### Journal: IET Power Electronics

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- Cites in {2012} to items published in: 2011 = 114
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Bifurcation and Lyapunov’s exponents characteristics of electrical scalar drive systems

R.A. Sangrody¹ J. Nazarzadeh² K.Y. Nikravesh³

¹Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
²Electrical Engineering, Shahed University, Tehran, Iran
³Electrical Engineering, Amirkabir University of Technology, Tehran, Iran
E-mail: Nazarzadeh@shahed.ac.ir

Abstract: In this study, bifurcation and chaos phenomena in scalar drives of induction machines are investigated. Modified Poincare’s map, Lyapunov exponents and bifurcation diagram are utilised for this purpose. The boundary related to bifurcated response and conditions of chaotic response is also acquired for this purpose using Poincare’s map. In addition, root–locus curve of the system for stability and chaos analysis is derived by changing the controller parameters in constant speed control. In order to prove the chaotic response of the system, the largest Lyapunov’s exponent is determined numerically. In addition, an experimental prototype is prepared to show these phenomena. It is shown that chaotic response of the system can be controlled by adjusting the critical values of the speed controller’s gain value.

1 Introduction

Strange responses like bifurcated or chaotic ones cause fluctuations in torque and oscillations in the speed response of electrical drive systems [1]. Also, electrical currents and supply voltages may become chaotic and will not be sinusoidal. Under these circumstances, passive or active filters are not beneficial in improving the electrical drive performance. A lot of papers studied these phenomena in permanent magnet, reluctance and dc machines [2–7]. However, induction machines have non-linear dynamical models with coupling between their state variables; thus, stability and chaos analysis of induction machines is more complicated. In [8, 9], the authors showed these responses for an open-loop supply of induction machines. However, when other non-linear dynamical or algebraical structures accompany the closed-loop drives, occurrence of these responses seems to be more probable. For instance, Nagy [10] and Suto et al. [11] found these phenomena when an inverter supplied the induction machines. Other researches investigated and analysed these phenomena in vector control drives of induction machines. They showed that chaos may appear in these systems, when an estimation error exists in the identification of the system’s parameters such as rotor or stator resistance [12–17]. With regard to the previous researches, it is specified that either uncertainties or applying actual inverters to the system can cause a chaotic response in the induction machine drivers. Moreover, scalar drives of induction machines may be a good solution for closed-loop speed control systems. In spite of more inferior dynamical response related to vector control drives, they are simpler and also do not need complex devices, sensors and processing operations. In addition, the scalar drivers are cheaper than field oriented vector control drives and their implementation is simpler.

On the contrary, there are a lot of researches [18, 19] which showed that their responses may oscillate in open-loop conditions with light load. In these situations, reactive current and therefore electromagnetic torque of induction machines will oscillate [20]. Also, in these researches, damping factors specially stator and rotor resistors are less, hence reactive current may oscillate in light load. Study of the stability in open-loop conditions is not difficult because synchronous frequency will be a constant value. As a result, the dynamical model of the machine will be linear and linear control methods like Bode plot etc. are sufficient to investigate the stability conditions. In closed-loop conditions, synchronous frequency is a function of the system variables depending on the control strategy and is not a constant value. Therefore the dynamical model of the drive will be non-linear and strange responses like bifurcation and chaos may happen in the system.

In these drives, the parameters of the machine are not used to control them directly; therefore if chaos occurs in these systems, estimation error will not be the reason for the chaotic response. In [21], dynamical behaviour of the scalar drivers was investigated by perturbation method; moreover, bifurcation diagrams were used for verifying the chaotic responses of the system. It should be mentioned that, this method is utilised for local stability analysis of the system and cannot show chaotic phenomena in the system. Poincare’s map is another tool to investigate the strange response and control of chaos in the non-linear periodic systems, in which the system response can be found by determining the system trajectories in one period of their limit cycles. For instance, in a dc–dc converter, the period
of the limit cycle is almost equal to the switching time period [22]: however, when the input supply source is periodic and input source frequency is not a proper common multiple of switching frequency, the limit cycle period will not be an integer. In these cases, instead of the Poincare’s map of the system, a modified Poincare’s map is recommended. In [23], this map was introduced for power factor correction (PFC) converters; however, because of its complexity, the analytical Poincare’s map was not calculated.

