Monitoring Correlated Profile and Multivariate Quality Characteristics

Amirhossein Amiri, a* Changliang Zou b and Mohammad H. Doroudyan a

Monitoring multivariate quality characteristics is very common in production and service environment. Therefore, many control charts have been suggested by authors for monitoring multivariate processes. In another side, profile monitoring is a new approach in the area of statistical process control. In this approach, the quality of a product or a process is characterized by a relation between one response variable and one or more independent variables. In practice, sometimes the quality of a product or a process is represented by a correlated profile and multivariate quality characteristics. To the best of our knowledge, there is no method for monitoring this type of quality characteristics. Note that monitoring correlated profile and multivariate quality characteristics separately leads to misleading results. In this article, we specifically focus on correlated simple linear profile and multivariate normal quality characteristics and propose a method using multivariate exponentially weighted moving average control chart to monitor the correlated profile and multivariate quality characteristics simultaneously. The performance of the proposed control chart is evaluated by simulation studies in terms of average run length criterion. Finally, the proposed method is applied to a real case in the electronics industry. Copyright © 2013 John Wiley & Sons, Ltd.

Keywords: statistical process control; simple linear profile; correlated profile and multivariate quality characteristics; MEWMA control chart; phase II

1. Introduction

O wadays, the quality of many products or processes is represented by two or more correlated quality characteristics. Hotelling1 showed that monitoring multivariate quality characteristics separately leads to misleading results. He proposed \( T^2 \) control chart for monitoring multivariate quality characteristics. Multivariate exponentially weighted moving average (MEWMA)2 and multivariate cumulative sum (MCUSUM)3 control charts are the other most common multivariate control charts. Reviews of the most usual methods in multivariate process monitoring have been performed by several authors such as Basseville and Nikiforov,4 Ryan,5 Frisen,6 Sonesson and Frisen,7 Bersimis et al.8 and Frisen.9 Recently, a new procedure was proposed by Butte and Tang10 in some common multivariate control charts to facilitate the identification of the source of out-of-control signal. Kim et al.11 proposed a non-parametric fault isolation approach based on a one-class classification algorithm and showed that their proposed method can detect source of variation better than \( T^2 \) decomposition in the presence of nonnormal processes. Some kinds of variable sampling rate in multivariate control charts, which lead to overall better performance rather than standard fixed sampling rate, were proposed by Reynolds and Cho.12 The applications of multivariate control charts in health care were studied by Waterhouse et al.13

Sometimes, the quality of a product or a process is characterized by a relation between a response variable and one or more independent variables, which is called profile. The most common type of profile is a simple linear profile in which a response variable has a linear relation with an explanatory variable. Simple linear profile monitoring was first investigated by Kang and Albin14 via proposing two approaches including \( T^2 \) and EWMA/R. Then, Kim et al.15 proposed EWMA-3, Mahmoud and Woodall16 proposed an F method, Mahmoud et al.17 Zou et al.18 and Zhang et al.19 suggested an LRT-based method, and Saghaie et al.20 used CUSUM-3 to monitor simple linear profile. Other complicated profiles such as multiple linear profile, polynomial profile, nonlinear profile, logistic profile, and multivariate linear profiles were also investigated by the authors. For example, multiple linear regression profile was studied by Zou et al.21 and Amiri et al.22. Kazemzadeh et al.23,24 proposed some methods in phases I and II of monitoring polynomial profiles, respectively. Nonlinear profile was monitored by Williams et al.25 and Vaghefi et al.26. Logistic profile was monitored by Yeh et al.27 and multivariate linear profile monitoring was investigated by Noorossana et al.28,29, Eyvazian et al.30 and Zou et al.31 These are some other issues considered in the literature of profile monitoring. In addition, Woodall et al.32 and Woodall33 reviewed common methods in profile monitoring. In addition, Noorossana et al.34 recently summarized major achievements in the area of profile monitoring.

*Correspondence to: Amirhossein Amiri, Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran.
E-mail: amiri@shahed.ac.ir

aLPMC and Department of Statistics, School of Mathematical Sciences, Nankai University, Tianjin, China
bIndustrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran.
In practice, sometimes the quality of a product or a process could be represented by a correlated profile and multivariate quality characteristics. For example, in some plastic manufacturing companies, the quality of a product or raw material can be characterized by a relationship between the hardness of a product as response variable and the distance from injection point as an explanatory variable in a simple linear regression profile. In addition, the weight of product has a correlation with the hardness of a product. As another example, in an aluminum electrolytic capacitor (AEC) manufacturing process, the quality characteristics of unfinished AEC product such as the capacitance and loss tangent are monitored in each stage. Moreover, the relationship between the quality characteristics of an AEC from one stage to another stage is described by a simple linear profile. Besides the monitoring of those important characteristics, the engineers are often rather concerned about the significant changes in such a profile. To the best of our knowledge, there is no method in monitoring this type of quality characteristics.

