Research Article

Quality and Reliability Engineering International

(wileyonlinelibrary.com) DOI: 10.1002/qre.1483

Published online 28 January 2013 in Wiley Online Library

Monitoring Correlated Profile and Multivariate Quality Characteristics

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Monitoring multivariate quality characteristics is very common in production and service environment. Therefore, many control charts have been suggested by authors for monitoring multivariate processes. In another side, profile monitoring is a new approach in the area of statistical process control. In this approach, the quality of a product or a process is characterized by a relation between one response variable and one or more independent variables. In practice, sometimes the quality of a product or a process is represented by a correlated profile and multivariate quality characteristics. To the best of our knowledge, there is no method for monitoring this type of quality characteristics. Note that monitoring correlated profile and multivariate quality characteristics separately leads to misleading results. In this article, we specifically focus on correlated simple linear profile and multivariate normal quality characteristics and propose a method using multivariate exponentially weighted moving average control chart to monitor the correlated profile and multivariate quality characteristics simultaneously. The performance of the proposed control chart is evaluated by simulation studies in terms of average run length criterion. Finally, the proposed method is applied to a real case in the electronics industry. Copyright © 2013 John Wiley & Sons, Ltd.

Keywords: statistical process control; simple linear profile; correlated profile and multivariate quality characteristics; MEWMA control chart; phase II

1. Introduction

Nowadays, the quality of many products or processes is represented by two or more correlated quality characteristics. Hotelling[1] showed that monitoring multivariate quality characteristics separately leads to misleading in results. He proposed T^2 control chart for monitoring multivariate quality characteristics. Multivariate exponentially weighted moving average (MEWMA)[2] and multivariate cumulative sum (MCUSUM)[3] control charts are the other most common multivariate control charts. Reviews of the most usual methods in multivariate process monitoring have been performed by several authors such as Basseville and Nikiforov,[4] Ryan,[5] Frisen,[6] Sonesson and Frisen,[7] Bersimis *et al.*,[8] and Frisen.[9] Recently, a new procedure was proposed by Butte and Tang[10] in some common multivariate control charts to facilitate the identification of the source of out-of-control signal. Kim *et al.*[11] proposed a non-parametric fault isolation approach based on a one-class classification algorithm and showed that their proposed method can detect source of variation better than T^2 decomposition in the presence of nonnormal processes. Some kinds of variable sampling rate in multivariate control charts, which lead to overall better performance rather than standard fixed sampling rate, were proposed by Reynolds and Cho.[12] The applications of multivariate control charts in health care were studied by Waterhouse *et al.*[13]

Sometimes, the quality of a product or a process is characterized by a relation between a response variable and one or more independent variables, which is called profile. The most common type of profile is a simple linear profile in which a response variable has a linear relation with an explanatory variable. Simple linear profile monitoring was first investigated by Kang and Albin[14] via proposing two approaches including T^2 and EWMA/R. Then, Kim *et al.*[15] proposed EWMA-3, Mahmoud and Woodall[16] proposed an F method, Mahmoud *et al.*,[17] Zou *et al.*,[18] and Zhang *et al.*[19] suggested an LRT-based method, and Saghaie *et al.*[20] used CUSUM-3 to monitor simple linear profile. Other complicated profiles such as multiple linear profile, polynomial profile, nonlinear profile, logistic profile, and multivariate linear profiles were also investigated by the authors. For example, multiple linear regression profile was studied by Zou *et al.*[21] and Amiri *et al.*[22]. Kazemzadeh *et al.*[23,24] proposed some methods in phases I and II of monitoring polynomial profiles, respectively. Nonlinear profile was monitored by Williams *et al.*[25] and Vaghefi *et al.*[26]. Logistic profile was monitored by Yeh *et al.*,[27] and multivariate linear profile monitoring was investigated by Noorossana *et al.*[28,29], Eyvazian *et al.*,[30] and Zou *et al.*[31] These are some other issues considered in the literature of profile monitoring. In addition, Woodall *et al.*[32] and Woodall[33] reviewed common methods in profile monitoring. In addition, Noorossana *et al.*[34] recently summarized major achievements in the area of profile monitoring.

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In practice, sometimes the quality of a product or a process could be represented by a correlated profile and multivariate quality characteristics. For example, in some plastic manufacturing companies, the quality of a product or raw material can be characterized by a relationship between the hardness of a product as response variable and the distance from injection point as an explanatory variable in a simple linear regression profile. In addition, the weight of product has a correlation with the hardness of a product. As another example, in an aluminum electrolytic capacitor (AEC) manufacturing process, the quality characteristics of unfinished AEC product such as the capacitance and loss tangent are monitored in each stage. Moreover, the relationship between the quality characteristics of an AEC from one stage to another stage is described by a simple linear profile. Besides the monitoring of those important characteristics, the engineers are often rather concerned about the significant changes in such a profile. To the best of our knowledge, there is no method in monitoring this type of quality characteristics.

In this article, we specifically focus on correlated simple linear profile and multivariate quality characteristics with multivariate normal distribution. This research is applied in phase II; hence, the distribution parameters of quality characteristics are known. In this article, we aim to monitor sample intercept and slope as parameters of the profile as well as multivariate quality characteristics. As a result, the control statistic consists of sample intercept and slope and sample variables, which should be monitored simultaneously. Therefore, the correlation coefficient between variables and sample intercept and slope are calculated based on statistical methods. Finally, MEWMA control chart is used to monitor mean vector. The performance of the proposed method is evaluated by using simulation studies in terms of average run length criterion.

