2012 – 9th International Multi-Conference on Systems, Signals and Devices

Improved Power Allocation in Parallel Poisson Channels

Razgar Rahimi
Technical Department, Shahed University
Tehran, Iran
r.rahimi@iccssco.com

Gholamreza Dadashzadeh
Technical Department, Shahed University
Tehran, Iran
gdadashzadeh@shahed.ac.ir

***Abstract*—The optimum power allocation for parallel Poisson channels which can minimize average power for obtaining maximum capacity is considered. In this paper a new power allocation scheme that outperforms previous schemes for more than enough powers is presented. Since in Poisson channels unbounded channel feeding with input power do not lead to a better utilization of the channel capacity, in the presented scheme extra powers that are more than required power for achieving maximum capacity are not used or if power dissipation is not possible, the extra power can be allocated in a new way which shows an improvement in the channel capacity. This approach is applied to the 2-fold parallel Poisson channels and is generalized to n-fold.**

Keywords- Poisson Channels; parallel channels; optimum power allocation; free-space optical communication; capacity

I. INTRODUCTION

Free-Space Optics (FSO) is emerging as a popular technology because of its low-cost, high data rate communication and its several applications [4, and references therein]. The Poisson Channel has been accepted as a standard model for optical communication channels [7]. Shot noise, thermal noise, and laser intensity noise are well-known noises in optical intensity-modulated communications [1] and [8]. As mentioned in a literature review of Poisson communication theory in [6], communication under Poisson regime was presented by I. Bar-David [9]. The capacity of Poisson channels under peak and average power constraints was proposed in [2]-[3], and the capacity region of Poisson multiple-access channel is investigated in [7]. The Poisson broadcast channels and Poisson multi-input multi-output channels are studied in [10]-[11], respectively. In free-space optical channels atmospheric turbulence can lead to random fluctuation in intensity of optical signal and form a Poisson fading channel which is considered in [12]. A new relationship between mutual information and conditional mean estimation in Poisson channels is found in [5]. A 2-fold and its generalized to n-fold parallel Poisson channel is considered in [1], in which a power allocation scenario that attempts to maximize the capacity of a peak and average power limited parallel Poisson channel is proposed. In the presented paper the above-mentioned power allocation for parallel Poisson channel is investigated and an improved scheme is proposed which shows that for powers that are more than sufficient, achievable capacities are higher than what has been stated in [1]. In this scheme despite the previous one the extra power can be dissipated or allocated in a different way to get better results.

The reminder of this paper is organized as follows. In section II, a simple Poisson channel model and parallel Poisson channel models are outlined. Discussion about the alleged optimum power allocation and the new improved scheme are presented in section III. Performance comparisons are provided in section IV, and conclusions are given in section V.

II. PARALLEL POISSON CHANNEL

A. Poisson Channel

In the Poisson channel model shown in Fig.1, according to [7] for channel input $x(t) \geq 0$ and constant $\lambda_0 \geq 0$, which represents both the dark current and background noise, the channel output $y(t)$ is a doubly stochastic Poisson process with instantaneous rate $y(t)=x(t)+\lambda_0$, which is the number of photoelectrons counted in the interval $[0,T]$ by the direct detection device (photo-detector). Channels under peak and average limits are constrained to satisfy:

$$0 \leq x(t) \leq A \qquad \frac{1}{T}\int_0^T x(\tau)d\tau \leq \sigma A . \tag{1}$$

Where the peak power $A>0$ and the ratio of average to peak power $0 \leq \sigma \leq 1$ are constant. In [2] and [3] the Shannon capacity of Poisson channel is given by:

$$\begin{aligned} C(A,\sigma,s) = A[&p(1+s)\ln(1+s)+(1-p)s\ln(s) \\ &-(p+s)\ln(p+s)] \qquad nats/s \end{aligned} \tag{2}$$

Where $s=\lambda_0 / A$ and $p=\min(\sigma, q(s)=\frac{(1+s)^{1+s}}{s^s e}-s)$

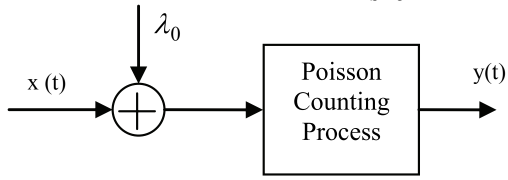

Figure 1. Poisson channel model.

