objectives, (Low, 2005). Solving results of MODM problems called Pareto optimal solution (i.e. non-dominated solution). A solution is said to be Pareto optimal if it is not dominated by any other solution in the solution space and the set of all feasible non-dominated solutions is referred to as the Pareto optimal set. The ultimate goal of a multi-objective optimization algorithm is to identify solutions in the Pareto optimal set and determine the best optimal solution from this set by decision-maker, (Deb, 2001). In this paper we seek only Pareto optimal solution and neither Pareto optimal set since we want to demonstrate validity of the proposed model, i.e. there are no any decision-maker for selection an optimal solution from between Pareto optimal set.

1.3.1 The Global Criterion Method

The GC method tries to minimize a distance from the optimal solution. The optimal solution is computed by solving all the problems with only one objective function. The optimal value for the function f_i is obtained by solving the problem with only f_i as objective function. In this way an optimal vector $f^* = (f_1^*, f_2^*, \ldots f_k^*)$ is obtained. Now a new problem is solved whose objective function is:

$$f(x) = \left(\sum_{i=1}^{k} w_i \left[\frac{f_i(X^*) - f_i(X)}{|f_i(X^*)|} \right]^P \right)^{\frac{1}{P}} , (1 \le P \le \infty) \tag{11}$$

Where parameter of P defines the type of distance and when reducing distance between an objective function and its optimal value is very important this parameter is considered very big and inverse, also vector of $w = (w_1, w_2, \ldots w_k)$ shows objectives importance. Solving result of such a problem is a non-dominance solution and with changing P and vector of w other optimal solutions will obtain that form Pareto optimal set, (Salukvadze, 1974).

1.3.2 The Goal-attainment Method

The GA method is a highly effective means of obtaining the best compromise solution to multi-objective problems, and it is not subject to convexity limitations of any kind. A vector of desired goals $g = [g_1, g_2, \ldots g_n]^T$, which is associated with a set of objectives $f(x) = \{f_1(x), f_2(x), \ldots f_n(x)\}$, is specified in the GA, (Chankong, 1983). The relative degree of under or over attainment of the desired goals is controlled by a vector of weights $w = [w_1, w_2, \ldots w_n]^T$. The best compromise solution is found by solving the following problem:

$$\mathit{Min} \quad \phi \tag{12}$$

St:

$$g + \phi w \ge f(x) \qquad x \in X, w \in \Omega \tag{13}$$

Where ϕ is a scalar variable unrestricted in sign, x denotes a set of desired parameters, X represents a feasible-solution region, and $\Omega = \{w \in R^n \ st\ w_i \ge \varepsilon, \sum_{i=1}^{n} |w_i| = 1 \ and\ \varepsilon \ge 0\}$. The term ϕw introduces a degree of slackness into the problem, which would otherwise impose that the goals be rigidly met. The weight vector w enables the decision maker (DM) to quantitatively express the tradeoffs among the objectives. The smaller w_i the ith objective prefers a smaller function value.

1.4 Numerical Examples and Analysis

Since, unfortunately, unlike the TSP and the SCP, no test problems for the CTP exists, experiments were conducted on a series of randomly generated instances. Three instances of problem with 10, 20 and 30 cities (set V) with 8, 15 and 22 demand points (set W) respectively using both two method, GC method and GA method, are ran and Pareto optimal solution for each instance is computed. Objective weights in both methods are same. It means vector w for GC method is (0.5,0.5) and for GA method is (-0.5,0.5). Since we want to compare two method results we considered parameter of P equal to 1. Desired goal for GA method are obtained by solving all the problems with only one objective function and results are considered for g vector. Other information for first instance is depicted in Table 1.1, Table 1.2 and Table 1.3. Note that each of fixed values in two methods (mean P, w and g) are changeable and one can perform sensitivity analysis and form Pareto optimal set. In this work we used CPLEX solver of GAMS 21.7 software for optimizing of the two constructed MODM models.
Results for three instances that are Pareto optimal solutions are depicted in Table 1.4.

Table 1.1. Distance between of the cities for first instance (c_{ij})

i \ j	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	18	12	9	11	12	4	6	21	6
2		6	5	5	9	7	12	15	9
3			3	4	20	13	18	18	12
4				11	8	9	13	16	9
5					4	5	10	17	24
6						2	15	5	7
7							8	9	8
8								11	19
9									20

Table 1.2. Binary coefficient (δ_{lk})

l \ k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	1	0	1	0	0	0	0	0	1
2	1	0	0	0	1	0	0	0	0	1
3	0	1	1	0	1	0	0	0	1	0
4	0	0	1	1	0	1	0	0	0	0
5	0	0	1	1	0	0	0	1	0	0
6	0	0	0	0	1	1	0	1	1	0
7	0	0	0	0	1	0	1	0	1	0
8	0	0	0	0	0	1	0	0	1	1

Table 1.3. Binary coefficient (δ_{lk})

l \ k	1	2	3	4	5	6	7	8	9	10
1	1	1	0	1	0	0	0	0	1	1
2	1	0	0	0	1	0	0	0	1	1
3	0	1	1	1	1	0	0	0	1	0
4	0	0	1	1	0	1	0	0	0	0
5	0	1	1	1	0	0	0	1	0	0
6	0	0	0	0	1	1	1	1	1	0
7	0	0	0	0	1	0	1	0	1	0
8	0	0	0	0	0	1	0	0	1	1

The column headings of Table 1.4 are defined as follows:

GA: goal-attainment method
GC: global criterion method
NO: number of instance
GO: objective goals vector (for GA method)
OBJT.1: computed value for first objective (tour length)
OBJT.2: computed value for second objective (backup coverage)
C.NO: number of visited cities
B.NO: number of demand points with backup coverage
GAP: relative distance between solutions of two methods

Table 1.4. GC and GA method results for three generated instances

NO	GO	OBJT.1		OBJT.2		C.NO		B.NO		GO	
		GC	GA	GC	GA	GC	GA	GC	GA	OBJT.1	OBJT.2
1	(28,30)	30	30	30	30	6	6	7	7	0%	0%
2	(55,77)	55	55	77	77	14	14	13	13	0%	0%
3	(82,90)	82	87	98	90	22	20	17	15	6%	8%

Table 1.4 show that both two methods generated almost same results and only in third instance a little margin is observable that it's because of deference in their approaches. For example we explain obtained results for third instance. In this case, tour length in GC method is 82, that is 5 units less than GA method and so its gap value is 6 percent and this gap for backup coverage objective is only 8 percent. Note that though number of visited cities in GC method is more than GA method, but length tour in this method is less that perhaps it is because of data structure of third instance, meanwhile in GC method number of visited cities is more 2 unite than GA method and this has led to more coverage supports. Results of two methods for two other instances are completely same and differences in third instance also are ignorable, so we conclude that proposed model (MBCTP) is a reliable model for our purpose. Note that you can see trade-off between two

objectives with changing *w* vector in both two methods and in each of instances but we don't want to tent to this problem in this work.

1.5 Conclusion and Future Research

In this work we have studied a generalization of the covering tour problem, namely maximal backup covering tour problem (MBCTP). The MBCTP is a multi-objective problem and aims to optimizing two objectives at a same time: (i) the minimization of the tour length and (ii) the maximization of the backup coverage. We used two powerful MODM methods for solve this problem. Finally three numerical examples illustrated the proposed model. Results showed that output of both two methods is almost same. Development of our model in capacitated form, considering the problem in a fuzzy or stochastic space and considering location-allocation problem for this problem can be discussed as future researches.

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