The structure of the article is as follows. In Section 2, the problem is defined. Section 3 illustrates the proposed method for monitoring correlated profile and multivariate quality characteristics. In Section 4, the performance of the proposed method is evaluated using simulation studies. The application of the problem is shown in Section 5 by introducing a real case. Our concluding remarks as well as some future suggestion are given in Section 6.

2. Problem definition

In some statistical process control applications, the quality of a product or a process can be represented by a vector of correlated variables such as $\mathbf{w} = (w_1 \quad w_2 \quad \ldots \quad w_p)^T$, or a relation between a response variable and one or more independent variables, such as a simple linear profile. In this research, we face a combination of correlated profile and multivariate quality characteristics. We specifically focus on correlated simple linear profile and multivariate quality characteristics. In the simple linear profile $z_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, the intercept ($\beta_0$) and the slope ($\beta_1$) are parameters and x_i's ($i = 1, 2 \ldots, n$) are the values of explanatory variables. Meanwhile, error terms (ε_i's) are independent and have normal distribution with mean zero and variance σ^2. We assume that multivariate quality characteristics $\mathbf{y} = (y_1 \quad y_2 \quad \ldots \quad y_m)^T$ follow multivariate normal distribution with mean vector μ and covariance matrix $\mathbf{\Sigma}$. The important issue in this article is the definition of correlation between profile and multivariate quality characteristics. The source of this correlation is the covariance between each level of profile (z_i) and each variable quality characteristic (y_j) as σ_{ij}, and all of them form the covariance matrix $\mathbf{\Sigma}_{zy}$ with size $n \times m$. This research is performed in phase II; hence, the parameters of profile including the values of intercept (β_0), slope (β_1), variance (σ^2) of error terms (ε_i's) and x_i's values are assumed to be known. Likewise, the value of mean vector (μ) and covariance matrix ($\mathbf{\Sigma}$) of multivariate normal quality characteristics as well as covariance matrix between different levels of profile and variables ($\mathbf{\Sigma}_{zy}$) will be known. In this article, we aim to monitor the mean vector. Therefore, the covariance matrix is assumed to be stable over time, and assignable causes just may lead to change in the mean vector of distribution of quality characteristics.

In simple linear profile monitoring, Kang and Albin[14] proposed two strategies including monitoring sample intercept $\left(\hat{\beta}_0\right)$ and slope $\left(\hat{\beta}_1\right)$ in each observation, or residuals that are differences between the reference line and the sample profile. In this article, we apply first strategy. Thus, to monitor a simple linear profile, one can estimate the values of intercept $\left(\hat{\beta}_0\right)$ and slope $\left(\hat{\beta}_1\right)$ in each sample using the least square method, which can be seen in Hines and Montgomery.[35] Equations (1) and (2) show these estimations as follows:

$$\hat{\beta}_0 = \bar{z} - \hat{\beta}_1 \bar{x}, \tag{1}$$

$$\hat{\beta}_1 = \frac{S_{xz}}{S_{xx}}, \tag{2}$$

where $\bar{z} = n^{-1}\sum_{i=1}^{n} z_i, \bar{x} = n^{-1}\sum_{i=1}^{n} x_i$, $S_{xx} = \sum_{i=l}^{n} (X_i - \bar{X})^2$ and $S_{xz} = \sum_{i=l}^{n} Z_i(x_i - \bar{x})$, respectively. The sample of $\hat{\beta}_0$ and $\hat{\beta}_1$ follow normal distribution with mean β_0 and β_1, respectively, and variances given in Equations (3) and (4), respectively,

$$\sigma^2_{\hat{\beta}_0} = \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right) \tag{3}$$

$$\sigma^2_{\hat{\beta}_1} = \sigma^2\left(\frac{1}{S_{xx}}\right) \tag{4}$$

Moreover, the covariance between sample intercept and slope are calculated by Equation (5) as follows:

$$Cov\left(\hat{\beta}_0, \hat{\beta}_1\right) = -\sigma^2 \frac{\bar{x}}{S_{xx}} \tag{5}$$

Despite of numerous studies in the field of profile and multivariate monitoring separately, to the best of our knowledge, there is no research in the case of correlated profile and multivariate quality characteristics. Therefore, in the next section, we propose a method to monitor this type of quality characteristics.

3. Proposed method

To observe correlated simple linear profile and multivariate quality characteristics, we propose a control statistic consists of sample intercept and slope as parameters of the simple linear profile and sample variables. Equation (6) shows the control statistic as follows:

$$\mathbf{w} = \begin{pmatrix} \hat{\beta}_0 & \hat{\beta}_1 & y_1 & y_2 & \dots & y_m \end{pmatrix}^T \tag{6}$$

