The Achievable Distortion of Relay-Assisted Block Fading Channels

Sayed Ali Khodam Hoseini, Soroush Akhlaghi, and Mina Baghani

***Abstract*—This letter concerns the achievable distortion of a Gaussian source over a relay-assisted block fading channel under mean square-error distortion measure. It is assumed the communication is occurred in two hops through the use of a relay node, where the Decode and Forward (DF) strategy is employed at this node. Also, the transmitter does not have the channel state information, while the channel gains associated with both hops are available at the relay. It is assumed the Gaussian source is hierarchically encoded through using scalable source coding approach, and is sent to the DF relay through the use of a multi-layer code, while the relay sends the retrieved information through a single layer code. In this regard, in a Rayleigh block fading environment, the optimal power allocation strategy across code layers is derived, showing the resulting distortion outperforms that of using AF relaying or exploiting a single layer code.**

***Index Terms*—Channel assignment, Hungarian algorithm, resource allocation.**

I. Introduction

The notion of relaying is mostly regarded as a promising solution to improve the coverage and reliability of wireless communication networks. In this regard, extensive researches are carried out to explore effective coding strategies at the relays, among them, Amplify-and-Forward (AF), Decode-and-Forward (DF), and Compress-and-Forward (CF) are mostly addressed in the literature, each performing well under certain conditions. For instance, when the direct link between the source and the destination is in poor condition and destination has only access to the information transmitted by the relay, the DF strategy is optimal in the terms of rate maximization at the destination [1].

We consider a point-to-point relay-assisted channel in which there is not a direct link between the transmitter and the affiliated receiver. The channel studied here is constant throughout one transmission block and varies independently for the next blocks. The relay is assumed to be simple, meaning it cannot do buffering, water-filling across time or coding over consecutive blocks. We assume the DF strategy is employed at the relay, where a special case in which the Channel State Information (CSI) is not available at the transmitter, while the CSI associated with both hops are available at the relay, is considered. As a practical implication, one can consider the case in which the relay is close to the destination, while is far from the transmitter. Thus, due to the large distance between the transmitter and the relay, the CSI at the transmitter (CSIT) may be outdated or too noisy, thereby is less likely to be incorporated at the transmitter [2].

Manuscript received January 23, 2012. The associate editor coordinating the review of this letter and approving it for publication was Z. Z. Lei.

The authors are with the Department of Engineering, Shahed University, Tehran, Iran (e-mail: {khodamhoseini, akhlaghi, baghani}@shahed.ac.ir).

Digital Object Identifier 10.1109/LCOMM.2012.051512.120173

In a point to point channel with Gaussian noise, when the CSIT is available, the use of a single layer code is optimal. However, in the lack of CSIT, it is demonstrated the so-called broadcast strategy which makes use of multi-layer coding approach is optimal in terms of maximizing the average achievable rate [3]. In fact, the original problem can be translated to a point to multi-point network encompassing a continuum of virtually ordered destinations, each corresponding to a channel strength. Accordingly, it is shown the superposition code can achieve the sum-rate capacity of such network. This problem is further extended to the relay-assisted networks, where the average achievable rate of using various coding strategies at the relay is studied [3], [4]. On the other hand, the successive refinement (SR) source coding approach is useful when trying to broadcast the source information to multiple receivers with different channel conditions. In SR method, the source information is hierarchically refined through multiple source layers such that the upper layers are refinements of lower layers and the base layer is the most protected one.

This motivated Tian *et al.* to concurrently incorporate SR source coding and multi-layer channel coding approaches to derive an achievable distortion of Gaussian source over a single hop channel when the CSIT is not available [5]. This motivated us to pursue addressing the achievable distortion of Gaussian source in two-hop relay-assisted networks, assuming the transmitter is unaware of channel gains, while the relay perfectly knows the channel gains associated with both hops.

This paper is organized as follows. The problem formulation for a single hop channel is presented in Section II. Section III extends the terminology to a two-hop network and presents the results for Rayleigh fading channel. Finally, Section IV summarizes findings.