Meanwhile, bifurcation diagram is another method for analysing the periodic systems. It can be calculated by sampling the steady-state response of the system by changing a system’s parameters called bifurcation parameters. Using this diagram, system’s responses (bifurcation or chaotic and their boundaries) can be achieved. Also, Lyapunov’s exponents are quantities that can evaluate the chaotic or stable response of the system. The sign and the value of Lyapunov’s exponents can be used for this analysis [24].

In this paper, a method for finding modified Poincare’s map in the scalar drives is introduced. For this purpose, algebraic and dynamical models of the scalar drive consisting of electrical machine, converter and controllers are presented. Then, a modified Poincare’s map of the system is formulated. Using this modified Poincare’s map, the boundary of chaotic response is achieved. Also, for evaluating the system’s response, the largest Lyapunov’s exponent and bifurcation diagram in the scalar drive systems are presented. Finally, chaotic response of the system is verified by experimental and numerical methods. The results show that we can avoid chaotic responses in the closed-loop scalar drive systems by adjusting the speed controller parameters to the specific values.

2 Modified Poincare’s map in non-linear perodical systems

Poincare’s map is a classical technique for analysing dynamical systems. It reduces the flow of an nth-order continuous system to an (n - 1)th-order discrete system \( P(x) \). This map, which is shown in Fig. 1, has the same stability properties, Lyapunov’s exponents and bifurcation behaviour of its related flow. An nth-order non-autonomous system with period \( T \) can be transformed into an (n - 1)th autonomous system in cylindrical state space by the definition of a new variable \( (\theta) \) as

\[
\dot{x} = f(x, \theta), \quad x(t_0) \quad \dot{\theta} = \frac{2\pi}{T}, \quad \theta(0) = \frac{2\pi}{T} \quad (1)
\]

The trajectory of the system intersects an n-dimensional hyper plane \( \Sigma \) each second that \( \Sigma \) is defined as

\[
\Sigma := \{(x, \theta) \in \mathbb{R}^n \times S^1; \theta = \theta_0\}
\]

The resulting map for the dynamical system in (1) with Poincare’s plane in (2) can be defined as

\[
P : \Sigma \rightarrow \Sigma, \quad (x, \theta_0) \mapsto \Phi_T(x, \theta_0)
\]

where \( \Phi_T(x, \theta_0) \) is a trajectory of the system.

Also, Poincare’s map can be defined for an nth-order autonomous system with the limit cycle \( \Gamma \). By considering an \((n-1)\)th-order hyperplane \( \Sigma \) that is transversal to \( \Gamma \) at a point like \( x^* \), the trajectory emanating from \( x^* \) will hit \( \Sigma \) at \( T \) second later where \( T \) is the period of the limit cycle. For other initial conditions, which are in a sufficiently small neighbourhood of \( x^* \), the trajectories will intersect \( \Sigma \) in approximately \( T \) second later.

Dynamical behaviour of the non-linear system can be evaluated by a limit set of the system on \( \Sigma \). The limit set of Poincare’s map is corresponding to the limit set of underlying flow. Period-one and kth-order subharmonic of a flow correspond to a fixed point and period \( k \) close orbit \( \{x_1^*, \ldots, x_k^*\} \) to the Poincare’s map of the system, respectively. If the system has a quasi-periodic solution that has two incommensurate frequencies \( (f_1, f_2) \), the limit set will be a circle. Also, for a \( k \)-periodic solution, the limit set of Poincare’s map is \((k - 1)\)th order. Limit set of Poincare’s map for a chaotic response is different. It is not a simple geometrical object as in the case of limit cycle and quasi-periodic responses. In addition, it has a fine structure that is characterised by fractional dimension.

For a non-linear system in (1), Poincare’s map can be expressed as

\[
x(t + T) = x(t) + \int_{t}^{t+T} f(x, \theta) \, dt = \Phi_T(x, \theta) \quad (4)
\]

Poincare’s map can be found by solving (4) in switching PWM systems when the non-linear system separates linear sub-systems. In these cases, the time period of the limit cycle is equal to the switching time period of the PWM system. However, since \( f(x, \theta) \) is non-linear, obtaining Poincare’s map through this method in high-order systems like electrical drives is cumbersome and probably impossible. Also, when different frequencies from input source are applied to the system, we have to utilise the modified Poincare’s map [23]. In these conditions, the modified Poincare’s map may be solved for the period related to the largest common multiple of PWM switching and main frequencies (Fig. 2). Then, the fixed point of the modified Poincare’s map can be calculated to investigate the stability of the corresponding limit cycle. Fig. 2 shows how linearisation of this map can be simplified.