In this article, we specifically focus on correlated simple linear profile and multivariate quality characteristics with multivariate normal distribution. This research is applied in phase II; hence, the distribution parameters of quality characteristics are known. In this article, we aim to monitor sample intercept and slope as parameters of the profile as well as multivariate quality characteristics. As a result, the control statistic consists of sample intercept and slope and sample variables, which should be monitored simultaneously. To the best of our knowledge, there is no method in monitoring this type of quality characteristics.

In this article, we specifically focus on correlated simple linear profile and multivariate quality characteristics with multivariate normal distribution. This research is applied in phase II; hence, the distribution parameters of quality characteristics are known. In this article, we aim to monitor sample intercept and slope as parameters of the profile as well as multivariate quality characteristics. As a result, the control statistic consists of sample intercept and slope and sample variables, which should be monitored simultaneously. Therefore, the correlation coefficient between variables and sample intercept and slope are calculated based on statistical methods. Finally, MEWMA control chart is used to monitor mean vector. The performance of the proposed method is evaluated by using simulation studies in terms of average run length criterion.

The structure of the article is as follows. In Section 2, the problem is defined. Section 3 illustrates the proposed method for monitoring correlated profile and multivariate quality characteristics. In Section 4, the performance of the proposed method is evaluated using simulation studies. The application of the problem is shown in Section 5 by introducing a real case. Our concluding remarks as well as some future suggestions are given in Section 6.

2. Problem definition

In some statistical process control applications, the quality of a product or a process can be represented by a vector of correlated variables such as \( w = (w_1, w_2, \ldots, w_p) \), or a relation between a response variable and one or more independent variables, such as a simple linear profile. In this research, we face a combination of correlated profile and multivariate quality characteristics. We specifically focus on correlated simple linear profile and multivariate quality characteristics. In the simple linear profile \( z_i = \beta_0 + \beta_1 x_i + \epsilon_i \), the intercept \( \beta_0 \) and the slope \( \beta_1 \) are parameters and \( x_i \)'s \( (i = 1, 2, \ldots, n) \) are the values of explanatory variables. Meanwhile, error terms \( \epsilon_i \)'s are independent and have normal distribution with mean zero and variance \( \sigma^2 \). We assume that multivariate quality characteristics \( y = (y_1, y_2, \ldots, y_m)^T \) follow multivariate normal distribution with mean vector \( \mu \) and covariance matrix \( \Sigma \). The important issue in this article is the definition of correlation between profile and multivariate quality characteristics. The source of this correlation is the covariance between each level of profile \( z_i \) and each variable quality characteristic \( y_j \) as \( \sigma_{\mu j} \), and all of them form the covariance matrix \( \Sigma_{yX} \) with size \( n \times m \). This research is performed in phase II; hence, the parameters of profile including the values of intercept \( \beta_0 \), slope \( \beta_1 \), variance \( \sigma^2 \) of error terms \( \epsilon_i \)'s and \( x_i \)'s values are assumed to be known. Likewise, the value of mean vector \( \mu \) and covariance matrix \( \Sigma \) of multivariate normal quality characteristics as well as covariance matrix between different levels of profile and variables \( (\Sigma_{Xy}) \) will be known. In this article, we aim to monitor the mean vector. Therefore, the covariance matrix is assumed to be stable over time, and assignable causes just may lead to change in the mean vector of distribution of quality characteristics.

