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Ferroresonance in Voltage Transformer Considering Linear and Nonlinear Core Losses Effect

Hamid Radmanesh, Mehrdad Rostami, and Jafar Khalilpour

Abstract—**Ferroresonance is a complex electrical phenomenon, which can cause overvoltages in the electrical power system and endangers the system reliability and continuous safe operating. This paper studies the effect of linear and nonlinear core losses on the chaotic ferroresonance in a voltage transformer. For confirmation this aspect simulation has been done on a one phase voltage transformer rated 100VA, 275kV. The magnetization characteristic of the transformer is modeled by a single-value two-term polynomial with q=7. The simulation results reveal that considering the nonlinear core losses on the voltage transformer, exhibits a great effect on ferroresonance overvoltages. Significant effect on the onset of chaos, the range of parameter values that may lead to chaos along with ferroresonance voltages has been obtained and presented.**

Index Terms—**Nonlinear core losses effect, chaos, bifurcation, ferroresonance, voltage transformers, linear core loss**

I. INTRODUCTION

Ferroresonance overvoltage on electrical power systems were recognized and studied as early as 1930s. Kieny first suggested applying chaos to the study of ferroresonance in electric power circuits [1]. He studied the possibility of ferroresonance in power system, particularly in the presence of long capacitive lines as highlighted by occurrences in France in 1982, and produced a bifurcation diagram indicating stable and unstable areas of operation. Then the combination of nonlinear iron core inductor with series capacitor has been investigated and shown that this core is the most possible case for occurring ferroresonance in the power system. These capacitances can be due to number of elements, such as the line-to-line capacitance, parallel lines, conductor to earth capacitance and circuit breaker grading capacitance. A special ferroresonance phenomenon on 3-phase66 kV VT-generation of 20Hz zero sequence continuous voltage is given in [2]. Typical cases of ferroresonance are reported in [3], [4], in these papers power transformer and VTs has been investigated due to ferroresonance overvoltages. Digital simulation of transient in power system has been done in [5]. Application of nonlinear dynamics and chaos to ferroresonance in the distribution systems can be found in [6]. The susceptibility of a ferroresonance circuit to a quasi periodic and frequency locked oscillations has been presented in [7], in this case, investigation of ferroresonance has been done upon the new branch of chaos theory that is quasi periodic oscillation in the power system and finally ferroresonance appears by this route. Modelling iron core nonlinearities has been illustrated in [8]. Mozaffari has been investigated the ferroresonance in power transformer and effect of initial condition on this phenomena. He analyzed condition of occurring chaos in the transformer and suggested the reduced equivalent circuit for power system including power switch and trans [9],[10]. The mitigating effect of transformer connected in parallel to a MOV arrester has been illustrated in [11]. Analysis of ferroresonance in voltage transformer has been investigated by Zahawi in [12] and [13]. Analysis of ferroresonance phenomena in the power transformers including neutral resistance effect has been reported in [14]. Ferroresonance conditions associated with a 13 kV voltage regulator during back-feed conditions are given in [15]. Performance of various magnetic core models in comparison with the laboratory test results of a ferroresonance test on a 33 kV voltage transformer investigated in [16]. Mitigating ferroresonance in voltage transformers in ungrounded MV networks has been reported in [17]. An approach for determining the subsystem experiencing and producing a bifurcation in a power system dynamic model has been reported in [18]. Effect of circuit breaker shunt resistance on chaotic ferroresonance in voltage transformer has been done in [19]. In all previous studies, possibility of occurring ferroresonance and nonlinear phenomena in power system had been studied, also the effects of nonlinear core losses on VT ferroresonance in the deeper case has not been investigated. Current paper studies the effect of linear and nonlinear core losses on the occurring of ferroresonance overvoltages in VT. It is shown that by considering these nonlinear core losses causes to change the behaviour of system and its behaviour completely goes to chaotic regions.