II. System Model

In a point-to-point block fading channel, the received signal at the destination can be written as $y_k = hx_k + n_k$ for $k = 1, \ldots, N$, where x_k and y_k denote the k^{th} complex channel input and the corresponding output at the k^{th} channel use, respectively. n_k is an additive complex white Gaussian noise of unit variance, i.e., $n_k \sim \mathcal{CN}(0, 1)$. Moreover, $h \in \mathbb{C}$ represents the current channel fading coefficient with strength $\gamma = |h|^2$, and is assumed to be constant within a block, while it varies independently across different blocks. Finally, N represents the number of channel uses in each transmission block.

Assuming K complex-valued samples of a Gaussian memoryless source are sent within each transmission block, the source channel mismatch factor is defined as $b = N/K$. Also, it is assumed each channel block to be long enough to approach the rate-distortion limit, while still shorter than the coherence time, i.e., the time interval in which the channel gain is approximately constant.

The transmitter uses a multi-layer code with infinite number of layers, in which the fractional power $\rho(\gamma)d\gamma$ is set to the layer indexed by γ, thus the corresponding fractional rate becomes $dR(\gamma) = \log\big(1 + \frac{\gamma\rho(\gamma)d\gamma}{1+\gamma T(\gamma)}\big) \simeq \frac{\gamma\rho(\gamma)d\gamma}{1+\gamma T(\gamma)}$, where $T(\gamma) = \int_{\gamma}^{\infty} \rho(u)du$ is the power allocated to the layers indexed by $\gamma + d\gamma$ to $+\infty$, hence, it is the sum power assigned to layers which are not decodable when the channel strength is γ. As a result, the total rate decoded at the destination with channel strength γ is the sum of all fractional rates allocated to the layers indexed below γ, i.e., $R(\gamma)= \int_0^{\gamma} \frac{u\rho(u)du}{1+uT(u)}$. Considering the complex channel model, the distortion of a Gaussian source represented by $R(\gamma)$ can be written as $D(\gamma) = \exp\big(-bR(\gamma)\big)$. Congruent to what is done in [5], we use an auxiliary function based on $D(\gamma)$ as $I(\gamma) = \exp\big(R(\gamma)\big)$. Using this function, the transmit power constraint can be written as $\int_0^{\infty} \frac{I(\gamma)}{\gamma^2}d\gamma \leq P_t$, where P_t represents the maximum transmitted power for transmitting each coding block. Noting $D(\gamma) = I(\gamma)^{-b}$, the minimum average achievable distortion and the aforementioned constraint can be formulated as,

$$D = \min_{I(.)} \int_0^{\infty} \frac{f(\gamma)}{I(\gamma)^b}d\gamma \quad \text{s.t.} \quad \int_0^{\infty} \frac{I(\gamma)}{\gamma^2}d\gamma \leq P_t \ , \qquad (1)$$

where $f(\gamma)$ denotes the pdf associated with the channel strength γ. Accordingly, the best function $I^{opt}(\gamma)$ which minimizes (1) is derived in [5], showing for each continues interval which meets the constraint $\frac{d}{d\gamma}\big(\gamma^2 f(\gamma)\big) > 0$, there is at most one positive interval with positive power allocation. For instance, in Rayleigh channel it is shown there is only one continues power allocation interval which meets the aforementioned constraint. Assuming the interval $\gamma \in [\gamma_1, \gamma_2]$ is the only positive power allocation region in this interval, the values of γ_1 and γ_2 as well as the function $I^{opt}(\gamma)$ are obtained.

III. Distortion Minimization for DF Relaying

In this work, the transmitter is assumed to be unaware of the channel state information, thus, it makes use of joint successive refinement source coding together with a multi-layer channel code to transmit the information to the relay. Then, during the second hop, the DF relay sends the retrieved information through a single layer code since the relay is aware of the channel gain associated with the second hop, thus the use of a single layer code is optimal. The problem is to find the optimum power allocation across the code layers of the first hop, leading to the minimum average achievable distortion at the destination. Note that as is argued in the preceding section, the instantaneous achievable distortion at the relay can be written as $D_r(\gamma) = I_t(\gamma)^{-b}$, where, γ is the instantaneous channel strength of the first hop and $I_t(.)$ is the auxiliary function, defined as $I_t(\gamma) = \exp\big(R_t(\gamma)\big)$.