In high-order systems, a recursive method can be utilised to obtain the modified Poincare’s map. For this purpose, (4) can

![Fig. 1 Poincare’s map of a system](image-url)
be approximated as \((N = T/T_s)\)
\[
x(t + T_s) = x(t) + T_s f(x(t), \theta_0) = g_1(x(t), \theta_0 + (2\pi/N))
\] (5)

for the second step, we have
\[
x(t + 2T_s) = x(t + T_s) + T_s f(x(t + T_s), \theta_0 + (2\pi/N))
\]
\[
= g_1(x(t), \theta_0 + (2\pi/N)) + T_s f(g_1(x(t), \theta_0) + (2\pi/N))
\]
\[
= g_2(x(t), \theta_0 + (4\pi/N))
\] (6)

By repeating (5) and (6) for the \(k\)th iteration, we can write
\[
x(t + (k + 1)T_s) = x(t + kT_s) + T_s f(g_{k-1}(x(t), \theta_k), \theta_k)
\] (7)

where
\[
\theta_k = \theta_0 + k \frac{2\pi}{N}
\] (8)

In the iteration equal to the main period \((k = N)\), we have
\[
x(t + T) = g_N(x(t), \theta_0) = P(x(t))
\] (9)

\(P(x)\) is the Poincare’s map of the system where the system’s trajectory collides with the \(\Sigma\) at \(t, t + T, t + 2T, \ldots\) (Fig. 1).

Perturbation method may be utilised in the system for stability analysis. However, finding an analytical solution for the Poincare’s maps in (9) is usually impossible in the non-linear dynamical systems. In these cases, the Poincare’s map can be linearised step by step (Fig. 2). This figure shows a typical steady-state trajectory \((x^*(t))\) in the first-order system such that \(x^*(t)\) is composed of two trajectories \(x_1^*(t)\) and \(x_2^*(t)\) with main and switching time period \((T \text{ and } T_s)\), respectively.

If the steady-state trajectory at \(t + kT_s\) is perturbed by the small variation \(\delta x(t + kT_s)\), using (7), a small variation of the system trajectory at \(t + (k + 1)T_s\) can be obtained by
\[
\delta x(t + (k + 1)T_s) = J(x^*(t + kT_s), \theta_k) \delta x(t + kT_s)
\] (10)

where
\[
J(x^*(t + kT_s), \theta_k) = \left( I + T_s \frac{\partial f(x, \theta_0)}{\partial x} |_{x=x^*(t + kT_s)} \right)
\] (11)

For \(N\)-sequential steps, we have
\[
\delta x(t + T) = \left( \prod_{k=0}^{N-1} J(x^*(t + kT_s), \theta_k) \right) \delta x(t)
\] (12)

If all multipliers of the linear system in (12) are inside the unit circle, the fixed point of Poincare’s map is stable and therefore the corresponding limit cycle related to the flow is stable.

### 3 Mathematical model of scalar drive

Fig. 3 shows a kind of closed-loop speed control drive, in which reference speed \((\omega_{ref})\) is compared with motor speed and the result after a proportional-integral (PI) slip controller gives reference of slip frequency \((\omega_{sl})\). By adding the slip frequency \((\omega_{sl})\) to the motor speed \((\omega_m)\), reference frequency is achieved \((\omega_v)\). Magnitude of the reference voltage \((V_{ref})\) is proportional to the angular frequency to have constant magnetising flux. In this drive, the torque is indirectly controlled by controlling the slip frequency because it is proportional to the slip frequency.