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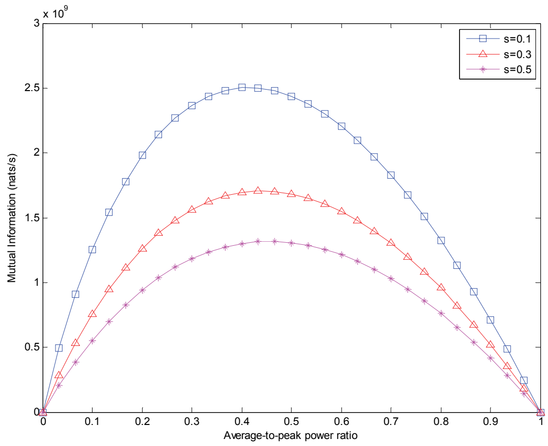

Figure 2. Poisson mutual information versus average-to-peak power ratio for different signal-to-noise-ratios ($A=10^{10}$)

According to [1], the derivative of Poisson channel capacity as function of average-to-peak power ratio is given by $dC/dp = A\ln[(1+s)^{1+s}/(s^s e(1+p))]$. Therefore, regardless of average power constraint the maximum capacity is independent of σ and is equal to:

$$C_{\max} = A[\frac{(1+s)^{1+s}}{s^s e} - s(1+s)\ln(1+\frac{1}{s})]. \quad nats/s \qquad (3)$$

As mentioned in [4], for OOK signaling scheme, the transmitted power has direct relationship to the duty cycle of the transmit aperture. Fig.2 illustrates the Poisson channel mutual information versus average-to-peak power ratio or duty cycle σ for three different signal-to-noise-ratios *(SNR=1/s)*. It depicts three important points. First, It shows that with higher *SNR* or $s=0.1$, higher amount for maximum mutual information is achievable. Second, it shows that for fixed peak power $A=10^{10}$ the maximum capacity for channels with lower signal-to-noise-ratio can be achieved with lower amount of channel input power. Finally, it reveals that by increasing σ the Poisson channel capacity raises up to a maximum value and then declines.

B. Parallel Poisson channel

A parallel Poisson channel with n-independent channels can be modeled as n-independent single Poisson channels. Fig.3 shows an n-fold parallel Poisson channel such that peak and average input power of channel i are restricted to A_i and $\sigma_i A_i$ respectively. Under peak and average power constraints, each independent channel could use a certain amount of total input power. Therefore, the total channel capacity is the summation of individual channel capacities under above-mentioned restrictions.

$$C_{total} = \sum_i C_i \qquad P_{total} = \sum_i \sigma_i A_i . \qquad (4)$$

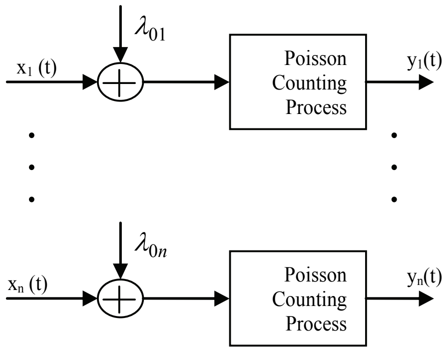

Figure 3. Parallel Poisson channel

Fig.4 illustrates the influence of different power allocations schemes on total capacity of a two parallel Poisson channels with $s_1=0.20$, $s_2=0.05$, and $A_1=A_2=10^9$. It shows that the maximum total rate 4.9163x10^8 is achievable for a certain case that the total input power is equal to the summation of required powers for obtaining maximum capacity on each individual channel. Therefore, for two channels with same peak power limits $A=A_1=A_2$ we have: $\sigma_{tot\_max}= \sigma_{1\_max}+ \sigma_{2\_max}$, where σ_{tot_max}, σ_{1_max}, and σ_{2_max} are optimum duty cycles for obtaining maximum capacity on two parallel channels, channel one, and channel two, respectively. Fig.4 shows black and red curves that depict the individual channel capacities. As mentioned before Poisson channel with higher *SNR* ($s_2=0.05$) needs lower power for attaining the maximum capacity. Note that like one single Poisson channel, the total capacity of parallel Poisson channel does not increase unboundedly by feeding higher amount of input power. These facts reveal the necessity of an optimum power allocation for improving the channel capacity.

III. Optimum Power Allocation

As mentioned before applying different power allocation schemes can change the attainable capacity of parallel Poisson channels considerably. In [1], a power allocation scheme is introduced as optimum power allocation for these channels. In this section we investigate the proposed scheme and show that by modifying it for powers more than sufficient power for reaching the maximum total capacity, the channel utilization can be improved. For finding the optimum power allocation following optimization problem should be solved:

$$\begin{aligned} &Max \quad C = C_{total}(\sigma_1,\sigma_2,...,\sigma_n) = \sum_i C_i(\sigma_i) \\ &Subject \quad to: \quad G(\sigma_1,\sigma_2,...,\sigma_n) = \sum_i \sigma_i A_i = P_{total}. \end{aligned} \qquad (5)$$

We use maximization method of Lagrange multipliers to find best amounts of duty cycles. For simplicity, according to [1], at first we try to solve the problem for 2-fold parallel Poisson channel, and then generalize it to the n-fold case. Following this method, we must find the multiplier γ such that $\nabla C = \gamma \nabla G$.