In simple linear profile monitoring, Kang and Albin\(^{14}\) proposed two strategies including monitoring sample intercept \( \hat{\beta}_0 \) and slope \( \hat{\beta}_1 \) in each observation, or residuals that are differences between the reference line and the sample profile. In this article, we apply first strategy. Thus, to monitor a simple linear profile, one can estimate the values of intercept \( \hat{\beta}_0 \) and slope \( \hat{\beta}_1 \) in each sample using the least square method, which can be seen in Hines and Montgomery.\(^{35}\) Equations (1) and (2) show these estimations as follows:

\[
\hat{\beta}_0 = \bar{z} - \hat{\beta}_1 \bar{x}, \tag{1}
\]

\[
\hat{\beta}_1 = \frac{S_{zx}}{S_{xx}}, \tag{2}
\]

where \( \bar{z} = n^{-1} \sum_{i=1}^{n} z_i, \bar{x} = n^{-1} \sum_{i=1}^{n} x_i, S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \) and \( S_{zx} = \sum_{i=1}^{n} z_i (x_i - \bar{x}) \), respectively. The sample of \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \) follow normal distribution with mean \( \beta_0 \) and \( \beta_1 \), respectively, and variances given in Equations (3) and (4), respectively,

\[
\sigma^2_{\hat{\beta}_0} = \sigma^2 \left( \frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right), \tag{3}
\]

\[
\sigma^2_{\hat{\beta}_1} = \sigma^2 \left( \frac{1}{S_{xx}} \right). \tag{4}
\]

Moreover, the covariance between sample intercept and slope are calculated by Equation (5) as follows:
Despite of numerous studies in the field of profile and multivariate monitoring separately, to the best of our knowledge, there is no research in the case of correlated profile and multivariate quality characteristics. Therefore, in the next section, we propose a method to monitor this type of quality characteristics.

3. Proposed method

To observe correlated simple linear profile and multivariate quality characteristics, we propose a control statistic consists of sample intercept and slope as parameters of the simple linear profile and sample variables. Equation (6) shows the control statistic as follows:

\[ w = \left( \hat{\beta}_0, \hat{\beta}_1, y_1, y_2, \ldots, y_m \right)^T \]  

(6)

Because the simple linear profile and the multivariate quality characteristics are correlated, we obtain the covariance matrix of the control statistic based on statistical methods. Then, we define

\[ c_i = \frac{x_i - \bar{x}}{S_{xx}} \]

(7)

\[ d_i = \frac{1}{n} - x c_i \]

(8)

Therefore,

\[ \hat{\beta}_1 = \frac{S_{yx}}{S_{xx}} = \frac{1}{n} \sum_{i=1}^{n} z_i (x_i - \bar{x}) = \frac{1}{n} \sum_{i=1}^{n} z_i \left( \frac{x_i - \bar{x}}{S_{xx}} \right) = \sum_{i=1}^{n} z_i c_i \]

(9)

\[ \hat{\beta}_0 = z - \hat{\beta}_1 \bar{x} = \frac{1}{n} \sum_{i=1}^{n} z_i - \bar{x} \sum_{i=1}^{n} z_i c_i = \frac{1}{n} \sum_{i=1}^{n} (z_i - x z_i) = \sum_{i=1}^{n} (z_i \left( \frac{1}{n} - x c_i \right)) = \sum_{i=1}^{n} z_i d_i \]

(10)

Hence, the covariance between sample intercept and variables is obtained using Equation (11), and the covariance between sample slope and variables is obtained using Equation (12), as follows:

\[ \text{Cov}(\hat{\beta}_0, y_i) = \text{Cov}(\sum_{i=1}^{n} d_i z_i, y_i) = \sum_{i=1}^{n} \text{Cov}(d_i z_i, y_i) = \sum_{i=1}^{n} d_i \text{Cov}(z_i, y_j) = \sum_{i=1}^{n} d_i \sigma_{y_i} \]

(11)

\[ \text{Cov}(\hat{\beta}_1, y_i) = \text{Cov}(\sum_{i=1}^{n} c_i z_i, y_i) = \sum_{i=1}^{n} \text{Cov}(c_i z_i, y_i) = \sum_{i=1}^{n} c_i \text{Cov}(z_i, y_j) = \sum_{i=1}^{n} c_i \sigma_{y_j} \]

(12)

Thereupon, vector \( w \) follows multivariate normal distribution with mean vector and covariance matrix as follows:

\[
\mu_w = (\beta_0, \beta_1, \mu_1, \mu_2, \ldots, \mu_m)^T
\]

\[
\Sigma_w = \begin{bmatrix}
\text{var}(\beta_0) & \text{cov}(\beta_0, \beta_1) & \text{cov}(\beta_0, y_1) & \text{cov}(\beta_0, y_2) & \cdots & \text{cov}(\beta_0, y_m) \\
\text{cov}(\beta_1, \beta_0) & \text{var}(\beta_1) & \text{cov}(\beta_1, y_1) & \text{cov}(\beta_1, y_2) & \cdots & \text{cov}(\beta_1, y_m) \\
\text{cov}(y_1, \beta_0) & \text{cov}(y_1, \beta_1) & \text{var}(y_1) & \text{cov}(y_1, y_2) & \cdots & \text{cov}(y_1, y_m) \\
\text{cov}(y_2, \beta_0) & \text{cov}(y_2, \beta_1) & \text{cov}(y_2, y_1) & \text{var}(y_2) & \cdots & \text{cov}(y_2, y_m) \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\text{cov}(y_m, \beta_0) & \text{cov}(y_m, \beta_1) & \text{cov}(y_m, y_1) & \text{cov}(y_m, y_2) & \cdots & \text{var}(y_m)
\end{bmatrix}
\]