The two axes theory may be applied for modelling induction motors in different reference frames. In a synchronous reference frame, voltage equations of an induction machine can be presented as

\[
L \frac{d}{dt} \tilde{r}(t) = V_s(t) - R(t) \tilde{r}(t)
\] (13)

where \(\tilde{r}(t), V_s(t)\) are \(d-q\) axes components of current and supply voltage vectors that can be written as

\[
\tilde{r}(t) = \begin{pmatrix} i_d & i_q & i_d & i_q \end{pmatrix}^T
\]

\[
V_s(t) = \begin{pmatrix} v_d & v_d & v_d & v_d \end{pmatrix}^T
\] (14)

Also, inductance and resistance system matrices \((L, R(t))\) are

\[
L = \begin{pmatrix} L_s & 0 & L_m & 0 \\
0 & L_s & 0 & L_m \\
L_r & 0 & L_r & 0 \\
0 & L_{m} & 0 & L_r \end{pmatrix}
\] (15)

\[
R(t) = \begin{pmatrix} r_s & -\omega L_s & 0 & -\omega L_m \\
-\omega_L_s & r_s & \omega L_m & 0 \\
0 & -\omega L_m & r_s & -\omega L_r \\
\omega L_m & 0 & \omega L_r & r_s \end{pmatrix}
\] (16)

where \(L_s, L_r \text{ and } L_m\) are stator, rotor and mutual inductances that are referred to the stator or per unit form, respectively. Also, \(r_s \text{ and } r_r\) present rotor and stator resistances. \(\omega_m \text{ and } \omega_{sl}\) are synchronous and slip speed of the systems that can be written as

\[
\omega_{sl} = \omega_m - \omega_r
\] (17)

where \(\omega_r\) is the rotor speed that can be obtained from

\[
\omega = \frac{d}{dt} \log \frac{I}{I_{ref}}
\]

\[
\text{IET Power Electron.}, 2012, Vol. 5, Iss. 7, pp. 1236–1244
torque balance equation of the scalar drive

\[
\frac{d}{dt} \omega_t = \frac{1}{J} (T_e - T_L - B \omega_t), \quad \frac{d}{dt} \theta_t = \omega_t \tag{18}
\]

where \( J, P, T_L \) and \( B \) are rotary inertia, number of machine poles, load torque and friction coefficient, respectively. Also, \( T_e \) is electromagnetic torque that for a machine with \( P \) poles can be expressed as

\[
T_e = \frac{3}{2} P L_m (i_{dq} - i_{dq}) \tag{19}
\]

Equations (13) and (18) are sixth-order and non-linear dynamical equations; however, because of non-linearity, this dynamical system may show a strange response like chaotic one. In a closed-loop control system, reference synchronous angular speed of machine is given

\[
\omega_{ref} = k_p \omega + k_i \int \omega dt + \omega_t \tag{20}
\]

where \( k_p, k_i \) and \( \delta \omega \) are speed integral, proportional slip controller gains and speed tracking error, respectively. If the integrate term in (20) is given as \( \omega_i \), by substituting (17) into (20), we will have

\[
\omega_{di} = k_p \delta \omega + k_i \omega_t \tag{21}
\]

where

\[
\frac{d}{dt} \omega_{di} = \delta \omega \tag{22}
\]

Referring to Fig. 3, three-phase sinusoidal signals with amplitude \( V_m \) have compared with a triangle carrier signal to generate three-phase sinusoidal pulse width modulation (SPWM) switching patterns. If the averaging method is utilised for the output voltage variables of the inverter and also rotor circuits are given as short circuits, the vector \( V_s(t) \) in (14) can be obtained as

\[
V_s(t) = \begin{pmatrix} 0 & -f(\omega_s) & 0 \end{pmatrix}^T \tag{23}
\]

where \( f(\omega_s) \) is the stator voltage vector that is a function of the frequency related to the \( V/\omega_s \) control method. In unsaturated conditions, this function is dependent on \( \omega_s \) linearly [i.e. \( f(\omega_s) = k \omega_s \)]. Thus, a dynamical model of the scalar system in Fig. 3 can be obtained by augmentation of dynamical equations (13), (18) and (22). The augmented equations can be rearranged as

\[
\frac{d}{dt} \begin{pmatrix} i(t) \\ \omega(t) \\ \omega_{i}\end{pmatrix} = \begin{pmatrix} \frac{-L^{-1}R(i(t)) + L^{-1}V_s(t)}{i_{di} - i_{dq}} - T_L - B \omega_t \\ \omega_{ref} - \omega_t \end{pmatrix} \tag{24}
\]

This system is sixth-order, non-linear, strongly coupled and is similar to dynamical equation (1); therefore a strange response is possible for it. In the next section, Poincare’s map of the system will be obtained and applied to chaos analysis of the scalar drive system.