As is stated above, the relay makes use of a single layer code with the maximum rate of $R_r(l) = \log(1 + lP_r)$ to transmit its retrieved information, where l and P_r denote, respectively, the channel strength of the second hop and the relay's maximum transmit power. Thus the instantaneous achievable distortion at the destination becomes $D_d(l) = \exp\big(-bR_r(l)\big) = (1 + lP_r)^{-b}$. However, it may happen the relay does not have that much information to transmit. In other words, the achievable distortion at the destination can not be lower than that of received at the relay, thus we have, $D_d\big(l|D_r(\gamma)\big) = \max\big(D_r(\gamma), (1 + lP_r)^{-b}\big)$ which can be simplified to

$$D_d\big(l|D_r(\gamma)\big) = \begin{cases} (1 + lP_r)^{-b} & \text{if } l < l_{th} \\ D_r(\gamma) & \text{o.w.} \end{cases} , \qquad (2)$$

where l_{th} can be readily computed from $(1 + l_{th}P_r)^{-b} = D_r(\gamma)$, leading to $l_{th} = \frac{1}{P_r}(D_r(\gamma)^{\frac{-1}{b}} - 1)$. Therefore the average distortion at the destination, considering the available distortion at the relay to be $D_r(\gamma)$, becomes

$$\begin{aligned} D_{ave|D_r(\gamma)} &= \int_0^{\infty} f_r(l)D_d\big(l|D_r(\gamma)\big)dl \\ &= \int_0^{l_{th}} f_r(l)(1 + lP_r)^{-b}dl + D_r(\gamma)\int_{l_{th}}^{\infty} f_r(l)dl \\ &= \mathcal{G}\big(D_r(\gamma)\big), \end{aligned} \qquad (3)$$

where $f_r(.)$ denotes the pdf associated with the channel strength of the second hop. Thus the average distortion at the destination can be computed as $D_{ave} = \int_0^{\infty} f_t(\gamma)\mathcal{G}\big(D_r(\gamma)\big)d\gamma$, where $f_t(.)$ is the pdf of channel strength associated with the first hop. The goal is to find the auxiliary function $I_t(.)$ such that the average achievable distortion at the destination is minimized, i.e.,

$$\begin{aligned} D_{ave} = \min_{I_t(.)} &\int_0^{\infty} f_t(\gamma)\mathcal{G}\big(D_r(\gamma)\big)d\gamma \\ \text{s.t.} &\begin{cases} \int_0^{\infty} \frac{I_t(\gamma)}{\gamma^2}d\gamma \leq P_t \\ I_t'(\gamma) \geq 0 \end{cases} . \end{aligned} \qquad (4)$$

The first constraint in (4) ensures the transmitted power not to exceed its maximum value and the second one guarantees $I_t(\gamma)$ to be monotonically non-decreasing function, i.e., to have a non-negative auxiliary (rate allocation) function ($R(\gamma) \geq 0$ for $\gamma \geq 0$). In what follows, we first relax the second constraint and derive the solution. Then, we will prove that the derived solution meets the second constraint, thus, is the optimal solution.

Let's assume the optimal power allocation falls within a single interval $\gamma \in [\gamma_1, \gamma_2]$. Defining $\mathcal{H}\big(\gamma, I_t(\gamma), I_t'(\gamma)\big) = f_t(\gamma)\mathcal{G}\big(D_r(\gamma)\big)$, and noting $\mathcal{G}\big(D_r(\gamma)\big) = 1$ for $\gamma \in [0, \gamma_1]$ as there is a zero rate allocation in this interval, and again noting the fact that $\mathcal{G}\big(D_r(\gamma)\big) = \mathcal{G}\big(D_r(\gamma_2)\big)$ for $\gamma \in [\gamma_2, \infty)$ due to the zero rate allocation in this region, the objective of (4) changes to,

$$\begin{aligned} D(I_t) &= \int_0^{\infty} \mathcal{H}\big(\gamma, I_t(\gamma), I_t'(\gamma)\big)d\gamma \\ &= \int_{\gamma_1}^{\gamma_2} \mathcal{H}\big(\gamma, I_t(\gamma), I_t'(\gamma)\big)d\gamma + \\ &\quad F_t(\gamma_1) + \big(1 - F_t(\gamma_2)\big)\mathcal{G}\big(D_r(\gamma_2)\big) \ , \end{aligned} \qquad (5)$$

where $F_t(\gamma)$ denotes the commutative distribution function (cdf) associated with the first hop. Defining