II. SYSTEM DESCRIPTION CONSIDERING LINEAR CORE LOSSES

During voltage transformer (VT) ferroresonance, an oscillation occurs between the nonlinear iron core inductance of the VT and existing capacitances of the network. In this case, energy is coupled to the nonlinear core of the voltage transformer via the open circuit breaker grading capacitance or system capacitance to sustain the resonance. The result may be saturation in the VT core and very high voltage up to

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4pu can theoretically gained in worst case conditions. The magnetizing characteristic of a typical 100VA VTs can be presented by 7 order polynomial [19]. These VTs fed through circuit breaker grading capacitance, and studied using nonlinear dynamics analysis and packages such as Rung kutta Fehlberg algorithm and MATLAB SIMULINK. Fig. 1 shows the single line diagram of the most commonly encountered system arrangement that can give rise to VT ferroresonance [19]. Ferroresonance can occur upon opening of disconnector *3* with circuit breaker open and either disconnector *1* or *2* closed. Alternatively it can also occur upon closure of both disconnector *1* or *2* with circuit breaker and disconnector *3* open.

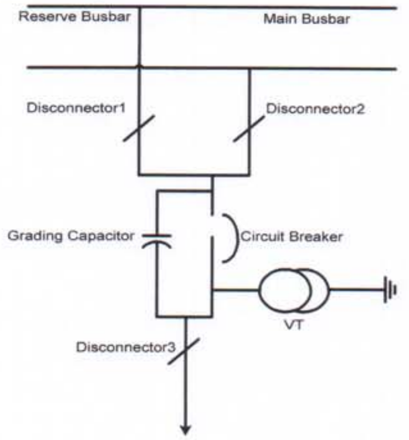

Fig. 1. System one line diagram arrangement resulting to VT ferroresonance

The system arrangement shown in Fig. 1 can effectively be reduced to an equivalent circuit as shown in Fig. 2.

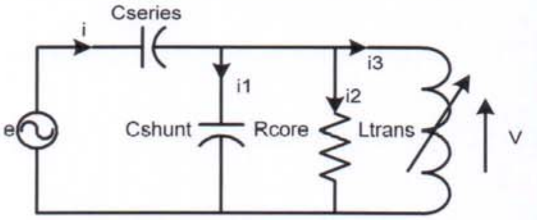

Fig. 2. Basic reduced equivalent ferroresonance circuit [13]

In Fig. 2, E is the RMS supply phase voltage, C_{series} is the circuit breaker grading capacitance and C_{shunt} is the total phase-to-earth capacitance of the arrangement. The resistor R represents a voltage transformer core loss that has been found to be an important factor in the initiation of ferroresonance. In the peak current range for steady-state operation, the flux-current linkage can be approximated by a linear characteristic such as $i_L = a\lambda$ where the coefficient of the linear term *(a)* corresponds closely to the reciprocal of the inductance $(a \equiv 1/L)$. However, for very high currents the iron core might be driven into saturation and the flux-current characteristic becomes highly nonlinear, here the $\lambda - i$ characteristic of the voltage transformer is modeled as in [8] by the polynomial

$$i = a\lambda + b\lambda^7 \tag{1}$$

where, $a = 3.14$, $b = 0.41$

The polynomial of the order seven and the coefficient b of equation (1) are chosen for the best fit of the saturation region that was obtained by the comparison between different approximations of the saturation regions against the true magnetization characteristic that was obtain by dick and Watson[5]. It was found that for adequate representation of the saturation characteristics of a voltage transformer core, the exponent q may acquire value 7 [19]. Fig. 3 shows simulation of these iron core characteristic for q=5, 7, 11. The basic voltage transformer ferroresonance circuit of Fig. 2 can be presented by a differential equation. Because of the nonlinear nature of the transformer magnetizing characteristics, the behavior of the system is extremely sensitive to the change in system parameter and initial conditions. A small change in the value of system voltage, capacitance or losses may lead to dramatic change in the behavior of it. A more suitable mathematical language for studying ferroresonance and other nonlinear systems is provided by nonlinear dynamic methods. Mathematical tools that are used in this analysis are phase plan diagram, time domain simulation and bifurcation diagram.