4 Poincare’s map of scalar drive

Using Poincare’s map, different parameter’s value of the system such as speed controller gain values, reference speed value and other parameters that cause bifurcation phenomena in the system can be calculated. Comparing (10) and (11) with augmented model (24), a linearised
model of the system as expressed in (24) can be achieved as

\[
\begin{pmatrix}
\Delta t \\
\Delta \omega_i \\
\Delta \theta_i \\
\end{pmatrix}
t + (k + 1)T_i = \begin{pmatrix}
J_{ii} & J_{ia} & J_{ia} \\
J_{iia} & 1 - BT_s & 0 \\
0 & T_s & 1 \\
\end{pmatrix}
\begin{pmatrix}
\Delta t \\
\Delta \omega_i \\
\Delta \theta_i \\
\end{pmatrix}
t + (k + 1)T_i,
\]

(25)

where \(J_{ii}, J_{ia}, \) and \(J_{ia}\) are Jacobian’s matrices of the system at \(t + kT_i\). Comparing (24) with (1) and (11), Jacobian’s matrix \(J_{ii}\) can be determined as (see (26))

\[\Delta = L_m^2 - L_s L_t \]

(27)

Also, other matrices can be obtained by

\[
J_{ia} = T_s((-1 - k_p) \Psi_{sq}^* - k_p (k_v + \Psi_{sd}^*)) - k_p (L_{sq}i_{sq} + L_{mq}i_{mq})^T
\]

\[J_{ia} = T_s((-1 - k_p) \Psi_{sq}^* - k_p (k_v + \Psi_{sd}^*)) - k_p (L_{sq}i_{sq} + L_{mq}i_{mq})^T
\]

\[J_{ia} = \frac{3T_s PL_m}{2J} (-i_{sq}^* r_{sq} i_{mq}^* - r_{sq} - i_{mq}^*)
\]

where \(\Psi_{sd}^* \) and \(\Psi_{sq}^* \) are \(d-q\) stator flux of the machine in equilibrium point that can be written as

\[
\Psi_{sd}^* = L_s i_{sd}^* + L_m i_{mq}^* \quad \Psi_{sq}^* = L_s i_{sq}^* + L_m i_{mq}^*
\]

(29)

Similar to (12), by repeating (25) for \(N\) steps, we can obtain a mapping model of the scalar drive system.

5 Lyapunov’s exponents and bifurcation diagram

Sensitivity of the system trajectories to initial conditions is an important characteristic of chaotic systems such that Lyapunov’s exponents may be used for sensitivity analysis of the system for initial conditions. If the sum of Lyapunov’s exponents is negative, the response of the system will be bounded and vice versa. Also, the largest and sign of sum of Lyapunov’s exponents are sufficient to prove the chaotic behaviour of the system. When the system trajectory is bounded, the summation will be negative, so the chaotic response can be evaluated by the largest Lyapunov’s exponent. The largest Lyapunov’s exponent can be calculated through the numerical method, which is shown in Fig. 4. In this method at first, initial conditions are considered. For instance, the initial conditions for (24) are given by

\[
x_0 = (i_{di}(0) \quad i_{dq}(0) \quad i_{qi}(0) \quad \omega_q(0) \quad \omega_d(0))^T
\]

(30)

The second initial conditions are chosen in the small neighbourhood of the first one with small deviation \(\|\Delta x\| \ll 1\)

\[
\dot{x}_0 = x_0 + \Delta x
\]

(31)

Equation (1) or (24) has to be solved with two initial conditions \(x_{i-1}\) and \(\tilde{x}_{i-1}\) for obtaining a system trajectory at the \(i\)th point. For this purpose, the system variables at the step \(i\)th can be written as

\[
x_i = x_{i-1} + h f(x_{i-1}, (i-1)h)
\]

(32)

where \(h\) is too small a time step. Similarly, for the second initial conditions, the system response would be obtained as

\[
\tilde{x}_i = \tilde{x}_{i-1} + h f(\tilde{x}_{i-1}, (i-1)h)
\]

(33)

In this step, the second initial conditions are modified for direction with constant deviation amplitude. Therefore we have

\[
\Delta x_i = \tilde{x}_i - x_i \quad x_i = x_i + \frac{\Delta x_0}{\Delta x_i} \Delta x_i
\]

(34)

Finally, the largest Lyapunov’s exponent at the \(i\)th iteration can be obtained by the following recursive relation as

\[
\lambda_{max}^i = \lambda_{max}^{i-1} + \ln \left| \frac{\Delta x_0}{\Delta x_i} \right|
\]

(35)

If the recursive equation (35) is repeated for long iterations,

![Method for finding the largest Lyapunov’s exponent](image-url)
$\lambda^\text{max}$ will be limited to a constant value which is called the largest Lyapunov’s exponent of the system.