$\mathcal{W}\big(\gamma, I_t(\gamma), I_t^{'}(\gamma)\big) = \frac{I_t(\gamma)}{\gamma^2}$, the power constraint in (4) becomes,

$$P(I_t) = \int_{\gamma_1}^{\gamma_2} \mathcal{W}\big(\gamma, I_t(\gamma), I_t^{'}(\gamma)\big)d\gamma + \frac{I_t(\gamma_2)}{\gamma_2} - \frac{1}{\gamma_1} \le P_t. \quad (6)$$

Using lagrange multipliers method, the problem in (5) and the constraint in (6) can be encapsulated into a single-letter formula as $\mathcal{L}(I_t) = D(I_t) + \lambda\big(P(I_t) - P_t\big)$, where we aim at finding the optimum auxiliary function $I_t(\gamma)$ with the constraint $I_t(\gamma_1) = 1$ to minimize $\mathcal{L}(I_t)$. Therefore, following the same approach as is done in [5], taking an arbitrary increment $\delta I_t(\gamma)$ on $I_t(\gamma)$ within the interval $\gamma \in [\gamma_1, \gamma_2]$, the lagrangian increment will be given by $\Delta(\delta I_t) = \mathcal{L}(I_t + \delta I_t) - \mathcal{L}(I_t)$. As a result, noting $\delta I_t(\gamma) = 0$ for $\gamma \notin [\gamma_1, \gamma_2]$ and $\delta I_t(\gamma_1) = 0$ ($I_t(\gamma_1) = I_t(\gamma_1) + \delta I_t(\gamma_1) = 1$), equating the linear part of the increment to zero, gives,

$$\begin{aligned}&\int_{\gamma_1}^{\gamma_2} \big[\mathcal{H}_{I_t} + \lambda\mathcal{W}_{I_t} - \frac{d}{d\gamma}[\mathcal{H}_{I_t'} + \lambda\mathcal{W}_{I_t'}]\big]\delta I_t(\gamma)d\gamma \\ &+\big(\mathcal{H}_{I_t'} + \lambda\mathcal{W}_{I_t'}\big)\Big|_{\gamma=\gamma_2}\delta I_t(\gamma_2) + \\ &\Big(\big(1 - F_t(\gamma_2)\big)\mathcal{G}'\big(D_r(\gamma_2)\big)\frac{\partial D_r(\gamma_2)}{\partial I_t(\gamma_2)} + \frac{\lambda}{\gamma_2}\Big)\delta I_t(\gamma_2) = 0,\end{aligned} \quad (7)$$

where $\mathcal{G}'\big(D_r(\gamma)\big)$ is the partial derivative of $\mathcal{G}\big(D_r(\gamma)\big)$ with respect to $D_r(\gamma)$. Due to the arbitrariness of the function $\delta I_t(.)$ and noting $\mathcal{H}_{I_t'} = \mathcal{W}_{I_t'} = 0$, and $D_r(\gamma) = I_t^{-b}(\gamma)$, we shall have the following equalities at the optimal point,

$$\begin{aligned}f_t(\gamma)\mathcal{G}^{'}\big(D_r(\gamma)\big)\frac{-b}{I_t(\gamma)^{b+1}} + \frac{\lambda}{\gamma^2} = 0,\\ \big(1 - F_t(\gamma_2)\big)\mathcal{G}^{'}\big(D_r(\gamma_2)\big)\frac{-b}{I_t(\gamma_2)^{b+1}} + \frac{\lambda}{\gamma_2} = 0.\end{aligned} \quad (8)$$