Because the simple linear profile and the multivariate quality characteristics are correlated, we obtain the covariance matrix of the control statistic based on statistical methods. Then, we define

$$c_i = \frac{x_i - \bar{x}}{S_{xx}} \tag{7}$$

$$d_i = \frac{1}{n} - \bar{x}c_i \tag{8}$$

Therefore,

$$\hat{\beta}_1 = \frac{S_{xz}}{S_{xx}} = \frac{\sum_{i=1}^{n} z_i(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sum_{i=1}^{n} z_i \left(\frac{x_i - \bar{x}}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right) = \sum_{i=1}^{n} z_i c_i \tag{9}$$

$$\hat{\beta}_0 = \bar{z} - \hat{\beta}_1 \bar{x} = \frac{1}{n}\sum_{i=1}^{n} z_i - \bar{x}\sum_{i=1}^{n} z_i c_i = \sum_{i=1}^{n}\frac{1}{n}(z_i) - \bar{x}z_i c_i = \sum_{i=1}^{n} z_i\left(\frac{1}{n} - \bar{x}c_i\right) = \sum_{i=1}^{n} z_i d_i \tag{10}$$

Hence, the covariance between sample intercept and variables is obtained using Equation (11), and the covariance between sample slope and variables is obtained using Equation (12), as follows:

$$\mathrm{Cov}\left(\hat{\beta}_0, y_j\right) = \mathrm{Cov}\left(\sum_{i=1}^{n} d_i z_i, y_j\right) = \sum_{i=1}^{n}\mathrm{Cov}\left(d_i z_i, y_j\right) = \sum_{i=1}^{n} d_i \mathrm{Cov}\left(z_i, y_j\right) = \sum_{i=1}^{n} d_i \sigma_{ij} \tag{11}$$

$$\mathrm{Cov}\left(\hat{\beta}_1, y_j\right) = \mathrm{Cov}\left(\sum_{i=1}^{n} c_i z_i, y_j\right) = \sum_{i=1}^{n}\mathrm{Cov}\left(c_i z_i, y_j\right) = \sum_{i=1}^{n} c_i \mathrm{Cov}\left(z_i, y_j\right) = \sum_{i=1}^{n} c_i \sigma_{ij} \tag{12}$$

Thereupon, vector w follows multivariate normal distribution with mean vector and covariance matrix as follows:

$$\boldsymbol{\mu}_{\mathbf{w}} = \begin{pmatrix} \beta_0 & \beta_1 & \mu_1 & \mu_2 & \dots & \mu_m \end{pmatrix}^T \tag{13}$$

$$\boldsymbol{\Sigma}_{\mathbf{w}} = \begin{pmatrix} \mathrm{var}(\beta_0) & \mathrm{cov}(\beta_0,\beta_1) & \mathrm{cov}(\beta_0,y_1) & \mathrm{cov}(\beta_0,y_2) & \cdots & \mathrm{cov}(\beta_0,y_m) \\ \mathrm{cov}(\beta_1,\beta_0) & \mathrm{var}(\beta_1) & \mathrm{cov}(\beta_1,y_1) & \mathrm{cov}(\beta_1,y_2) & \cdots & \mathrm{cov}(\beta_1,y_m) \\ \mathrm{cov}(y_1,\beta_0) & \mathrm{cov}(y_1,\beta_1) & \mathrm{var}(y_1) & \mathrm{cov}(y_1,y_2) & \cdots & \mathrm{cov}(y_1,y_m) \\ \mathrm{cov}(y_2,\beta_0) & \mathrm{cov}(y_2,\beta_1) & \mathrm{cov}(y_2,y_1) & \mathrm{var}(y_2) & \cdots & \mathrm{cov}(y_2,y_m) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(y_m,\beta_0) & \mathrm{cov}(y_m,\beta_1) & \mathrm{cov}(y_m,y_1) & \mathrm{cov}(y_m,y_2) & \cdots & \mathrm{var}(y_m) \end{pmatrix} \tag{14}$$

Let us take a more careful look at this correlation from statistical point. Under the assumption, $z_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ and x_i's are the fixed (not random as in our assumption) design point. Hence,

$$\begin{aligned} \mathrm{Cov}\left(z_i, y_j\right) &= \mathrm{Cov}\left(\beta_0 + \beta_1 x_i + \varepsilon_i, y_j\right) \\ &= \mathrm{Cov}\left(\beta_0 + \beta_1 x_i, y_j\right) + \mathrm{Cov}\left(\varepsilon_i, y_j\right) = 0 + \mathrm{Cov}\left(\varepsilon_i, y_j\right) \end{aligned} \tag{15}$$

Therefore, the covariance between z_i and y_j is just that between ε_i and y_j. We always assume that ε_i's are independent identically distributed (i.i.d.), and thus it is maybe unreasonable to make an assumption that $\mathrm{Cov}(\varepsilon_i, y_j) \neq \mathrm{Cov}(\varepsilon_k, y_j)$ for $i \neq k$ except for the case that

one sometimes $\varepsilon_i \equiv \varepsilon_i(x_i)$ (the distribution of ε_i may depend on x_i). Thus, it is possible to impose an assumption that $\mathrm{Cov}(\varepsilon_i, y_j)$ is constant for each variable quality characteristic (y_j), say $\mathrm{Cov}(\varepsilon_i, y_j) = \sigma_{zj}$. Accordingly, $\mathrm{Cov}(z_i, y_j) = \sigma_{zj}$ and since $\sum_{i=l}^{n} C_i = 0$ and $\sum_{i=1}^{n} d_i = 1$

$$\mathrm{Cov}\left(\hat{\beta}_0, y_j\right) = \sum_{i=1}^{n} d_i \mathrm{Cov}\left(z_i, y_j\right) = \sum_{i=1}^{n} d_i \sigma_{ij} = \sigma_{zj} \sum_{i=1}^{n} d_i = \sigma_{zj} \tag{16}$$

$$\mathrm{Cov}\left(\hat{\beta}_1, y_j\right) = \sum_{i=1}^{n} c_i \mathrm{Cov}\left(z_i, y_j\right) = \sum_{i=1}^{n} c_i \sigma_{ij} = \sigma_{zj} \sum_{i=1}^{n} c_i = 0 \tag{17}$$