(13)

(14)

Let us take a more careful look at this correlation from statistical point. Under the assumption, \( z_i = \beta_0 + \beta_1 x_i + \epsilon_i \) and \( x_i \)'s are the fixed (not random as in our assumption) design point. Hence,

\[ \text{Cov}(z_i, y_j) = \text{Cov}(\beta_0 + \beta_1 x_i + \epsilon_i, y_j) \]

\[ = \text{Cov}(\beta_0, y_j) + \beta_1 \text{Cov}(x_i, y_j) + \text{Cov}(\epsilon_i, y_j) = 0 + \text{Cov}(\epsilon_i, y_j) \]

(15)

Therefore, the covariance between \( z_i \) and \( y_j \) is just that between \( \epsilon_i \) and \( y_j \). We always assume that \( \epsilon_i \)'s are independent identically distributed (i.i.d.), and thus it is maybe unreasonable to make an assumption that \( \text{Cov}(\epsilon_i, y_j) \neq \text{Cov}(\epsilon_k, y_j) \) for \( i \neq k \) except for the case that
one sometimes $e_i \equiv e_i(x_i)$ (the distribution of $e_i$ may depend on $x_i$). Thus, it is possible to impose an assumption that $\text{Cov}(e_i, y) = \sigma_{e_i}$. Accordingly, $\text{Cov}(z_i, y) = \sigma_{z_i}$ and since $\sum_{i=1}^{n} c_i = 0$ and $\sum_{i=1}^{n} d_i = 1$

$$\text{Cov} \left( \beta_0, y \right) = \sum_{i=1}^{n} d_i \text{Cov}(z_i, y) = \sum_{i=1}^{n} d_i \sigma_{z_i} = \sigma_{z_i} \sum_{i=1}^{n} d_i = \sigma_{z_i}$$

(16)

$$\text{Cov} \left( \beta_1, y \right) = \sum_{i=1}^{n} c_i \text{Cov}(z_i, y) = \sum_{i=1}^{n} c_i \sigma_{z_i} = \sigma_{z_i} \sum_{i=1}^{n} c_i = 0$$

(17)

The values of correlations obtained from Equations (11) and (12) are $\sigma_{z_i}$ and 0, respectively. Thus, the given covariance matrix in Equation (14) is changed to Equation (18), in which $\mathbf{\Sigma}_w = (\sigma_{z_1} \quad \sigma_{z_2} \quad \ldots \quad \sigma_{z_m})^T$ and $\mathbf{0}$ is an $m$-dimensional zero vector.

$$\mathbf{\Sigma}_w = \begin{pmatrix}
\sigma^2 \left( 1 + \frac{x^2}{s_{xx}} \right) & -\sigma^2 \frac{x}{s_{xx}} & \mathbf{\alpha}_{\mathbf{z}} \\
-\sigma^2 \frac{x}{s_{xx}} & \sigma^2 \left( \frac{1}{s_{xx}} \right) & 0 \\
\mathbf{\alpha}_{\mathbf{z}}^T & 0 & \mathbf{\Sigma}
\end{pmatrix}$$

(18)

Then, the use of the MEWMA control chart by Lowry et al. is suggested. The control statistic in MEWMA is as follows:

$$T_i = \mathbf{v}_i^T \sum_{i=1}^{-1} \mathbf{v}_i$$

(19)

In this equation, $\mathbf{v}_i$ is the smoothed value of $\mathbf{w}_i$, and obtained from:

$$\mathbf{v}_i = \lambda (\mathbf{w}_i - \mathbf{\mu}_w) + (1 - \lambda) \mathbf{v}_{i-1}$$

(20)

where $\lambda$ is a smoothing parameter (chosen such a way that $0 < \lambda \leq 1$) and $\mathbf{v}_0$ is an $m + 2$-dimensional zero vector. Likewise, the steady-state covariance matrix of control statistic is equal to:

$$\sum_{i=2}^{\infty} \frac{\lambda^i}{1 - \lambda} \sum_{i=0}$$

(21)

The upper control limit (UCL) for the MEWMA statistic in Equation (19) is obtained by using a simulation study such that a desired in-control ARL is achieved. In addition, based on a signal rule, the control chart signals whenever the control statistic exceeds the UCL.