Knowing $I_t(\gamma_1) = 1$ and setting $\gamma = \gamma_1$ in the first equation of (8), we get $\lambda = b\gamma_1^2 f_t(\gamma_1)\mathcal{G}^{'}\big(D_r(\gamma_1)\big)$, thus it can be concluded that,

$$I_t(\gamma) = \left(\frac{\gamma^2 f_t(\gamma)\mathcal{G}^{'}\big(D_r(\gamma)\big)}{\gamma_1^2 f_t(\gamma_1)\mathcal{G}^{'}\big(D_r(\gamma_1)\big)}\right)^{\frac{1}{b+1}}. \quad (9)$$

Taking partial derivative of $\mathcal{G}\big(D_r(\gamma)\big)$ with respect to $D_r(\gamma)$[1] and solving equation (9), one can derive the optimal value of $I_t^{opt}(\gamma)$. However, the values of γ_1 and γ_2 have yet to be addressed. To this end, setting $\gamma = \gamma_2$ in the first equation of (8), one can find λ as a function of γ_2. Then, plugging λ into the second equation of (8), γ_2 can be found from the following identity,

$$\gamma_2 f_t(\gamma_2) = 1 - F_t(\gamma_2). \quad (10)$$

Also, γ_1 can be readily found through substituting $I_t^{opt}(\gamma)$ into the power constraint in (6). Finally, plugging $I_t^{opt}(\gamma)$ into (5), one can compute the achievable distortion.

Now, we are going to state the condition in which there is just one single interval $\gamma \in [\gamma_1, \gamma_2]$ with positive power allocation. To this end, the lagrangian function associated with (4) can be written as,

$$\mathcal{L}(I_t) = \int_0^{\infty} \Big(f_t(\gamma)\mathcal{G}\big(D_r(\gamma)\big) + \lambda\frac{I_t(\gamma)}{\gamma^2} - I_t'(\gamma)\phi(\gamma)\Big)d\gamma, \quad (11)$$

[1]Note that l_{th} in (3) is a function of $D_r(\gamma)$

where λ is the corresponding lagrange multiplier of the first constraint and $\phi(\gamma)$ is an arbitrary non-negative function, ensuring $I_t'(\gamma)$ is monotonically non-decreasing at each point (the second constraint). Now using the variational method, the best function $I_t(.)$ can be found from the following equation [6],

$$-b\mathcal{G}'\big(D_r(\gamma)\big)\frac{f_t(\gamma)}{I_t(\gamma)^{b+1}} + \frac{\lambda}{\gamma^2} + \phi'(\gamma) = 0. \quad (12)$$

where $\phi'(\gamma) = \frac{d\phi(\gamma)}{d\gamma}$. Note that according to the slackness condition, in the region which there is a positive power allocation, i.e., $I_t'(\gamma) > 0$, we should have $\phi(\gamma) = 0$. In this case, referring to (12), it follows,

$$I_t(\gamma) = \left(\frac{\gamma^2 f_t(\gamma)\mathcal{G}'\big(D_r(\gamma)\big)}{\lambda}\right)^{\frac{1}{b+1}}. \quad (13)$$

Taking derivation of (13) w.r.t. γ and considering the definition of $\mathcal{G}\big(D_r(\gamma)\big)$, it can be concluded that the necessary condition to have $I_t'(\gamma) > 0$ is,

$$\frac{d}{d\gamma}\big(\gamma^2 f_t(\gamma)\big) > 0. \quad (14)$$

Moreover, in the sequel, we are going to show that there is at most one single interval in any interval which (14) holds. Suppose otherwise; we assume in the region $[l, u]$ the constraint (14) meets, while there are two disjoint positive power allocation intervals $[m_1, n_1]$ and $[m_2, n_2]$ ($m_1 < n_1 < m_2 < n_2$) with zero power allocation between them, i.e., $I_t'(\gamma) = 0$ for $\gamma \in (n_1, m_2)$. In [6], it is shown for a piecewise smooth continuous extremal solution, the following corner condition at each corner point γ_c, i.e., $\gamma_c = n_1, m_2$, must be satisfied [6].

$$\mathcal{L}_{I_t'}\big|_{\gamma=\gamma_c^-} = \mathcal{L}_{I_t'}\big|_{\gamma=\gamma_c^+}. \quad (15)$$