The values of correlations obtained from Equations (11) and (12) are σ_{zj} and 0, respectively. Thus, the given covariance matrix in Equation (14) is changed to Equation (18), in which $\boldsymbol{\sigma}_{zy} = \begin{pmatrix} \sigma_{z1} & \sigma_{z2} & \dots & \sigma_{zm} \end{pmatrix}^T$ and $\mathbf{0}$ is an m-dimensional zero vector.

$$\boldsymbol{\Sigma}_{\mathbf{w}} = \begin{pmatrix} \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right) & -\sigma^2 \frac{\bar{x}}{S_{xx}} & \boldsymbol{\sigma}_{zy} \\ -\sigma^2 \frac{\bar{x}}{S_{xx}} & \sigma^2\left(\frac{1}{S_{xx}}\right) & 0 \\ \boldsymbol{\sigma}_{zy}{}^T & 0^T & \boldsymbol{\Sigma} \end{pmatrix} \tag{18}$$

Then, the use of the MEWMA control chart by Lowry *et al.*[2] is suggested. The control statistic in MEWMA is as follows:

$$T_i = \mathbf{v}_i^T \sum{}^{-1} \mathbf{v}_i \tag{19}$$

In this equation, $\mathbf{v}_i$ is the smoothed value of $\mathbf{w}_i$, and obtained from:

$$\mathbf{v}_i = \lambda(\mathbf{w}_i - \boldsymbol{\mu}_w) + (1 - \lambda)\mathbf{v}_{i-l} \tag{20}$$

where λ is a smoothing parameter (chosen such a way that $0 < \lambda \leq 1$) and $\mathbf{v}_0$ is an $m+2$-dimensional zero vector. Likewise, the steady-state covariance matrix of control statistic is equal to:

$$\sum \frac{\lambda}{2 - \lambda} \sum{}_{\mathbf{w}} \tag{21}$$

The upper control limit (UCL) for the MEWMA statistic in Equation (19) is obtained by using a simulation study such that a desired in-control ARL is achieved. In addition, based on a signal rule, the control chart signals whenever the control statistic exceeds the UCL.

To compare the proposed method with the traditional methods, one may consider a scheme that uses two independent control charts simultaneously for monitoring the location parameters of simple linear profile and multivariate quality characteristics. This method surely neglects the correlation between profile and multivariate quality characteristics. Thus, it leads to poor performance of the procedure. To show this reality, in simulation study, we consider using an MEWMA control chart to monitor the intercept and slope estimators in combination with an MEWMA control chart to monitor the mean of variable quality characteristics and to compare the results of this method with the proposed method.

4. Simulation study

In this section, the performance of the proposed method is evaluated by using simulation studies through a numerical example in terms of average run length criterion. In this example, the quality of a product or a process is represented by a simple linear relation between a response variable (z_i) and one explanatory variable (x_i) as a simple linear profile as well as a variable quality characteristic. Note that in this example, because of simplicity and without loss of generality, only one variable quality characteristic is considered.

The relation of the simple linear profile is $z_i = 3 + 2x_i + \varepsilon_i$, where the x_i values are equal to 2, 4, 6, and 8. Also, error terms (ε_i' s) in the profile are independent and follow normal distribution with mean zero and variance one. Meanwhile, the variable quality characteristic is normally distributed with mean 0 and variance 1. Moreover, the response variable (z_i) in different levels of the explanatory variable (x_i) has correlation with the variable quality characteristic (y). The covariance between the response variable in different levels of the explanatory variable and the variable quality characteristic is the same and equal to 0.35.

As mentioned previously, in the traditional method, profile and multivariate quality characteristics are monitored separately. Therefore, an MEWMA control chart is designed to monitor sample intercept and slope of the profile. The variable quality characteristic is also monitored using an EWMA control chart. We set the smoothing parameters in both control charts equal to 0.2. Also, the control limits of the MEWMA and EWMA control charts are determined equal to 11 and ± 0.95, respectively, using 10000 simulation runs such that the overall ARL_0 becomes equal to 200.

On the basis of the proposed method, the control statistic is a vector consisting of sample intercept, slope, and variable quality characteristic as $\mathbf{w}_i = \begin{pmatrix} \hat{\beta}_0 & \hat{\beta}_1 & y \end{pmatrix}^T$. This statistic has multivariate normal distribution with mean vector $\boldsymbol{\mu}_{\mathbf{w}} = \begin{pmatrix} 3 & 2 & 0 \end{pmatrix}^T$. To compute the covariance matrix of the statistic, variances of sample intercept and slope as well as their covariance are calculated

