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DIRECT ADAPTIVE FUZZY CONTROL WITH MEMBERSHIP FUNCTION TUNING

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ABSTRACT

In this paper, a direct self-structured adaptive fuzzy control is introduced for the class of nonlinear systems with unknown dynamic models. Control is accomplished by an adaptive fuzzy system with a fixed number of rules and adaptive membership functions. The reference signal and state errors are used to tune the membership functions and update them instantaneously. The Lyapunov synthesis method is also used to guarantee the stability of the closed loop system. The proposed control scheme is applied to an inverted pendulum and a magnetic levitation system, and its effectiveness is shown via simulation.

Key Words: Adaptive fuzzy control, fuzzy control, nonlinear system, self structure algorithm.

I. INTRODUCTION

In recent years, fuzzy systems, such as fuzzy reasoning, fuzzy modeling, and fuzzy logic controllers, have been utilized in many fields for engineering and even social sciences. Some fuzzy control systems can already be seen in home appliances, transportation systems, manufacturing systems, and so on. The technological attempts over the recent century have been based on the capability of providing accurate, optimal, and powerful tools for controlling complex nonlinear systems [1–3].

A fuzzy system is characterized by presenting human knowledge or experience as fuzzy rules. Nevertheless, the fuzzy system has some problems. First, determining the number and shapes of membership functions. So, it takes time to design a fixed structure fuzzy system in detail.

These problems are more serious when the fuzzy system is applied to a more complex system. In order to solve these problems, the designer has two choices: (i) he or she needs to tune the rules using numerous time-consuming trial-and-error cycles; or (ii) some kind of optimization technique needs to improve the performance. For this, one could use reinforced learning [4], genetic algorithm [5], *etc*.

The second problem is the inability of fuzzy systems to produce a suitable control signal for making a smooth output. Hence, different methods, such as PID control [6], neural network [7–9], predictive control [10], and adaptive control [11, 12], are combined with the fuzzy method to make a reliable closed loop system.

The third problem has been considered less often. In systems with an unknown dynamic model or a known complex dynamic model, if state variables are employed as the input of the fuzzy system, it is complicated and time consuming to choose an efficient constant span for fuzzy sets until the span of fuzzy sets covers all the variations of state variables and all of the membership functions are applied optimally. In this case, if inputs will be out of the span or will be small with respect to the span, the output of the fuzzy sets will be near zero or have small output variations around one main point.

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Also, in some cases, some membership functions are less efficient in producing output. Only the memory of hardware is occupied.

To solve the above problems, one can use the aforementioned methods; though, for solving the choice of the number of rules, the self-structure algorithm was proposed [13]. The main problem of this proposed method, however, is the considerable number of rules, for example, in the case of controlling a magnetic levitation system, 128 rules were used. When the system has high complexity, using this method increases the computational load. In our method, however, the least number of rules are employed, *i.e.* four rules are used in an inverted pendulum system and nine rules are used in a magnetic levitation system. Also, the triangular membership function, which is the simplest shape for membership functions, is used to design the fuzzy system. To have a smooth output and solve the second problem that was mentioned, the adaptive method is combined with the fuzzy method [14, 15], and to have a stable controller, the robust adaptive method is considered [16–19]. Different methods have been proposed to tune the membership functions [20–27]. Nevertheless, as our goal in design is to use a simple method that needs less computation, all membership functions are employed optimally and fuzzy sets can cover all variations of input variables.

In our proposed method, references and state errors are employed as the main materials to tune the membership functions. We use references because, when the reference varies, the membership function varies, thus, one can guarantee that inputs are in a span of fuzzy sets at this time only by tuning the span with the state error. In addition, using references causes a reference change but has no impact on output production. As a result, the rate of tuning the adaptive parameters and decreasing the output errors is increased. For instance, in a magnetic levitation system, when the domain of reference changes from 0.5 to 3 cm, by updating the membership function with respect to the new reference value, the fuzzy system works the same as before and does not need to define more membership functions to cover the 3 cm and larger domains. So, using the least number of rules, the controller can produce an efficient control signal, cover the input variations, and use all of the membership functions efficiently.

