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ON THE STRUCTURE OF EDGES OF GOMORY-HU TREE

A. DOLATI[†,*], M. RAHIMI[‡], S. VARVAEI [††]

[†] DEPARTMENT OF MATHEMATICS, SHAHED UNIVERSITY, TEHRAN.
DOLATI@SHAHED.AC.IR.
[‡] DEPARTMENT OF MATHEMATICS, SHAHED UNIVERSITY, TEHRAN.
M.RAHIMI@SHAHED.AC.IR.
[††] DEPARTMENT OF MATHEMATICS, SHAHED UNIVERSITY, TEHRAN.
S.VARVAEI@SHAHED.AC.IR.

Abstract. For capacited graph $G\left(V,E\right)$, a tree $T\left(V\left(G\right),E_T\right)$ is a Gomory-Hu tree if for all $st \in E_T$, $\delta\left(W\right)$ is a minimum $s-t$ cut in G where W is one component of $T-st$.
In this paper, we prove that any pairs of vertices V can not be adjacent in Gomory-Hu tree T and present sufficient condition for a pairs of vertices which are not adjacent.

1. Introduction

Let $G=(V,E)$ be an undirected graph with positive edge capacities defined by $c:E\rightarrow R^{+}$. A Gomory-Hu tree is a weighted tree T on V, with the property that the pairwise edge connectivity between any two vertices s and t in the graph equals the minimum weight of an edge on the unique s-t path in T. Further, the partition of the vertices produced by removing this edge from T is a minimum s-t cut in the graph. An undirected graph has at least one Gomory-Hu tree, but it has been shown that Gomory-Hu trees need not exist for directed graphs [7].
Gomory-Hu trees have many applications in multi-terminal network flows [1] , mnimum T-cut problem [5] , Cut problems in graphs with a budget constraint problem [6].

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2. Main Results

In this section some results about the structure of the edges of a Gomory-Hu tree are presented. A capacited graph and its Gomory-Hu tree are depicted in Figure 1. The edges of Gomory-Hu tree are not edges of graph G essentially. An edge could

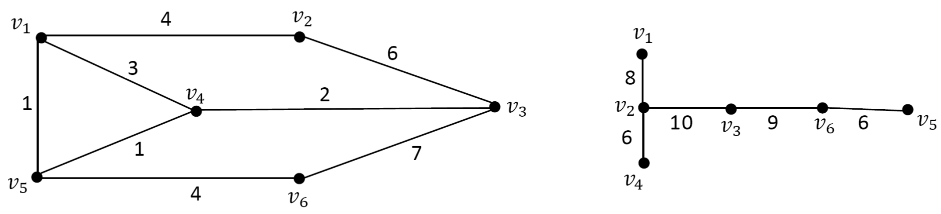

Figure 1. A capacited graph and its Gomory-Hu tree

be in the Gomory-Hu tree while not in G and vice versa.
The following theorem presents a sufficient condition under which a pair of vertices can not be able to be adjacent in Gomory-Hu trees.

Theorem 2.1. *Suppose that v is a cut vertex in graph G and K_1, K_2 are two components of graph $G - v$. If $x \in K_1$ and $y \in K_2$ then they can not be able to be adjacent in Gomory-Hu trees of G.*

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