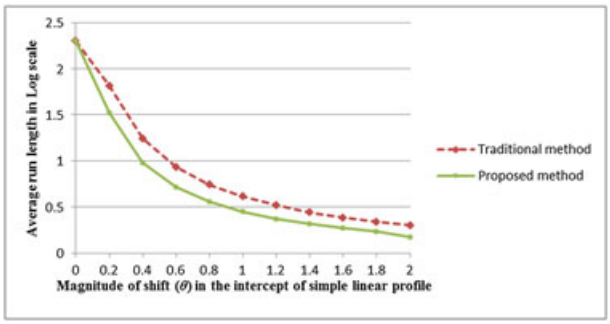

Figure 1. Average run length in log scale for different shifts in the intercept of simple linear profile from β_0 to $\beta_0+\theta\sigma$

based on Equations (3)–(5) equal to $\sigma^2_{\hat{\beta}_0}=1.5$, $\sigma^2_{\hat{\beta}_1}=0.05$, and $\mathrm{Cov}\left(\widehat{\beta}_0,\widehat{\beta}_1\right)=-0.25$, respectively. Furthermore, as explained in the previous section, covariance values between the sample intercept and slope of profile and the variable quality characteristic are equal to $\mathrm{Cov}\left(\widehat{\beta}_0,y\right)=0.35$ and $\mathrm{Cov}\left(\widehat{\beta}_1,y\right)=0$, respectively, based on Equations (16) and (17). Thus, the covariance matrix of the control statistic is as follows:

$$\mathbf{\Sigma_w}=\begin{pmatrix}1.5 & -0.25 & 0.35\\ -0.25 & 0.05 & 0\\ 0.35 & 0 & 1\end{pmatrix}$$

Then, an MEWMA control chart is designed to monitor the control statistic. Similar to the traditional method, the smoothing parameter is set equal to 0.2. Moreover, UCL is obtained equal to 11.875 using simulation study with 10,000 replications to achieve in-control average run length (ARL_0) equal to 200.

The performance of the proposed method in comparison with the traditional method is evaluated in terms of out-of-control average run length (ARL_1) criterion under different shift scenarios. These scenarios are step shifts in the intercept of the simple linear profile from β_0 to $\beta_0+\theta\sigma$, step shifts in the slope of the simple linear profile from β_1 to $\beta_1+\delta\sigma$, and step shifts in the mean of variable quality characteristic from μ_y to $\mu_y+\gamma\sigma_y$. The ARL_1 values of both methods under different shifts are computed by 10000 simulation runs and the average run length curves of both methods under different shifts in the intercept and the slope of simple linear profile, and the mean values of variable quality characteristics are depicted in Figures 1–3 in log scale, respectively. The ARL values for these

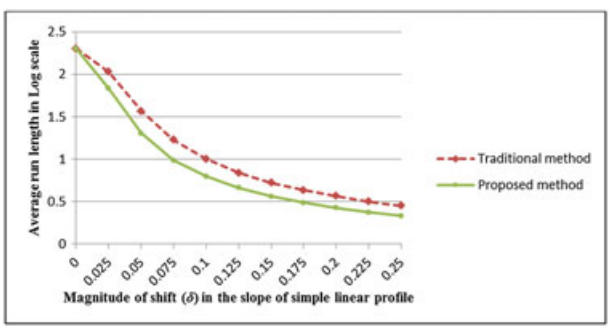

Figure 2. Average run length in log scale for different shifts in the slope of simple linear profile from β_1 to $\beta_1+\delta\sigma$

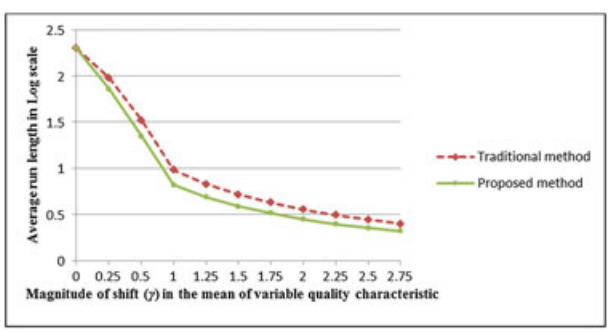

Figure 3. Average run length in log scale for different shifts in the mean of variable quality characteristic from μ_y to $\mu_y+\gamma\sigma_y$

three figures are available from authors upon request. The results reveal that the proposed method performs uniformly better than the combined EWMA–MEWMA control chart, which neglects the correlation between profile and variable quality characteristic.

5. Case study

In this section, we apply the proposed method to a real data set from an AEC manufacturing process (provided by ENW Electronics Ltd; see Figure 4) discussed generally in the introduction section. As shown in Figure 4, the entire manufacturing process, which is a typical multistage process (cf. Shi[36]), includes a sequence of operations, such as clenching, rolling, soaking, assembly, cleaning, aging, and classifying. The aim of the process is to transform the raw materials (anode aluminum foil, cathode aluminum foil, guiding pin, electrolyte sheet, plastic cover, aluminum shell, and plastic tube) into AECs with certain specifications. Many types of defects may be induced through the manufacturing process, which can be classified into cosmetic defects, capacity defects, leakage current defects, and dissipation factor defects. Several types of defects may be induced in each stage. For example, in the clenching and rolling stage, the possible defects