The paper is organized as follows. The control problem is explained in Section II. The direct adaptive fuzzy control (AFC) scheme is introduced in Section III. Section IV describes the fuzzy tuning algorithm followed by simulating two applications of the proposed method in Section V. Finally, the conclusion is drawn in Section VI.

II. PROBLEM STATEMENT

Consider a class of nonlinear single input single output (SISO) systems that is presented in the controllable canonical form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\;\;\cdots. \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f(\boldsymbol{x})+g(\boldsymbol{x})u \\ y &= x_1 \end{aligned} \tag{1}$$

where u is the system input, y is the system output, $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are two unknown continuous functions, and $\boldsymbol{x}=[x_1, x_2, \ldots, x_n]^T$ is the state vector of the system assumed to be available for measurement.

The objective of the control is to design an adaptive fuzzy controller such that:

- The closed-loop system is stable, *i.e.* all of its variables must be bounded.
- The output of the system $y(t)$ follows a continuous reference signal $r(t) \subset C^n$.

To design a controller to satisfy the above objectives, the following assumptions are made:

Assumption 1. $g(\boldsymbol{x})$ is continuous, and its sign is known for all X_s where $\boldsymbol{x} \in \Omega_x$. Ω_x is the controllability region. System (1) is controllable and $g(\boldsymbol{x})$ is continuous. Therefore, without a loss of generality, it can be assumed that $g(\boldsymbol{x})>0$ for all $\boldsymbol{x} \in \Omega_x$.

Assumption 2. A reference vector $\boldsymbol{r} := [r, \dot{r}, \ddot{r}, \ldots, r^{(n-1)}]^T$ is defined that satisfies: $\|\boldsymbol{r}\| < r_0$ and $\|r^{(n)}\| < r_1$, in which r_0 and r_1 are some bounded, known, real, and positive constants.

III. DIRECT ADAPTIVE FUZZY CONTROL

A zero-order Takagi–Sugeno fuzzy system with point fuzzification method, Mamdani product type inference, and center-average defuzzification approach is used in the proposed method.

First, $M(a, b; x)$ is defined to be a nonzero membership function for $x \in (a, b)$, and zero for

$x \notin (a,b)$. Then, the i-th rule of the proposed fuzzy system is considered as:

$$R^{(i)}: \text{IF } x_1 \text{ is } A_1^i, \text{ and } x_2 \text{ is } A_2^i, \text{ and } \ldots, \text{ and } x_n \text{ is } A_n^i, \text{ THEN } y \text{ is } \theta_i$$

in which $\boldsymbol{x}=[x_1,x_2,\ldots,x_n]^T \in U \subset R^n$ and $y \in V \subset R$ are the crisp input and output; A_{js}^i are fuzzy sets with membership functions $\mu_{A_j^i}(x_j)=M(a_{j1}^i, a_{j2}^i; x_j)$ for some $a_{j1}^i < a_{j2}^i$, $i=1,2,\ldots,m$, and $j=1,2,\ldots,n$; m is the number of rules; n is the system order; and $\boldsymbol{\theta}=[\theta_1,\theta_2,\ldots,\theta_m]^T$ is the output vector of the rules. The output of a Takagi–Sugeno fuzzy system is defined as $\hat{y} := \hat{f}(\boldsymbol{x},\boldsymbol{\theta})$, which is obtained by a weighted average of the rule outputs:

$$\hat{y} := \hat{f}(\boldsymbol{x},\boldsymbol{\theta}) = \frac{\sum_{i=1}^m \theta_i \mu_i(\boldsymbol{x})}{\sum_{i=1}^m \mu_i(\boldsymbol{x})} = \sum_{i=1}^m \theta_i \xi_i(\boldsymbol{x}) \tag{2}$$

in which, $\mu_i(\boldsymbol{x}) = \prod_{j=1}^n \mu_{A_j^i}(x_j)$, $\xi_i(\boldsymbol{x}) = \frac{\mu_i(\boldsymbol{x})}{\sum_{i=1}^m \mu_i(\boldsymbol{x})}$.