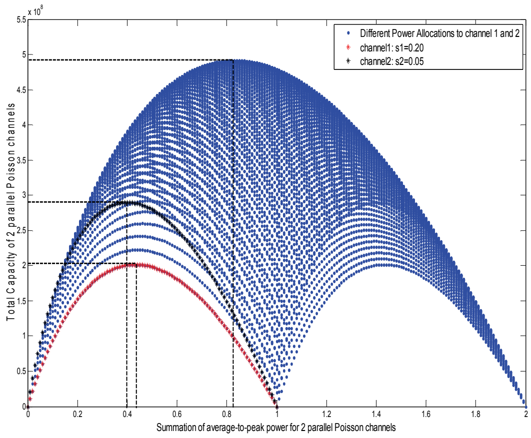

Figure 4. Total capacity of 2 parallel Poisson channels for $s_1=0.20$ and $s_2=0.05$ and $A=10^{10}$ with different power allocations.

We have:

$$\frac{\partial C}{\partial \sigma_1} = \gamma \frac{\partial G}{\partial \sigma_1} = \gamma A_1, \qquad \frac{\partial C}{\partial \sigma_2} = \gamma \frac{\partial G}{\partial \sigma_2} = \gamma A_2 \tag{6}$$

Solving above equations the following results obtain [1]:

$$\begin{aligned} \sigma_1 &= (e^{-\gamma} q(s_1) - (1 - e^{-\gamma}) s_1)^+ \\ \sigma_2 &= (e^{-\gamma} q(s_2) - (1 - e^{-\gamma}) s_2)^+ \end{aligned} \tag{7}$$

Where $(x)^+=max(x,0)$, and

$$e^{-\gamma} = \frac{s_1 A_1 + s_2 A_2 + P}{(s_1 + q(s_1))A_1 + (s_2 + q(s_2))A_2} \tag{8}$$

From equation (7) and considering Fig.4 as an example we may have three different conditions. First, when one of duty cycle equations $(e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i)$ become lower than zero. In this case as both equation (7) and Fig.4 suggests we must dedicate all input power to the other channel. Second is the condition that none of duty cycles are lower than zero, and the total power is smaller than enough power for reaching the maximum total capacity which is $P_{enough}=q(s_1)A_1+q(s_2)A_2$. in this case as suggested in [1], we allocate the available power to the channels according to equation (7). The third condition which is related to the main contribution of this paper is when the total available power is more than enough power P_{enough}. In this condition unlike the suggested approach in [1] we have two different approaches if total power allocation is not mandatory, we can highly improve the channel capacity with dissipating the non-required power which is more than enough and declines the total channel capacity. Utilizing this approach can maintain the capacity in the maximum value. Therefore, for $P_{total}>q(s_1)A_1+q(s_2)A_2$ we suggest $\sigma_i=q(s_i)$. If it is not possible to dissipate this extra power there is another power allocation scheme which although has not significant increase in capacity but shows that the alleged optimum power allocation in parallel Poisson channel can be improved slightly. In this case proposed values for duty cycles are same as equation (7). For n-fold parallel Poisson channel this approach can be simply generalized. For these channels if first condition happens, channels with negative equations for duty cycle are excluded and the total power could be dedicated to the remained channels. For second and third conditions, the approach is similar to that for 2-fold case. Supposing an n-fold parallel Poisson channel, the abovementioned approach is summarized as follows:

- If we have $(e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i) \le 0$, for $i \in L,\ \|L\| < n-1$, we set $\sigma_i = 0$ and for remained channels that $(e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i) > 0$, we allocate total power such that only $n - \|L\|$ channels exist. Therefore we have:

$$\begin{cases} \sigma_i = 0 & i \in L \\ \sigma_i = (e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i)^+ & i \notin L \end{cases} \tag{9}$$

Where

$$e^{-\gamma} = \frac{\sum_{i \notin L} s_i A_i + P}{\sum_{i \notin L} (s_i + q(s_i)) A_i} \tag{10}$$

- If we have $i \in L,\ \|L\| = n-1$, such that $(e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i) \le 0$, we set $\sigma_i = 0$ and for remained channel that $(e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i) > 0$, we allocate total power to this channel. Therefore we have:

$$\begin{cases} \sigma_i = 0 & i \in L \\ \sigma_i = P_{total} / A_i & i \notin L \end{cases} \tag{11}$$

- If $P_{total} \ge q(s_1)A_1 + q(s_2)A_2$ and power dissipation is allowed we set: $\sigma_i = q(s_i)$
- If $P_{total} \ge q(s_1)A_1 + q(s_2)A_2$ and total available power allocation is mandatory, we set: $\sigma_i = (e^{-\gamma} q(s_i) - (1 - e^{-\gamma}) s_i)^+$.