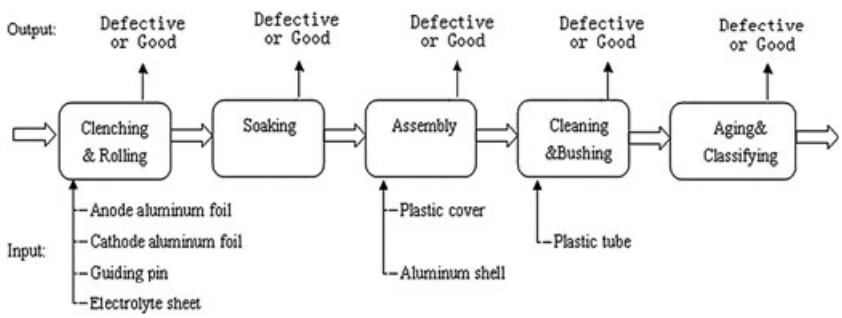

Figure 4. The manufacturing process for AECs

include improper compound thickness, aluminum foil cracking, scored aluminum foil, ragged margins, and so forth. In the soaking stage, dissipation factor defects may be found. Therefore, the quality character of AEC representing its appearance condition and functional performance condition at upstream stages will influence that of downstream stages, and then the different stages are correlated.

To this end, the quality of unfinished AEC products, which are called capacitor elements, in terms of appearance and functional performance, is inspected by sampling after each stage. At each stage, certain important characteristics of an AEC, such as the capacitance and loss tangent (or equivalently dissipation factor), are automatically calibrated by an electronic device at some given measuring voltage, frequency, and temperature. The relationship between the characteristics of an AEC from one stage to another stage can often be described by linear models as demonstrated in the literature (cf. Shi[36]). Besides the monitoring of those important quality characteristics, the engineers are often rather concerned about significant changes in such relationships (or profiles), which may indicate some assignable causes occurred in the process.

In this real case, we consider dissipation factor values in the aging stage as the response variable (z_i) and one input variable (x_i) from the soaking stage as explanatory variable. Simultaneously, the values of capacitance and dissipation factor observations (denoted as y_1 and y_2) from the soaking stage are monitored. The data set comprises 243 profile samples of size $n = 10$. Among them, 16 profiles are classified as inferior profiles based on the physical knowledge and experience of the engineers. We first use 200 correlated profiles and multivariate quality characteristics as the historical sample to calibrate the necessary parameters in phase I. Then, we monitor the other 43 samples in phase II based on the estimated parameters from phase I analyses.

Phase I analysis confirmed the adequacy of simple linear profile as $z_i = -758.92 + 200.81x_i + \varepsilon_i$ in which error terms (ε_i's) are independent and has normal distribution with mean zero and variance 2.934. Note that the x_i values are 3.82, 3.84, 3.86, 3.88, 3.90, 3.92, 3.94, 3.96, 3.98, and 4. In the profile, the values of -758.92, 200.81, and 2.934 are estimated by using equations $\bar{\hat{\beta}}_0 = \frac{1}{200}\sum_{j=1}^{200}\hat{\beta}_{0j}$, $\bar{\hat{\beta}}_1 = \frac{1}{200}\sum_{j=1}^{200}\hat{\beta}_{1j}$, and $\hat{\sigma}^2 = \frac{1}{200}\sum_{j=1}^{200}\mathrm{MSE}_j$, respectively.

In addition, variable quality characteristics follow multivariate normal distribution. The mean vector and the covariance matrix of the variable quality characteristics are, respectively, estimated by $\hat{\boldsymbol{\mu}}_{\mathbf{y}} = \frac{1}{200}\sum_{j=1}^{200}\mathbf{y}_j$ and $\widehat{\Sigma}_{\mathbf{y}} = \frac{1}{199}\sum_{j=1}^{200}\left(\mathbf{y}_j - \boldsymbol{\mu}_{\mathbf{y}}\right)\left(\mathbf{y}_j - \boldsymbol{\mu}_{\mathbf{y}}\right)^T$ equal to

$$\boldsymbol{\mu}_{\mathbf{y}} = \begin{pmatrix} -0.8989 & -2.0734 \end{pmatrix}^T$$

$$\boldsymbol{\Sigma}_{\mathbf{y}} = \begin{pmatrix} 0.0031 & -0.0001 \\ -0.0001 & 0.0065 \end{pmatrix}$$

Moreover, the covariance between all levels of profile (z_i) and the variable quality characteristics (y_1 and y_2) are roughly equal to $\mathrm{Cov}(z_i, y_1) = \sigma_{z1} = 0.272$ and $\mathrm{Cov}(z_i, y_2) = \sigma_{z2} = 0.350$, respectively. Note that these covariances are estimated by Equation (22) as follows:

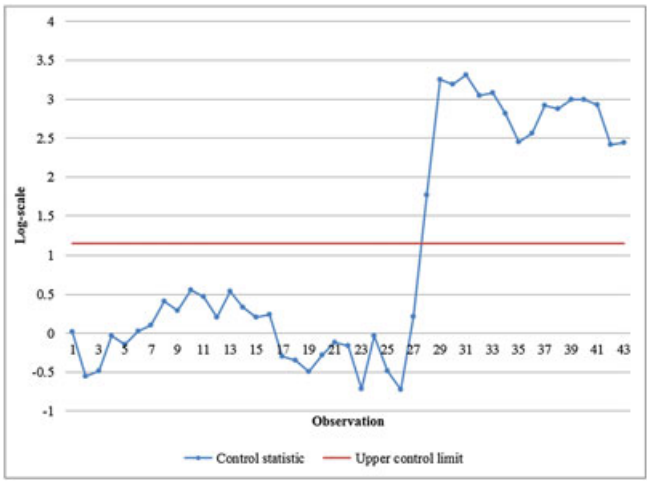

Figure 5. MEWMA control chart for correlated profile and multivariate quality characteristics