This class of fuzzy logic systems has universal approximation properties, *i.e.* it is able to approximate any continuous function with an arbitrary accuracy [15, 16].

If $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are supposed to be known functions, the input of the system can be written as

$$u^*(\boldsymbol{x}) = \frac{1}{g(\boldsymbol{x})}(-f(\boldsymbol{x}) + \boldsymbol{k}^T \boldsymbol{e} + r^{(n)}) \tag{3}$$

in which $\boldsymbol{e}=[e,\dot{e},\ddot{e},\ldots,e^{(n-1)}]^T$, $e=r-y$ and $\boldsymbol{k}=[k_1,k_2,\ldots,k_n]^T$. Substituting (3) into (1) results in

$$e^{(n)} = -\boldsymbol{k}^T \boldsymbol{e} = -k_1 e - k_2 \dot{e} \ldots - k_n e^{(n-1)} \tag{4}$$

and the fuzzy input is chosen as:

$$\begin{aligned} \boldsymbol{x} &= [x_1,x_2,\ldots,x_n]^T \\ \Omega_x &= \{\boldsymbol{x} | \boldsymbol{x} \in \Omega_x, \|\boldsymbol{r}\| \le r_0, \|r^{(n)}\| \le r_1\} \end{aligned} \tag{5}$$

On the other hand, if $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are considered to be unknown, a fuzzy logic controller is employed to approximate $u^*(\boldsymbol{x})$. Therefore, the input can be defined as:

$$u = \hat{u}(\boldsymbol{x},\boldsymbol{\theta}) = \sum_{i=1}^m \theta_i \xi_i(\boldsymbol{x}) \tag{6}$$

Substituting (6) into (1) and subtracting it from (3), the following equation is gained:

$$e^{(n)} = -\boldsymbol{k}^T \boldsymbol{e} + [g(\boldsymbol{x})u^*(\boldsymbol{x}) - g(\boldsymbol{x})\hat{u}(\boldsymbol{x},\boldsymbol{\theta})] \tag{7}$$

ε^* can be chosen arbitrarily. $\boldsymbol{\xi}(\boldsymbol{x})=[\xi_1(\boldsymbol{x}),\xi_2(\boldsymbol{x}),\ldots,\xi_m(\boldsymbol{x})]^T$ and an ideal parameter vector $\boldsymbol{\theta}^*=[\theta_1^*,\theta_2^*,\ldots,\theta_m^*]^T$ can be found so that:

$$g(\boldsymbol{x})u^*(\boldsymbol{x}) - g(\boldsymbol{x})\hat{u}(\boldsymbol{x},\boldsymbol{\theta}) = \sum_{j=1}^m c^j(\theta_j^* - \theta_j)\xi_j(\boldsymbol{x}) + \varepsilon \tag{8}$$

in which $|\varepsilon| \le \varepsilon^*$ and c_s^j are some positive constants. See [13]. Defining $\tilde{\boldsymbol{\theta}} = (\boldsymbol{\theta}^* - \boldsymbol{\theta})$, $C = diag([c^1,c^2,\ldots,c^m]^T)$, (7) results in:

$$\begin{aligned} e^{(n)} &= -\boldsymbol{k}^T \boldsymbol{e} + \tilde{\boldsymbol{\theta}}^T C \boldsymbol{\xi}(\boldsymbol{x}) + \varepsilon \\ &\Rightarrow \dot{\boldsymbol{e}} = A_c \boldsymbol{e} + \boldsymbol{b}_c(\tilde{\boldsymbol{\theta}}^T C \boldsymbol{\xi}(\boldsymbol{x}) + \varepsilon) \end{aligned} \tag{9}$$

In which A_c, $\boldsymbol{b}_c$ are:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -k_1 & -k_2 & -k_3 & \ldots & -k_n \end{bmatrix}$$

$$b_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{10}$$

$\boldsymbol{k}$ is chosen so that A_c becomes stable, resulting in an $n \times n$, positive, definite, and symmetric matrix p that satisfies the Lyapunov equation as:

$$A_c^T p + p A_c = -Q \tag{11}$$

in which Q is an arbitrary, $n \times n$, positive, and definite matrix chosen so that $\lambda_{min}(Q) > 1$.