To compare the proposed method with the traditional methods, one may consider a scheme that uses two independent control charts simultaneously for monitoring the location parameters of simple linear profiles and multivariate quality characteristics. This method surely neglects the correlation between profile and multivariate quality characteristics. Thus, it leads to poor performance of the procedure. To show this reality, in simulation study, we consider using an MEWMA control chart to monitor the intercept and slope estimators in combination with an MEWMA control chart to monitor the mean of variable quality characteristics and to compare the results of this method with the proposed method.

## 4. Simulation study

In this section, the performance of the proposed method is evaluated by using simulation studies through a numerical example in terms of average run length criterion. In this example, the quality of a product or a process is represented by a simple linear relation between a response variable ($z_i$) and one explanatory variable ($x_i$) as a simple linear profile as well as a variable quality characteristic. Note that in this example, because of simplicity and without loss of generality, only one variable quality characteristic is considered.

The relation of the simple linear profile is $z_i = 3 + 2x_i + e_i$, where the $x_i$ values are equal to 2, 4, 6, and 8. Also, error terms ($e_i$’s) in the profile are independent and follow normal distribution with mean zero and variance one. Meanwhile, the variable quality characteristic is normally distributed with mean 0 and variance 1. Moreover, the response variable ($z_i$) in different levels of the explanatory variable ($x_i$) has correlation with the variable quality characteristic ($y$). The covariance between the response variable in different levels of the explanatory variable and the variable quality characteristic is the same and equal to 0.35.

As mentioned previously, in the traditional method, profile and multivariate quality characteristics are monitored separately. Therefore, an MEWMA control chart is designed to monitor sample intercept and slope of the profile. The variable quality characteristic is also monitored using an EWMA control chart. We set the smoothing parameters in both control charts equal to 0.2. Also, the control limits of the MEWMA and EWMA control charts are determined equal to 11 and ±0.95, respectively, using 10000 simulation runs such that the overall ARL0 becomes equal to 200.

On the basis of the proposed method, the control statistic is a vector consisting of sample intercept, slope, and variable quality characteristic as $\mathbf{w}_i = \left( \beta_0 \quad \beta_1 \quad y \right)^T$. This statistic has multivariate normal distribution with mean vector $\mathbf{\mu}_w = (3 \quad 2 \quad 0)^T$. To compute the covariance matrix of the statistic, variances of sample intercept and slope as well as their covariance are calculated...
based on Equations (3)–(5) equal to \( \sigma_{\hat{\beta}_0}^2 = 1.5, \sigma_{\hat{\beta}_1}^2 = 0.05, \) and \( \text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = -0.25, \) respectively. Furthermore, as explained in the previous section, covariance values between the sample intercept and slope of profile and the variable quality characteristic are equal to \( \text{Cov}(\hat{\beta}_0, y) = 0.35 \) and \( \text{Cov}(\hat{\beta}_1, y) = 0, \) respectively, based on Equations (16) and (17). Thus, the covariance matrix of the control statistic is as follows:

\[
\Sigma_w = \begin{pmatrix}
1.5 & -0.25 & 0.35 \\
-0.25 & 0.05 & 0 \\
0.35 & 0 & 1
\end{pmatrix}
\]

Then, an MEWMA control chart is designed to monitor the control statistic. Similar to the traditional method, the smoothing parameter is set equal to 0.2. Moreover, UCL is obtained equal to 11.875 using simulation study with 10,000 replications to achieve in-control average run length (ARL) equal to 200.