$$\widehat{\mathrm{COV}}(z_i, y_k) = \frac{1}{199}\sum_{j=l}^{200}\left(z_{ij} - \bar{z}_i \quad y_{kj} - \bar{y}_k\right)\ \left(z_{ij} - \bar{z}_i \quad y_{kj} - \bar{y}_k\right)T;$$

$$\forall\, i = 1, 2 \ldots, 10 \text{ and } \forall k = 1, 2, \tag{22}$$

where $\bar{z}_i = \frac{1}{200}\sum_{j=1}^{200} z_{ij}$; $\forall\, i = 1, 2, \ldots, 10$.

On the basis of the proposed method, the control statistic is a vector consisting of sample intercept, slope, and variable quality characteristics as $w_i = \left(\widehat{\beta}_0 \quad \widehat{\beta}_1 \quad y_1 \quad y_2\right)^T$. This statistic has multivariate normal distribution with mean vector and covariance matrix as follows:

$$\boldsymbol{\mu}_{\mathbf{w}} = \left(-758.92 \quad 200.81 \quad -0.8989 \quad -2.0734\right)^T$$

$$\boldsymbol{\Sigma}_{\mathbf{w}} = \begin{pmatrix} 1359.5 & -347.63 & 0.272 & 0.350 \\ -347.63 & 88.909 & 0 & 0 \\ 0.272 & 0 & 0.0031 & -0.0001 \\ 0.350 & 0 & -0.0001 & 0.0065 \end{pmatrix}$$

Then, an MEWMA control chart is designed to monitor the mean vector of statistic. Hence, we set smoothing parameter equal to 0.2 and determine UCL by using simulation study equal to 13.874 for $\mathrm{ARL}_0 = 200$. In phase II, for each 43 samples (given in Appendix A), the intercept and slope of each profile are estimated using least square method and a vector of size 4 alongside variable quality characteristics y_1 and y_2 is formed. On the basis of this sample vector, the MEWMA control statistic for each sample is calculated by using Equation ((19)) and plotted on the control chart shown in Figure 5 in log scale. Note that $\mathbf{v}_0$ is set equal to zero vector as the initial vector of the MEWMA control chart statistic.

As illustrated in Figure 5, the first 27 samples are the in-control state. However, the values of control statistics for the other 16 samples are higher than UCL. On the basis of the signal rule, these samples are in the out-of-control state. This state is confirmed by quality engineers as well. Therefore, the results show that the proposed method can distinguish the in-control and out-of-control states correctly.

6. Conclusions and future studies

Multivariate quality characteristics and profile monitoring are widely applied in production and service environment and investigated by authors, separately. In this article, a method for monitoring correlated simple linear profile and multivariate quality characteristics was proposed. First, a multivariate statistic is proposed to consider the correlation between them, and then MEWMA control chart is applied to monitor the statistic. The performance of the proposed method is evaluated in comparison with the traditional methods by using simulation studies in terms of average run length criterion. In addition, the performance of the proposed method is evaluated through a real case in the electronic industry. This method can be easily extended to the correlated general linear profiles and variables, or attributes quality characteristics.

Acknowledgements

The authors are thankful to the anonymous reviewer for his precious comments which led to improvement in the paper. Dr. Amiri's research is partially supported by a grant from Shehed University. Dr. Zou's research was supported by the NNSF of China Grants 11001138, 11071128, 11131002, 11101306, the RFDP of China Grant 20110031110002, the Fundamental Research Funds for the Central Universities, and the PAPD of Jiangsu Higher Education Institutions.

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Appendix A

Table A1. The 43 samples of correlated profile and multivariate quality characteristics