Substituting (11) into (15), we arrive at,

$$\phi(\gamma_c^-) = \phi(\gamma_c^+). \quad (16)$$

According to the slackness condition, as we have positive power allocation in $[m_1, n_1]$, we have $\phi(n_1^-) = 0$. By the same token, it follows $\phi(m_2^+) = 0$. Noting this and referring to (16), it follows,

$$\begin{aligned}\phi(n_1^-) = \phi(n_1^+) = 0,\\ \phi(m_2^-) = \phi(m_2^+) = 0.\end{aligned} \quad (17)$$

On the other hand, noting we have assumed $I_t'(\gamma) = 0$ within the interval (n_1, m_2), we get $I_t(n_1^+) = I_t(m_2^-)$. Also, from (12), the following holds in the interval $\gamma \in (n_1, m_2)$,

$$\phi'(\gamma) = \mathcal{G}\big(D_r(n_1)\big)\frac{bf_t(\gamma)}{I_t(n_1)^{b+1}} - \frac{\lambda}{\gamma^2}. \quad (18)$$

This is due to the fact that $D_r(\gamma)$ and $I_t(\gamma)$ are constant in this interval as we have a zero power allocation. Also, noting $\phi'(n_1) = 0$ ($\phi(\gamma) = 0$ for $\gamma \in [m_1, n_1]$), the equation (12) at point $\gamma = n_1$ becomes, $\mathcal{G}\big(D_r(n_1)\big)\frac{bf_t(n_1)}{I_t(n_1)^{b+1}} - \frac{\lambda}{{n_1}^2} = 0$. Replacing λ from this equation into (18), we get $\phi'(\gamma) = \frac{b\mathcal{G}\big(D_r(n_1)\big)}{\gamma^2 I_t(n_1)^{b+1}}\big(\gamma^2 f_t(\gamma) - n_1^2 f_t(n_1)\big)$. As a result, noting this equation and (14), it follows $\phi'(\gamma) > 0$ in the interval $\gamma \in (n_1, m_2)$, meaning $\phi(\gamma)$ should be an increasing function.

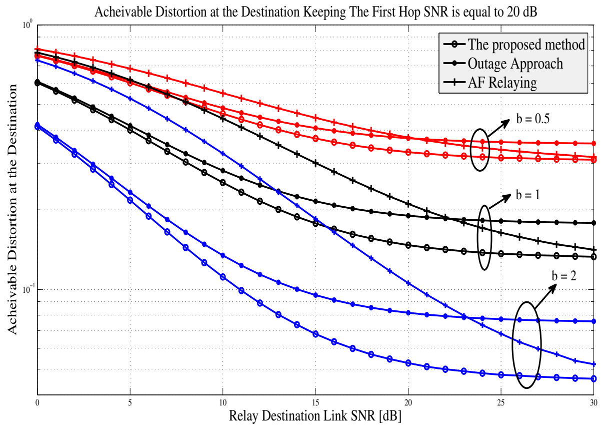

Fig. 1. Comparison results.

This, however, contradicts the corner condition in (17) which states $\phi(n_1) = \phi(m_2)$. Thus, there is only one continues single interval in the region which (14) holds.

For Rayleigh channel, assuming the channel strength associated with both hops are i.i.d. and follow exponential distributions, i.e. $f_t(\gamma) = \exp(-\gamma), f_r(l) = \exp(-l)$, one can readily verify that to have (14), it follows $\gamma \in (0, 2)$. Thus, as is argued earlier, there is at most one positive power allocation interval in $\gamma \in (0, 2)$. On the other hand, referring to (10), it follows $\gamma_2 = 1$, confirming the positive power allocation interval $[\gamma_1, \gamma_2]$ falls within the interval $\gamma \in (0, 2)$. Note that γ_1 as well as $I_t^{opt}(\gamma)$ have yet to be addressed. To this end, plugging $f_r(l) = \exp(-l)$ into (3), and taking the partial derivative of $\mathcal{G}\big(D_r(\gamma)\big)$ w.r.t. $D_r(\gamma)$, it follows, $\mathcal{G}'\big(D_r(\gamma)\big) = \exp(-\frac{D_r(\gamma)^{\frac{-1}{b}}-1}{P_r})$. Substituting this into (9), gives $I_t^{opt}(\gamma) = (b+1)P_r W_L\left(\frac{\left(\frac{\gamma^2\exp(\frac{1}{P_r}-\gamma+\gamma_1)}{\gamma_1^2}\right)^{\frac{1}{b+1}}}{(b+1)P_r}\right)$, where $W_L(.)$ is the omega function and is the inverse of the function $f(W) = We^W$. Finally, substituting $I_t^{opt}(\gamma)$ and $\gamma_2 = 1$ into the power constraint (6), gives the value of γ_1.