III. SYSTEM DYNAMIC AND EQUATION

In Mathematical analysis of equivalent circuit by applying KVL and KCL has been done and equations of system can be presented as below:

$$\lambda_{peak} = \sqrt{2}\,\frac{v_{RMS}}{\omega} \tag{2}$$

$$v_L = \frac{d\lambda}{dt} \tag{3}$$

$$i = C_{ser}\frac{d(e - v_L)}{dt} = C_{ser}\left(\dot{e} - \frac{d^2\lambda}{dt^2}\right) \tag{4}$$

$$i = i_1 + i_2 + i_3 \Rightarrow \frac{C_{sereis}}{(C_{ser} + C_{sh})}\left(\sqrt{2}E\cos\omega t\right) = \frac{1}{\omega}\frac{d^2\lambda}{dt^2} + \frac{1}{R\omega(C_{ser} + C_{sh})}\frac{d\lambda}{dt} + \frac{1}{\omega(C_{ser} + C_{sh})}\left(a\lambda + b\lambda^7\right) \tag{5}$$

where, ω is supply frequency, and E is the rms supply phase voltage, C_{series} is the circuit breaker grading capacitance and C_{shunt} is the total phase-to-earth capacitance of the arrangement and in equation (1) $a=3.4$ and $b=0.41$ are the seven order polynomial sufficient [19].

IV. METAL SYSTEM DESCRIPTION CONSIDERING NONLINEAR CORE LOSSES

In this case, system under study is similar with the case above, but the model of core losses has been changed. Equivalent thevenin circuit of this case has been illustrated in Fig. 4.

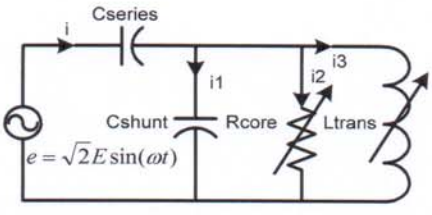

Fig. 4. Basic reduced equivalent ferroresonance circuit

V. System Modelling With MOV

Connecting MOV to the system in Fig. 2, circuit can be driven in Fig. 5.

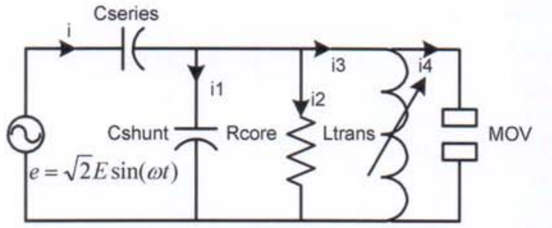

Fig. 5. Basic reduced equivalent ferroresonance circuit connecting MOV

In this paper, the core loss model adopted is described by a third order power series whose coefficients are fitted to match the hysteresis and eddy current nonlinear characteristics given in [6]:

Nonlinear equation of this circuit is as below:

$$\frac{C_{series}}{(C_{ser}+C_{sh})}\left(\sqrt{2}E\cos\omega t\right)=\frac{1}{\omega}\frac{d^2\lambda}{dt^2}+\frac{1}{\omega(C_{ser}+C_{sh})}\times$$
$$(\mathrm{h}_0+\mathrm{h}_1\frac{d\lambda}{dt}+\mathrm{h}_2(\frac{d\lambda}{dt})^2+\mathrm{h}_3(\frac{d\lambda}{dt})^3+a\lambda+b\lambda^7) \tag{6}$$

In this model of circuit, per unit value of i_2 is given as below [6]:

$$i_2=-.000001+.0047v_L-.0073{v_L}^2+.0039{v_L}^3$$

Other parameters of system are similar with the case 1.

VI. Simulation Results

Equation (15) contains a nonlinear term and does not have simple analytical solution. So the equations were solved numerically using an embedded Runge-Kutta-Fehlberg algorithm with adaptive step size control. Values of E and ω were fixed at 1pu, corresponding to AC supply voltage and frequency. In this analysis C_{series} is fixed at 0.5nF and C_{shunt} vary between 0.1nF and 3nF.solutions are obtained for initial values of $v_L(t)=\sqrt{2},\ \lambda(t)=0$ at t=0, representing circuit breaker operation at maximum voltage. In this state, system for both cases, with linear core losses and with nonlinear core losses has been simulated for E=3pu. it shows that the system under study has chaotic behaviour for E=3pu while in the case of considering nonlinear core losses, system behaviour remain more chaotic for E=3pu. The simulation result for E=3pu in the case of considering nonlinear core losses has shown the effect of nonlinear core losses on system behaviour which is presented in Figs. 5 and 6.