Where

$$e^{-\gamma} = \frac{\sum_i s_i A_i + P}{\sum_i (s_i + q(s_i)) A_i} \tag{12}$$

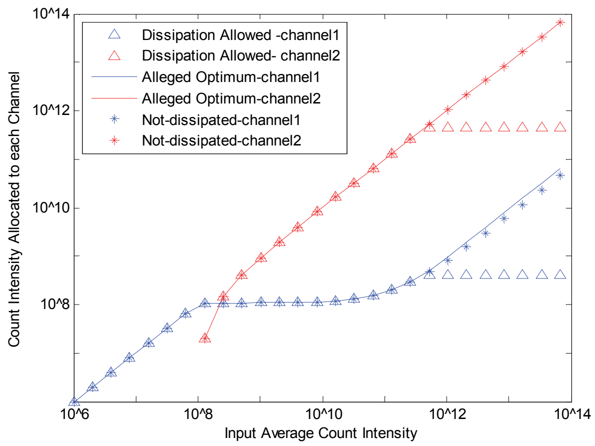

Figure 5. Comparison among three different count intensity allocation schemes with parameters $A_1=10^9$, $A_2=10^{12}$ photons/s, $s_1=0.1$ and $s_2=0.3$.

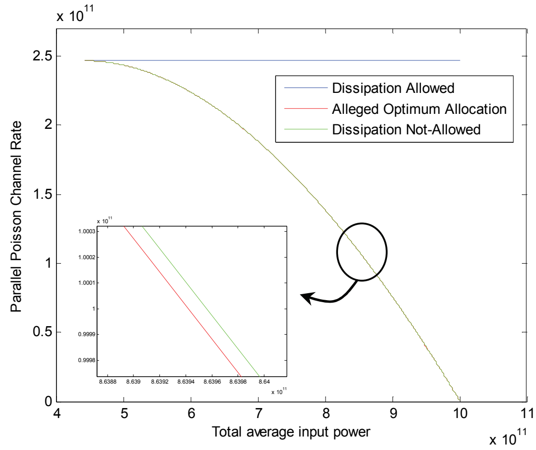

Figure 6. Parallel Poisson channel rate comparison for three different power allocation schemes.

IV. Performance Comparison

In this section, we analytically compare the performance of alleged optimum power allocation [1], with performance of two new proposed schemes for powers which are more than enough for obtaining maximum capacity. For proper comparison values are same as those used in [1]. Therefore, we have a 2-fold parallel Poisson channel with $A_1=10^9$, $A_2=10^{12}$, $s_1=0.1$, and $s_2=0.3$. Fig.5 illustrates three different count intensity allocations to individual channels versus input average count intensity. It can be observed that the alleged optimum scheme and the new scheme without power dissipation allocate all the input power to the individual channels, while in the third scheme which power dissipation is allowed only the enough power is allocated to the individual channels and the remained power is not used. Also, in comparison between new scheme without power dissipation and the referred one [1], it can be seen that the portion of input average count intensity that is dedicated to the stronger channel ($s_1=0.1$, $A_1=10^9$) in first scheme is lower than that for latter one. Instead this power is assigned to the weak channel ($s_2=0.3$, $A_2=10^{12}$). Fig.6 depicts the effect of these three different power allocation schemes on channel rate, in which the parallel Poisson channel rate for average input powers more than optimum power are presented. As expected the channel rate remained fixed with the scheme that dissipates the extra power, and a closer observation of Fig.6 indicates that the new scheme which does not dissipate the extra power but dedicates it in a different way, outperforms the alleged optimum power allocation scheme slightly.

V. Conclusion

In this paper, an improved power allocation scheme with peak and average power constraints for parallel Poisson channel is considered and it is shown that this scheme with two different approaches can obtain more channel rates than that for previous alleged optimum power allocation. The main difference occurs for input average powers that are more than required power for reaching the maximum channel capacity. Considering the performance results depending on the dissipation possibility, these approaches can substitute the previous schemes for power allocation in parallel Poisson channels.

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