Assumption 3. The upper and lower bounds of the ideal control signal can be determined as $u_L \le u^*(\boldsymbol{x}) \le u_U$. Consequently, the control signal is defined as:

$$\hat{u}(\boldsymbol{x},\boldsymbol{\theta}) = \begin{cases} u_L & \text{if } \sum_{i=1}^m \xi_i(\boldsymbol{x})\theta_i < u_L \\ u_U & \text{if } \sum_{i=1}^m \xi_i(\boldsymbol{x})\theta_i > u_U \\ \sum_{i=1}^m \xi_i(\boldsymbol{x})\theta_i & \text{if } u_L < \sum_{i=1}^m \xi_i(\boldsymbol{x})\theta_i < u_U \end{cases} \tag{12}$$

The above definition does not restrict the plan. It is a reasonable assumption because, in practice, it is essential to choose an actuator that is capable of performing the required control action. For a better application, θ_i is tuned by the following σ-modification robust adaptive law:

$$\dot{\theta}_i = \gamma(\boldsymbol{e}^T p\boldsymbol{b}_c \xi_i(\boldsymbol{x}) - \sigma\theta_i) \tag{13}$$

Theorem 1. Assuming System (1) satisfies Assumptions 1–3, a controller (6) with the adaptive law (13) guarantees that:

(i) The tracking error is bounded by

$$\|\boldsymbol{e}\| \leq \sqrt{\frac{\rho}{\lambda_{min}(Q)-1}}, \quad \forall t>0$$

in which ρ is a positive constant.

(ii) The system is uniformly and ultimately bounded, *i.e.* the tracking error converges to the compact set Ω_e in a finite time:

$$\Omega_{\boldsymbol{e}} = \left\{ \boldsymbol{e}(t) \,|\, \|\boldsymbol{e}(t)\| \leq \sqrt{\frac{\rho}{\lambda_{min}(Q)-1}} \right\}$$

Proof. Consider the Lyapunov function candidate as:

$$V(t) = \frac{1}{2}\boldsymbol{e}^T p\boldsymbol{e} + \frac{1}{2\gamma}\tilde{\boldsymbol{\theta}}^T C\tilde{\boldsymbol{\theta}} \tag{14}$$

$$\dot{V} = \frac{1}{2}\dot{\boldsymbol{e}}^T p\boldsymbol{e} + \frac{1}{2}\boldsymbol{e}^T p\dot{\boldsymbol{e}}^T + \frac{1}{2\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T C\tilde{\boldsymbol{\theta}} + \frac{1}{2\gamma}\tilde{\boldsymbol{\theta}}^T C\dot{\tilde{\boldsymbol{\theta}}} \tag{15}$$

Where $\dot{V}$ shows the derivative of $V(t)$. Substituting (9) into (15) and using the fact that $\boldsymbol{\xi}(\boldsymbol{x})^T C\tilde{\boldsymbol{\theta}}\boldsymbol{b}_c^T p\boldsymbol{e} = \boldsymbol{e}^T p\boldsymbol{b}_c\tilde{\boldsymbol{\theta}}^T C\boldsymbol{\xi}(\boldsymbol{x})$, $\dot{\tilde{\boldsymbol{\theta}}}^T C\tilde{\boldsymbol{\theta}} = \tilde{\boldsymbol{\theta}}^T C\dot{\tilde{\boldsymbol{\theta}}}$, the following equation results:

$$\begin{aligned}\dot{V} &= \frac{1}{2}\boldsymbol{e}^T A_c^T p\boldsymbol{e} + \frac{1}{2}\boldsymbol{e}^T pA_c\boldsymbol{e} + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T C\dot{\tilde{\boldsymbol{\theta}}} \\ &\quad + \boldsymbol{e}^T p\boldsymbol{b}_c\tilde{\boldsymbol{\theta}}^T C\boldsymbol{\xi}(\boldsymbol{x}) + \boldsymbol{e}^T p\boldsymbol{b}_c\varepsilon\end{aligned} \tag{16}$$