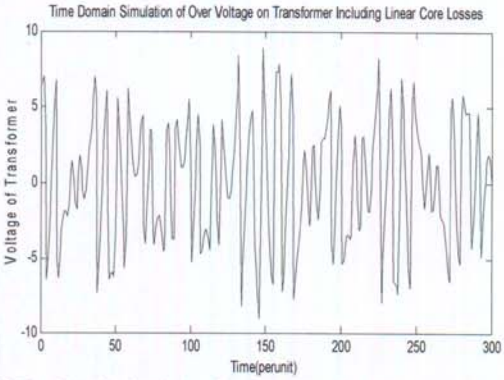

Fig. 5. Time domain simulation for chaotic motion considering linear core losses, $E=3pu$

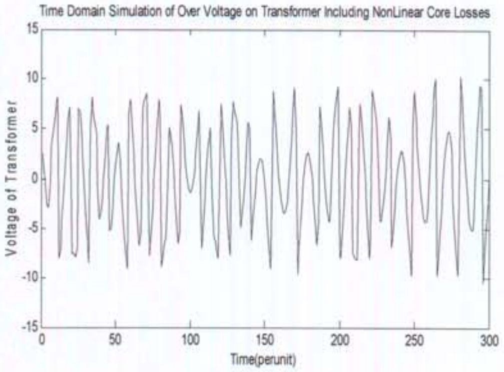

Fig. 6. Time domain simulation for the chaotic motion considering nonlinear core losses, $E=3pu$

In Fig. 5 system behavior has been simulated without considering nonlinear core losses. Time domain simulation is completely chaotic and ferroresonance overvoltages reach to 10pu. In the equal condition, by considering nonlinear core losses, this overvoltage has been changed and behavior of system goes to 10pu. Corresponding phase plan diagrams has been shown the clearance effect of considering nonlinear core losses to the system and it is shown in Figs. 7 and 8 for E=3pu. In the next state, by considering the nonlinear core losses effect, it is shown that the voltage of transformer reach to 11pu that has been shown in Fig. 8. Due to the abnormal condition such as switching action or other cases that may cause transient phenomena, When input voltage of power system goes up to 3pu, in the case of nonlinear core losses effect, ferroresonance overvoltage on voltage transformer reaches to 9pu, this state has been shown by phase plan diagram in Fig. 7.

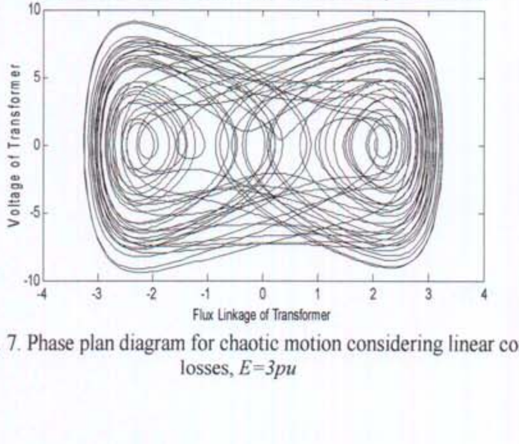

Fig. 7. Phase plan diagram for chaotic motion considering linear core losses, $E=3pu$

By applying nonlinear core losses to the system, ferroresonance overvoltages reach to 10pu which is presented in Fig. 8.

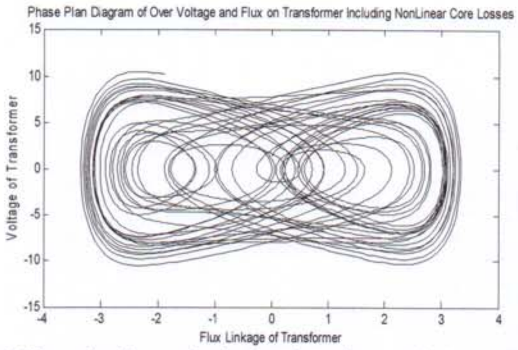

Fig. 8. Phase plan diagram for the chaotic motion considering nonlinear core losses, $E=3pu$

System parameters which are considered for simulation are listed in Table I.

TABLE I: PARAMETER VALUE FOR SIMULATION

Parameter	Actual value	Per unit value
E	275kv	1 pu
ω	377 rad/sec	1 pu
C_{series}	0.5 nf	39.959 pu
C_{shunt}	1.25nf	99 pu

Another tool that was used for solving the nonlinear equation of studied system is bifurcation diagram. In this paper, it is shown the effect of variation in the voltage of system on the ferroresonance overvoltage in the VT, and finally the effect of applying nonlinear core losses on this overvoltage by bifurcation diagrams.