On the other hand, the bifurcation diagram is another method, which is used for analysis of the dynamical systems with $N$-periods response. In this method, bifurcation diagrams can be obtained to show the steady state or equilibrium point of the system by using the Poincare’s map of the system. If the steady-state responses of the system are one, two or $N$-periods, the equilibrium points will be one, two or $N$ points. For a chaotic response, these responses are fractal objects.

### 6 Experimental and simulation results

An experimental set-up of an induction motor is used for modelling and analysis of the scalar drive system such that its parameters are shown in Table 1. An AT91SAM7S64 microcontroller is used for the implementation of the speed controller, PWM and other processes. Also, a 2000 pulse per rotation speed encoder that is coupled to the shaft of the motor provides speed feedback. For dc bus, a diode bridge, capacitors and negative temperature coefficient (NTC) resistance that provide 530 V is employed to supply the inverter. In addition, several boards are utilised for logic operations, speed feedback, reference speed and speed controller gain adjustment, inverter and so on.

Two equilibrium points of the system are presented in Table 2 in order to find the Poincare’s map and its multipliers. For example, if initial conditions are given by

$$x^* = \begin{pmatrix} 0.461 \\ 0.101 \\ 0.0053 \\ -0.043 \\ 203.57 \\ 3.57 \end{pmatrix}^T$$

\hspace{1cm} (36)

### Table 1 System parameters

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$r_s$</td>
<td>stator resistor</td>
<td>43.1 Ω</td>
</tr>
<tr>
<td>$r_r$</td>
<td>rotor resistor</td>
<td>72 Ω</td>
</tr>
<tr>
<td>$L_s$</td>
<td>stator inductance</td>
<td>1.995 H</td>
</tr>
<tr>
<td>$L_r$</td>
<td>rotor inductance</td>
<td>1.995 H</td>
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<td>$B$</td>
<td>friction coefficient</td>
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<tr>
<td>$T_L$</td>
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### Table 2 State variables of the system in steady-state conditions

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<th>$\omega_{st}$</th>
<th>$k_p$</th>
<th>$i_d^*$</th>
<th>$i_q^*$</th>
<th>$i_d$</th>
<th>$i_q$</th>
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<th>$\omega_q^*$</th>
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<td>0.101</td>
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<td>3.57</td>
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<td>5–15</td>
<td>0.468</td>
<td>0.086</td>
<td>0.004</td>
<td>-0.047</td>
<td>303.81</td>
<td>3.81</td>
</tr>
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</table>

for $\omega_{st} = 200$ rad/s, $k_p = 15$, $T_s = 3.087 \mu$s and using (25)–(29) and (36), the numerical Poincare’s map of the system can be obtained as (see (37))

then by substituting (37) into (12), the modified Poincare’s map can be obtained as (see (38))

Multiplier values of (38) are

$$\alpha(P(x^*)) = \begin{pmatrix} 1.000 & 0.704 & -0.53 & j0.28 \\ 2.49 \times 10^{-13} & 4.86 \times 10^{-14} \end{pmatrix}$$

All these multipliers are in the unit circle, therefore the system is stable for these set points. Using a similar approach, we can find bifurcation and chaotic response boundary for different values of reference speed, speed proportional controller gain, supply voltage, pole pairs and load values which are shown in Fig. 5. Also, for each reference speed, there are two speed proportional controller gains between which, the system response will be chaotic (Fig. 5a). For a special drive, this region can be selected exactly and an unstable response can be prevented. Figs. 5b and c show chaotic area in $V_s - \omega_{ref}$ plane for different pole pairs and different loads, respectively. The chaotic area will be decreased when the drive is connected to light loads or the pole pairs of the machine are decreased. Fig. 6 shows simulated and experimental current response of the motor when reference speed, voltage amplitude, speed proportional and integral controller gains are set to 250 rad/s, 400 V, 10 and 1, respectively. As seen in Figs. 5b and 5c, these set points are in the chaotic region area.