The performance of the proposed method in comparison with the traditional method is evaluated in terms of out-of-control average run length (ARL) criterion under different shift scenarios. These scenarios are step shifts in the intercept of the simple linear profile from \( \beta_0 \) to \( \beta_0 + \theta \), step shifts in the slope of the simple linear profile from \( \beta_1 \) to \( \beta_1 + \delta \), and step shifts in the mean of variable quality characteristic from \( \mu_y \) to \( \mu_y + \gamma \). The ARL values of both methods under different shifts are computed by 10000 simulation runs and the average run length curves of both methods under different shifts in the intercept and the slope of simple linear profile, and the mean values of variable quality characteristics are depicted in Figures 1–3 in log scale, respectively. The ARL values for these
5. Case study

In this section, we apply the proposed method to a real data set from an AEC manufacturing process (provided by ENW Electronics Ltd; see Figure 4) discussed generally in the introduction section. As shown in Figure 4, the entire manufacturing process, which is a typical multistage process (cf. Shi26), includes a sequence of operations, such as clenching, rolling, soaking, assembly, cleaning, aging, and classifying. The aim of the process is to transform the raw materials (anode aluminum foil, cathode aluminum foil, guiding pin, electrolyte sheet, plastic cover, aluminum shell, and plastic tube) into AECs with certain specifications. Many types of defects may be induced through the manufacturing process, which can be classified into cosmetic defects, capacity defects, leakage current defects, and dissipation factor defects. Several types of defects may be induced in each stage. For example, in the clenching and rolling stage, the possible defects include improper compound thickness, aluminum foil cracking, scored aluminum foil, ragged margins, and so forth. In the soaking stage, dissipation factor defects may be found. Therefore, the quality character of AECs representing its appearance condition and functional performance condition at upstream stages will influence that of downstream stages, and then the different stages are correlated.

To this end, the quality of unfinished AEC products, which are called capacitor elements, in terms of appearance and functional performance, is inspected by sampling after each stage. At each stage, certain important characteristics of an AEC, such as the capacitance and loss tangent (or equivalently dissipation factor), are automatically calibrated by an electronic device at some given performance condition at upstream stages will influence that of downstream stages, and then the different stages are correlated.

In this real case, we consider dissipation factor values in the aging stage as the response variable (z) and one input variable (x) from the soaking stage as explanatory variable. Simultaneously, the values of capacitance and dissipation factor observations (denoted as y1 and y2) from the soaking stage are monitored. The data set comprises 243 profile samples of size n = 10. Among them, 16 profiles are classified as inferior profiles based on the physical knowledge and experience of the engineers. We first use 200 correlated profiles and multivariate quality characteristics as the historical sample to calibrate the necessary parameters in phase I. Then, we monitor the other 43 samples in phase II based on the estimated parameters from phase I analyses.

Phase I analysis confirmed the adequacy of simple linear profile as $z = -758.92 + 200.81x + e_i$ in which error terms ($e_i$'s) are independent and has normal distribution with mean zero and variance 2.934. Note that the $x_i$ values are 3.82, 3.84, 3.86, 3.88, 3.90, 3.92, 3.94, 3.96, 3.98, and 4. In the profile, the values of $-758.92$, 200.81, and 2.934 are estimated by using equations

$$\hat{\beta}_0 = \frac{1}{200} \sum_{j=1}^{200} \hat{e}_j, \hat{\beta}_1 = \frac{1}{200} \sum_{j=1}^{200} \hat{e}_j, \text{ and } \sigma^2 = \frac{1}{200} \sum_{j=1}^{200} MSE_j,$$

In addition, variable quality characteristics follow multivariate normal distribution. The mean vector and the covariance matrix of the variable quality characteristics are, respectively, estimated by $\mu_y = \frac{1}{199} \sum_{j=1}^{199} y_i$ and $\Sigma_y = \frac{1}{199} \sum_{j=1}^{199} (y_j - \mu_y)(y_j - \mu_y)^T$ equal to

$$\mu_y = (-0.8989, -2.0734)^T$$

$$\Sigma_y = \begin{pmatrix} 0.0031 & -0.0001 \\ -0.0001 & 0.0065 \end{pmatrix}$$

Moreover, the covariance between all levels of profile (z) and the variable quality characteristics ($y_1$ and $y_2$) are roughly equal to $Cov(z, y_1) = \sigma_{z1} = 0.272$ and $Cov(z, y_2) = \sigma_{z2} = 0.350$, respectively. Note that these covariances are estimated by Equation (22) as follows:

---

Figure 4. The manufacturing process for AECs
\[
\text{COV}(z_i, y_k) = \frac{1}{199} \sum_{j=1}^{200} (z_{ij} - \bar{z}_i)(y_{kj} - \bar{y}_k)^T;
\]
\[
\quad \forall i = 1, 2, \ldots, 10 \text{ and } \forall k = 1, 2,
\]
(22)

where \( \bar{z}_i = \frac{1}{200} \sum_{j=1}^{200} z_{ij}; \forall i = 1, 2, \ldots, 10. \)