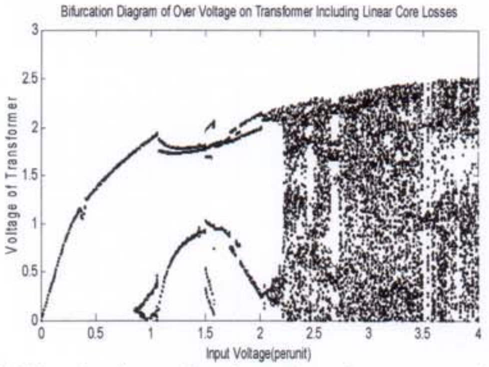

Fig. 9. Bifurcation diagram for voltage of transformer versus voltage of system, with linear core losses

System parameters are listed in Table II.

Fig. 9 clearly shows the ferroresonance overvoltages on VT when the voltage of system increases to 2.5pu. Parameters values of the system in this case are listed in Table II.

TABLE II: PARAMETER VALUE FOR SIMULATION

Parameter	Actual value	Per unit value
E	275kv	1 pu
ω	377 rad/sec	1 pu
C_{series}	0.5 nf	39.959 pu
C_{shunt}	0.1nf	7.929 pu

In Fig. 9 when E=0.25pu, voltage of VT has a period-1 behavior and system works is normal operation condition. In E=0.9pu, which is shown by point1, its behavior is still period1 and after this voltage, suddenly crisis takes place and system behavior goes to the chaotic region. After that, when the input voltage reach to 1.2pu, system comes out of chaotic region, again in E=2.7pu, bifurcation takes place. By this route system behavior goes to chaos. It is shown that system behavior has period doubling bifurcation logic and there are many resonances in the system behavior. Bifurcation diagram with the same parameter in the case of considering nonlinear core losses is shown in Fig. 10.

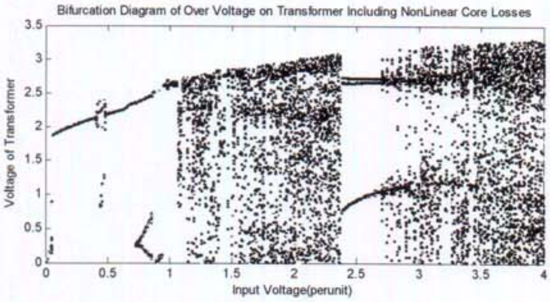

Fig. 10. Bifurcation diagram for voltage of transformer versus voltage of system considering nonlinear core losses

It is shown that by applying nonlinear core losses, system behaviors being more chaotic and overvoltage reaches to 3pu. In this case there is a jump in the voltage of transformer when the input voltage reaches up to 0.5pu. In the real systems, maximum overvoltage that VT can stand is 4pu. If overvoltage amplitude increases more, it can exactly cause VT failure.

VII. CONCLUSION

Low capacity VTs fed through circuit breaker grading capacitance have been shown to exhibit fundamental frequency and chaotic ferroresonance conditions similar to high capacity power transformers fed via capacitive coupling from neighbouring sources. Repeated simulation of the system's nonlinear differential equation has shown that a change in the value of the equivalent circuit capacitance to earth, possibly as a result of a change in system configuration, can give rise to different types of ferroresonance overvoltage. Nonlinear core losses can cause ferroresonance overvoltages. A comprehensive understanding of the possibilities that exist for ferroresonance is very desirable for engineers so that they can operate their systems outside dangerous regions and can plan the expansion of systems without enhancing the possibility of ferroresonance.

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REFERENCES

[1] C. Kieny, "Application of the bifurcation theory in studying and understanding the global behavior of a ferroresonant electric power

circuit," *IEEE Transactions on Power Delivery*, vol. 6, 1991, pp. 866-872.

[2] S. Nishiwaki, T. Nakamura, and Y .Miyazaki, "A Special Ferro-resonance Phenomena on 3-phase 66kV VT-generation of 20Hz zero sequence continuous voltage," *Presented at the International Conference on Power Systems Transients (IPST'07)*, 2007.

[3] E. J. Dolan, D. A. Gillies, and E. W. Kimbark, "Ferroresonance in a transformer switched with an EVH line," *IEEE Transactions on Power Apparatus and Systems*, 1972, pp. 1273-1280.