For the sake of comparison, the achievable distortion of the proposed method is compared to that of the AF strategy as well as the case of using a single layer code at the first hop, called the outage approach. In AF strategy, it is assumed the relay amplifies the received signal and retransmits it to the destination. In this case, the pdf of the equivalent channel between the source and the destination can be computed and the optimal power allocation can be found as if there is just one point-to-point channel [7]. On the other hand, in outage approach, it is assumed the source makes use of a single layer code and the DF relay can successfully decode the information as long as the channel strength of the first hop (γ) exceeds a certain threshold. This threshold, i.e., γ_{opt}, should be optimized such that the average achievable distortion at the destination is minimized. In this case, the achievable distortion at the destination can be formulated as follows[2],

$$D_{ave} = 1 \times F_t(\gamma) + \big(1 - F_t(\gamma)\big)\mathcal{G}\big(D_r(\gamma)\big)\ . \tag{19}$$

Also, from (2) and noting $D_r(\gamma) = (1+\gamma P_t)^{-b}$, it follows $l_{th} = \frac{P_t}{P_r}\gamma$. Taking derivative of D_{ave} with respect to γ and equating to zero, the optimal value of γ, i.e., γ_{opt}, can be found from the following,

$$1 - \int_0^{\frac{P_t}{P_r}\gamma_{opt}} \exp(-l)(1+lP_r)^{-b}dl - (1+\gamma_{opt}P_t)^{-b}\big(1+\frac{bP_t}{1+\gamma_{opt}P_t}\big)\exp(\frac{-P_t}{P_r}\gamma_{opt}) = 0. \tag{20}$$

Fig. 1 is provided to compare the performance of the proposed approach to that of using a single layer code as well as the AF strategy, when the transmit SNR is set to 20dB and the SNR at the relay changes from 0dB to 30dB.

IV. Conclusion

This paper aims at investigating the average achievable distortion of a two-hop DF relay-assisted network through using joint successive refinement source coding and multi-layer channel coding approach. To this end, the optimal power allocation policy across code layers is derived, leading to the minimum achievable distortion. The result is also compared to the case of using AF relaying as well as using a single layer code at the first hop, showing at moderate to high SNR region of the second hop the achievable distortion of the proposed approach has a sizable gap to the single layer code, while the gap to the AF relaying approaches to zero.

V. Acknowledgement

The authors would like to acknowledge the financial support provided by the Research Institute for ICT, Tehran, Iran.

References

[1] T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.
[2] Y. Ma, Y. Zhang, and R. Tafazolli, "Modulation-adaptive cooperation schemes for wireless networks," in *2008 IEEE Conf. Vehicular Technol.*
[3] A. Steiner and S. Shamai (Shitz), "Single-user broadcasting protocols over a two-hop relay fading channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 4821–4838, Nov. 2006.
[4] V. Pourahmadi, A. Bayasteh, and A. K. Khandani, "Multilevel coding strategy for two-hop single-user networks," in *2008 Biennial Symp. Commun.*
[5] C. Tian, A. Steiner, S. Shamai (Shitz), and S. N. Diggavi, "Successive refinement via broadcast: optimizing expected distortion of Gaussian source over a Gaussian fading channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 2903–2918, July 2008.
[6] I. M. Gelfand and S. V. Formin, *Calculus of Variations*. Prentice-Hall, 1963.
[7] S. Shamai (Shitz) and A. Steiner, "A broadcast approach for a single user slowly fading MIMO channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2617–2635, Oct. 2003.

[2]Note that in (19), when the channel strength of the first hop is lower than γ, according to the mean-square distortion measure, the resulting distortion is equal to the variance of Gaussian source which is one.