On the basis of the proposed method, the control statistic is a vector consisting of sample intercept, slope, and variable quality characteristics as

\[
w_i = (\beta_0, \beta_1, y_1, y_2)^T.
\]

This statistic has multivariate normal distribution with mean vector and covariance matrix as follows:

\[
\mu_w = \begin{pmatrix}
-758.92 \\
200.81 \\
-0.8989 \\
-2.0734
\end{pmatrix},
\]

\[
\Sigma_w = \begin{pmatrix}
1359.5 & -347.63 & 0.272 & 0.350 \\
-347.63 & 88.909 & 0 & 0 \\
0.272 & 0 & 0.0031 & -0.0001 \\
0.350 & 0 & -0.0001 & 0.0065
\end{pmatrix}.
\]

Then, an MEWMA control chart is designed to monitor the mean vector of statistic. Hence, we set smoothing parameter equal to 0.2 and determine UCL by using simulation study equal to 13.874 for \( \text{ARL}_0 = 200. \) In phase II, for each 43 samples (given in Appendix A), the intercept and slope of each profile are estimated using least square method and a vector of size 4 alongside variable quality characteristics \( y_1 \) and \( y_2 \) is formed. On the basis of this sample vector, the MEWMA control statistic for each sample is calculated by using Equation (19) and plotted on the control chart shown in Figure 5 in log scale. Note that \( \mathbf{v}_0 \) is set equal to zero vector as the initial vector of the MEWMA control chart statistic.

As illustrated in Figure 5, the first 27 samples are the in-control state. However, the values of control statistics for the other 16 samples are higher than UCL. On the basis of the signal rule, these samples are in the out-of-control state. This state is confirmed by quality engineers as well. Therefore, the results show that the proposed method can distinguish the in-control and out-of-control states correctly.

6. Conclusions and future studies

Multivariate quality characteristics and profile monitoring are widely applied in production and service environment and investigated by authors, separately. In this article, a method for monitoring correlated simple linear profile and multivariate quality characteristics was proposed. First, a multivariate statistic is proposed to consider the correlation between them, and then MEWMA control chart is applied to monitor the statistic. The performance of the proposed method is evaluated in comparison with the traditional methods by using simulation studies in terms of average run length criterion. In addition, the performance of the proposed method is evaluated through a real case in the electronic industry. This method can be easily extended to the correlated general linear profiles and variables, or attributes quality characteristics.
Acknowledgements

The authors are thankful to the anonymous reviewer for his precious comments which led to improvement in the paper. Dr. Amiri’s research is partially supported by a grant from Shehed University. Dr. Zou’s research was supported by the NNSF of China Grants 11001138, 11071128, 11131002, 11101306, the RFDP of China Grant 2011031110002, the Fundamental Research Funds for the Central Universities, and the PAPD of Jiangsu Higher Education Institutions.

References

## Appendix A

### Table A1. The 43 samples of correlated profile and multivariate quality characteristics

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_3$</td>
<td>16.393</td>
<td>17.102</td>
<td>15.866</td>
<td>15.092</td>
<td>15.111</td>
<td>16.133</td>
<td>15.034</td>
<td>15.623</td>
</tr>
<tr>
<td>$n_7$</td>
<td>32.856</td>
<td>32.414</td>
<td>31.15</td>
<td>31.298</td>
<td>31.132</td>
<td>29.669</td>
<td>31.687</td>
<td>31.58</td>
</tr>
<tr>
<td>$n_8$</td>
<td>35.307</td>
<td>35.765</td>
<td>36.203</td>
<td>35.548</td>
<td>35.281</td>
<td>36.237</td>
<td>35.334</td>
<td>35.839</td>
</tr>
<tr>
<td>$n_9$</td>
<td>41.295</td>
<td>41.292</td>
<td>40.813</td>
<td>40.178</td>
<td>40.049</td>
<td>41.49</td>
<td>40.955</td>
<td>41.225</td>
</tr>
<tr>
<td>$n_{10}$</td>
<td>45.604</td>
<td>47.96</td>
<td>47.214</td>
<td>44.382</td>
<td>46.137</td>
<td>46.486</td>
<td>46.532</td>
<td>45.908</td>
</tr>
</tbody>
</table>