[4] R. P. Aggarwal, M. S. Saxena, B. S. Sharma, S. Kumer, and S. Krishan, "Failure of electromagnetic voltage transformer due to sustained overvoltage on switching*/an in-depth field investigation and analytical study," *IEEE Transactions on Power Apparatus and Systems*, vol. 5 , 1981, pp. 4448-4455.

[5] E. P. Dick and W. Watson, "Transformer models for transient studies based on field measurements," *IEEE Trans*, 1981, PAS-100, pp. 409417.

[6] H. W. Dommel, A. Yan, R. J. O. D. Marcano, and A. B. Miliani, "Tutorial Course on Digital Simulation of Transients in Power Systems," in *IISc*, Bangalore, 1983, pp. 17-38.

[7] B. A. Mork and D. L. Stuehm, "Application of nonlinear dynamics and chaos to ferroresonance in distribution systems," *IEEE Transactions on Power Delivery*, vol. 9, 1994, pp. 1009-1017.

[8] S. K. Chkravarthy and C.V. Nayar, "Frequency-locked and quasi periodic (QP) oscillations in power systems," *IEEE Transactions on Power Delivery*, vol. 13, 1997, pp. 560-569.

[9] W. L. A. Neves and H. Dommel, "on modeling iron core nonlinearities," *IEEE Transactions on Power Systems*, vol. 8, 1993, pp. 417-425.

[10] S. Mozaffari, M. Sameti, and A. C. Soudack, "Effect of initial conditions on chaotic ferroresonance in power transformers," *IEE Proceedings*/Generation, Transmission and Distribution*, vol. 144, 1997, pp. 456-460.

[11] S. Mozaffari, S. Henschel, and A. C. Soudack, "Chaotic ferroresonance in power transformers," in *Proc. IEE Generation, Transmission Distrib*, vol. 142, 1995, pp. 247-250.

[12] K. A. Anbarri, R. Ramanujam, T. Keerthiga, and K. Kuppusamy, "Analysis of nonlinear phenomena in MOV connected Transformers," *IEE Proceedings*/Generation Transmission and Distribution1*, vol. 48, 2001, pp. 562-566.

[13] B. A. T. A. Zahawi, Z. Emin, and Y. K. Tong, "Chaos in ferroresonant wound voltage transformers: effect of core losses and universal circuit behavioral," *IEE Proceedings*/Sci. Measurement Technology*, vol. 145, 1998, pp. 39-43.

[14] Z. Emin, B. A. T. A. Zahawi, D. W. Auckland, Y. K. Tong, "Ferroresonance in Electromagnetic Voltage Transformers: A Study Based on Nonlinear Dynamics," *IEE Proc. on Generation, Transmission, Distribution*, vol. 144, 1997, pp. 383-387.

[15] H. Radmanesh, A. Abassi, and M. Rostami, "Analysis of Ferroresonance Phenomena in Power Transformers Including Neutral Resistance Effect," *IEEE 2009 conference*, Georgia, USA,.

[16] D. Shoup, J. Paserba, and A. Mannarino, "Ferroresonance Conditions Associated with a 13 kV Voltage Regulator During Back-feed Conditions," *Presented at the International Conference on Power Systems Transients (IPST'07)*, vol. 2, 2007, pp. 1212-1215.

[17] A. R. Zare, H. Mohseni, M. S. Pasand, S. Farhangi, and R. Iravani, "Performance of Various Magnetic Core Models in Comparison with the Laboratory Test Results of a Ferroresonance Test on a 33 kV Voltage Transformer," *Presented at the International Conference on Power Systems Transients (IPST'07)*, in Lyon, France on June 4-7, 2007.

[18] W. Piasecki, M. Florkowski,, M. Fulczyk, P. Mahonen, and W. Nowak, "Mitigating Ferroresonance in Voltage Transformers in Ungrounded MV Networks," *IEEE Transaction on power delivery*, vol. 22, no. 4, 2007.

[19] K. B. Kilani and R. A. Schlueter, "An Approach for Determining the Subsystem Experiencing and Producing a Bifurcation in a Power System Dynamic Model," *IEEE Transaction on power systems*, vol. 15, no. 3, 2000, pp. 1053-1061 .

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