### Fig. 7

Fig. 7 shows dominant multipliers of Poincare’s map for different reference speeds by different values of speed proportional gain values. As seen for each reference speed, there is a range of proportional speed controller values in which multipliers will be outside the unit circle, so a chaotic response will occur in these areas. By changing the reference speed ($\omega_{ref}$) with constant controller gain, the dominant multiplier of the system may be located in

$$J_1(x^*, \theta_0) = \begin{pmatrix} 0.998 & 0.017 & 0.003 & 0.017 & 4.89 \times 10^{-6} & 0 \\ -0.017 & 0.998 & -0.017 & 0.003 & -0.00021 & 0.000043 \\ 0.002 & -0.017 & 0.997 & -0.018 & -4.98 \times 10^{-6} & 0 \\ 0.017 & 0.0023 & 0.018 & 0.997 & 0.00021 & -0.000042 \\ 0.00033 & 0.00004 & 0.00076 & -0.0035 & 0.9999 & 0 \\ 0 & 0 & 0 & 0 & -3.087 \times 10^{-6} & 1 \end{pmatrix}$$

\hspace{1cm} (37)

$$P(x^*) = \begin{pmatrix} 0.59 & -0.697 & 0.586 & -0.687 & -0.0118 & 0.0051 \\ 1.648 & 3.536 & 1.636 & 3.575 & -0.0837 & 0.0228 \\ -1.244 & -0.223 & -1.236 & -0.242 & 0.0324 & -0.0105 \\ -1.348 & -3.552 & -1.339 & -3.585 & 0.0754 & -0.0199 \\ 16.864 & -15.31 & 16.828 & -15.48 & 0.328 & 0.102 \\ 0.231 & 0.8598 & 0.228 & 0.875 & -0.0296 & 1.001 \end{pmatrix}$$

\hspace{1cm} (38)
the unstable region. The type of bifurcation is hopf because multipliers on the circle have both real and imaginary parts. This fact can help in adjusting the speed proportional gain value in order to work in stable operations like adjustable speed drives or in chaotic operations like washing machines or mixers.

Figs. 8a and b show simulated and experimental current signals of the motor when speed reference value equals 300 rad/s and speed proportional controller gain is 12, respectively. As can be seen, the motor response is chaotic.

Figs. 9a and b show the bifurcation diagram of the stator direct and quadrature currents with \( \omega_{\text{ref}} = 300 \) and \( k_i = 1 \), respectively. Using this figure, the range of \( k_p \), in which the response of the drive is bifurcated or chaotic can be obtained. For instance, chaotic response occurs with \( 11 < k_p < 47 \).

Fig. 5 Bifurcation boundary region
a \( k_p, \omega_{\text{ref}} \) plane
b \( V, \omega_{\text{ref}} \) plane with different poles
c \( V, \omega_{\text{ref}} \) plane with different load

Fig. 6 Current and voltage responses for \( V_m = 400 \text{ V} \), \( \omega_{\text{ref}} = 250 \text{ rad/s} \), \( k_p = 10 \) and \( k_i = 1 \)
a Simulated result
b Experimental result
Moreover, the chaotic response of the system can be proved by calculating the largest Lyapunov’s exponent. For this purpose, we give an initial condition and small perturbation as

\[ x_0 = (0.51, 0.08, 0.003, -0.046, 300, 3.7)^T \]

Using (30)–(35) and for \( \omega_{ref} = 300 \) rad/s and \( k_p = 12 \), after \( k = 10^6 \) iterations; the largest Lyapunov’s exponents can be obtained as

\[ \lambda_{max} = 4.8005 \]  

Fig. 10 shows the largest Lyapunov’s exponents in each
iteration. For $k > 50 \times 10^5$, it converges to $+0.015$ that shows the system response is chaotic.

7 Conclusion

In this paper, chaotic and bifurcation response related to the scalar drives of the induction machine were analysed and a modified Poincaré’s map was explained. Using this map, the largest Lyapunov’s exponent was introduced to prove the chaotic response in the systems. Moreover, the effect of changing speed proportional gain for different reference speeds was investigated and it was shown that adjusting this value can cause chaotic or stable responses. Therefore by setting the speed proportional gain in scalar drive systems, chaotic behaviour can be avoided.

8 References