### Variables

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>1</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_1$</td>
<td>9.565</td>
<td>-0.735</td>
<td>-0.903</td>
<td>-0.921</td>
<td>-0.936</td>
<td>-0.921</td>
<td>-0.871</td>
<td>-0.843</td>
</tr>
<tr>
<td>$n_2$</td>
<td>-0.965</td>
<td>-0.827</td>
<td>-0.805</td>
<td>-0.873</td>
<td>-0.87</td>
<td>-0.876</td>
<td>-0.845</td>
<td>-0.832</td>
</tr>
</tbody>
</table>

### Variables

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>1</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_7$</td>
<td>31.595</td>
<td>32.221</td>
<td>31.731</td>
<td>30.747</td>
<td>30.272</td>
<td>31.165</td>
<td>30.496</td>
<td>30.581</td>
</tr>
<tr>
<td>$n_8$</td>
<td>35.626</td>
<td>35.401</td>
<td>34.696</td>
<td>36.678</td>
<td>35.167</td>
<td>35.064</td>
<td>36.794</td>
<td>34.73</td>
</tr>
<tr>
<td>$n_9$</td>
<td>40.706</td>
<td>42.003</td>
<td>41.587</td>
<td>40.694</td>
<td>40.743</td>
<td>41.244</td>
<td>40.81</td>
<td>41.81</td>
</tr>
<tr>
<td>$n_{10}$</td>
<td>45.931</td>
<td>44.616</td>
<td>45.415</td>
<td>46.34</td>
<td>47.768</td>
<td>45.495</td>
<td>47.144</td>
<td>45.876</td>
</tr>
</tbody>
</table>

### Variables

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>1</th>
<th>2</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_1$</td>
<td>-0.948</td>
<td>-0.905</td>
<td>-0.93</td>
<td>-0.981</td>
<td>-0.907</td>
<td>-0.913</td>
<td>-0.924</td>
<td>-0.822</td>
</tr>
</tbody>
</table>

### Variables

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>1</th>
<th>2</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_6$</td>
<td>26.635</td>
<td>28.6</td>
<td>28.03</td>
<td>28.466</td>
<td>29.298</td>
<td>27.01</td>
<td>27.717</td>
<td>27.253</td>
</tr>
<tr>
<td>$n_7$</td>
<td>32.681</td>
<td>30.65</td>
<td>32.538</td>
<td>32.572</td>
<td>39.002</td>
<td>31.98</td>
<td>34.008</td>
<td>30.67</td>
</tr>
<tr>
<td>$n_8$</td>
<td>35.02</td>
<td>34.709</td>
<td>35.622</td>
<td>38.584</td>
<td>48.447</td>
<td>39.388</td>
<td>42.299</td>
<td>34.793</td>
</tr>
<tr>
<td>$n_9$</td>
<td>40.613</td>
<td>40.059</td>
<td>41.86</td>
<td>43.027</td>
<td>59.194</td>
<td>45.731</td>
<td>47.868</td>
<td>41.178</td>
</tr>
<tr>
<td>$n_{10}$</td>
<td>47.389</td>
<td>48.412</td>
<td>45.615</td>
<td>52.827</td>
<td>70.527</td>
<td>52.001</td>
<td>56.942</td>
<td>47.273</td>
</tr>
</tbody>
</table>

(Continues)
Authors’ biographies

Amirhossein Amiri is an assistant professor at Shahed University of Iran. He holds a BS, an MS, and a PhD in Industrial Engineering from Khajeh Nasir University of Technology, Iran University of Science and Technology, and Tarbiat Modares University, respectively. He is a member of the Iranian Statistical Association. His research interests are statistical quality control, profile monitoring, and Six Sigma.

Changliang Zou is an associate professor at the Department of Statistics, Nankai University. He obtained his BS, MS, and PhD degrees in statistics from the School of Mathematical Sciences, Nankai University, in 2003, 2006, and 2008, respectively. He worked as post-doctoral researcher at Quality Lab, Department of IELM, Hong Kong University of Science and Technology. His research interests include statistical process control and quality improvement, nonparametric regression, and dimension reduction. Special focuses are profile monitoring, multivariate statistical process control, semiparametric modeling, and nonparametric lack of fit tests. He has authored more than 30 refereed journal publications in *JASA*, *Annals of Statistics*, *Technometrics*, *Journal of Quality Technology*, *Statistics Sinica*, *IIE Transactions*, *Annals of Operations Research*, and other technical journals.

Mohammad Hadi Doroudyan is a PhD student in Industrial Engineering at Yazd University in Iran. He holds MS degree in Industrial Engineering from Shahed University and BS degree in the same field from Islamic Azad University, South Tehran Branch. His research interests are statistical quality control and design of experiments.