Sample no.		1	2	3	4	5	6	7	8
Profile	n_1	9.565	10.383	10.189	10.684	10.181	10.858	10.435	11.867
	n_2	13.361	13.647	12.215	14.99	13.33	13.28	12.029	12.157
	n_3	16.393	17.102	15.866	15.092	15.111	16.133	15.034	15.623
	n_4	19.867	18.396	19.562	18.452	19.984	20.959	19.816	19.159
	n_5	24.249	22.813	22.164	22.473	22.501	24.473	24.278	23.714
	n_6	24.408	26.019	27.734	25.59	26.006	27.57	26.789	27.178
	n_7	32.856	32.414	31.15	31.298	31.132	29.669	31.687	31.58
	n_8	35.307	35.765	36.203	35.548	35.281	36.237	35.334	35.839
	n_9	41.295	41.292	40.813	40.178	40.049	41.49	40.955	41.225
	n_{10}	45.604	47.96	47.214	44.382	46.137	46.486	46.532	45.908
Variables	y_1	−0.735	−0.903	−0.921	−0.936	−0.921	−0.871	−0.843	−0.866
	y_2	−1.919	−2.031	−2.12	−1.9	−2.15	−2.19	−2.093	−2.157
Sample no.		9	10	11	12	13	14	15	16
Profile	n_1	11.24	9.754	11.878	10.232	10.406	11.723	11.002	9.862
	n_2	12.195	12.542	12.512	12.804	13.846	12.253	13.41	14.233
	n_3	16.59	16.787	15.031	15.035	15.59	16.953	15.714	17.12
	n_4	20.234	19.397	19.645	18.971	18.28	19.665	19.238	19.56
	n_5	23.639	22.991	22.5	21.665	21.792	22.114	22.127	21.645
	n_6	26.862	26.435	25.235	28.21	26.752	30.603	26.425	27.247
	n_7	31.595	32.221	31.731	30.747	30.272	31.165	30.496	30.581
	n_8	35.626	35.401	34.696	36.678	35.167	35.064	36.794	34.73
	n_9	40.706	42.003	41.587	40.694	40.743	41.244	40.81	41.81
	n_{10}	45.931	44.616	45.415	46.34	47.768	45.495	47.144	45.876
Variables	y_1	−0.955	−0.872	−0.805	−0.873	−0.87	−0.876	−0.845	−0.832
	y_2	−2.13	−2.188	−1.953	−2.04	−2.202	−2.001	−2.022	−2.011
Sample no.		17	18	19	20	21	22	23	24
Profile	n_1	10.162	11.165	9.791	10.272	10.245	11.033	9.736	10.39
	n_2	12.91	13.28	13.794	12.044	13.248	13.417	12.815	12.516
	n_3	16.55	15.146	15.824	15.834	15.775	16.909	16.53	14.608
	n_4	18.906	18.531	18.975	19.022	20.283	18.059	19.277	17.552
	n_5	22.15	23.382	22.562	23.146	20.543	24.044	21.334	24.302
	n_6	26.345	26.485	27.258	27.921	26.483	26.484	26.691	27.059
	n_7	29.376	31.712	34.882	32.426	30.902	31.707	30.159	29.344
	n_8	36.373	36.603	35.414	36.349	35.731	35.376	35.905	36.915
	n_9	40.384	41.243	41.063	41.247	41.874	39.624	41.228	40.895
	n_{10}	47.225	46.299	48.003	47.84	47.038	46.791	45.57	44.71
Variables	y_1	−0.948	−0.905	−0.93	−0.981	−0.907	−0.913	−0.924	−0.822
	y_2	−1.99	−2.026	−2.063	−2.151	−2.037	−2.046	−2.161	−2.191
Sample no.		25	26	27	28	29	30	31	32
Profile	n_1	9.704	9.603	10.035	6.676	−12.505	4.198	1.475	10.827
	n_2	13.157	12.676	12.294	9.981	−6.475	9.671	4.478	14.623
	n_3	16.423	16.022	15.594	10.885	2.479	12.346	11.172	14.376
	n_4	19.025	20.069	19.287	16.7	11.357	16.18	15.919	18.803
	n_5	22.507	22.755	23.966	21.902	19.24	22.236	19.972	21.535
	n_6	26.635	28.6	28.03	28.466	29.298	27.01	27.717	27.253
	n_7	32.681	30.65	32.538	32.572	39.002	31.98	34.008	30.67
	n_8	35.02	34.709	35.622	38.584	48.447	39.388	42.299	34.793
	n_9	40.613	40.059	41.86	43.027	59.194	45.731	47.868	41.178
	n_{10}	47.389	48.412	45.615	52.827	70.527	52.001	56.942	47.273

(*Continues*)

Table A1. (Continued)

Variables	y_1	−1.004	−0.965	−0.853	−0.915	−0.969	−0.959	−0.858	−0.915
	y_2	−2.149	−2.121	−2.222	−3.136	−8.016	−2.922	−4.162	−1.52
Sample no.		33	34	35	36	37	38	39	40
Profile	n_1	3.264	11.445	13.611	6.994	0.588	5.77	4.606	5.4
	n_2	7.912	13.463	14.485	9.605	4.343	9.909	7.21	7.538
	n_3	11.541	17.38	16.344	13.407	9.477	14.795	11.376	14.117
	n_4	17.027	17.694	19.518	15.541	14.818	17.906	14.422	17.252
	n_5	21.581	22.039	22.498	21.344	20.961	21.406	21.295	21.581
	n_6	26.465	26.153	26.328	26.24	27.367	26.701	27.469	27.13
	n_7	33.088	31.437	31.201	31.672	34.101	33.182	33.047	32.88
	n_8	39.14	36.44	34.136	39.381	42.553	43.292	40.717	39.804
	n_9	46.978	41.497	39.652	46.556	49.488	44.785	47.465	43.797
	n_{10}	53.672	46.66	46.616	52.798	58.164	51.702	55.585	52.495
Variables	y_1	−0.975	−0.844	−0.803	−0.816	−0.9	−0.965	−0.815	−0.894
	y_2	−3.296	−1.552	−1.248	−2.74	−4.155	−2.665	−3.378	−2.948
Sample no.		41	42	43					
Profile	n_1	6.648	13.512	8.239					
	n_2	9.389	15.985	10.458					
	n_3	14.7	17.256	13.355					
	n_4	17.391	20.959	19.05					
	n_5	21.003	23.542	21.383					
	n_6	27.353	26.342	27.666					
	n_7	31.5	28.546	32.028					
	n_8	37.209	33.745	35.987					
	n_9	45.81	37.886	43.281					
	n_{10}	52.197	43.098	50.909					
Variables	y_1	−0.977	−0.989	−0.844					
	y_2	−2.619	−0.842	−2.592					

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