Using the adaptive law (13), results in:

$$\begin{aligned}\dot{V} &= -\tfrac{1}{2}\boldsymbol{e}^T Q\boldsymbol{e} - \tilde{\boldsymbol{\theta}}^T C\boldsymbol{e}^T p\boldsymbol{b}_c\boldsymbol{\xi}(\boldsymbol{x}) + \tilde{\boldsymbol{\theta}}^T C\sigma\boldsymbol{\theta} \\ &\quad + \tilde{\boldsymbol{\theta}}^T C\boldsymbol{e}^T p\boldsymbol{b}_c\boldsymbol{\xi}(\boldsymbol{x}) + \boldsymbol{e}^T p\boldsymbol{b}_c\varepsilon\end{aligned} \tag{17}$$

$$\dot{V} = -\tfrac{1}{2}\boldsymbol{e}^T Q\boldsymbol{e} + \tilde{\boldsymbol{\theta}}^T C\sigma\boldsymbol{\theta} + \boldsymbol{e}^T p\,\boldsymbol{b}_c\varepsilon.$$

Considering

$$\begin{aligned}&\tilde{\boldsymbol{\theta}}^T C\boldsymbol{\theta} = \tfrac{1}{2}\boldsymbol{\theta}^{*T} C\boldsymbol{\theta}^* - \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^T C\tilde{\boldsymbol{\theta}} - \tfrac{1}{2}\boldsymbol{\theta}^T C\boldsymbol{\theta}, \\ &\boldsymbol{e}^T p\boldsymbol{b}_c\varepsilon \leq |k_e p\boldsymbol{b}_c\varepsilon^*|, \lambda_{min}(Q)\|\boldsymbol{e}\|^2 < \boldsymbol{e}^T Q\boldsymbol{e}\end{aligned} \tag{18}$$

in which k_e, ε^* are arbitrary and positive constants that satisfy $k_e > |\boldsymbol{e}(t)|$, $\forall t$, $\varepsilon^* > |\varepsilon|$, the following inequality is gained:

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}\lambda_{min}(Q)\|\boldsymbol{e}\|^2 - \frac{\sigma}{2}\tilde{\boldsymbol{\theta}}^T C\tilde{\boldsymbol{\theta}} + \omega, \\ \omega &= \frac{\sigma}{2}\boldsymbol{\theta}^{*T} C\boldsymbol{\theta}^* + |k_e p\boldsymbol{b}_c\varepsilon^*|.\end{aligned} \tag{19}$$

It is easy to find a $\eta>0$ that satisfies the following inequality:

$$\lambda_{min}(Q) > \lambda_{max}(p)\eta, \quad \sigma > \frac{\eta}{\gamma}. \tag{20}$$

$\dot{V}$ can be written as:

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}\eta\lambda_{max}(p)\|\boldsymbol{e}\|^2 - \frac{\eta}{2\gamma}\tilde{\boldsymbol{\theta}}^T C\tilde{\boldsymbol{\theta}} + \omega \\ &\leq -\eta V + \omega\end{aligned} \tag{21}$$

$$V(t) \leq e^{-\eta t}\left(V(0) - \frac{\omega}{\eta}\right) + \frac{\omega}{\eta}$$

$$\Rightarrow V(t) \leq max\left(V(0), \frac{\omega}{\eta}\right) \forall t>0. \tag{22}$$

Substituting (22) into (14) results in:

$$\|e\| \leq \sqrt{\frac{2max(V(0), \omega/\eta)}{\lambda_{min}(P)}}, \quad \forall t>0. \tag{23}$$

Using the fact that $\boldsymbol{e}^T p\boldsymbol{b}_c\varepsilon \leq \frac{1}{2}\|\boldsymbol{e}\|^2 + \frac{1}{2}\|p\boldsymbol{b}_c\|^2\|\varepsilon^*\|^2$, results in:

$$\begin{aligned}\dot{V} &\leq -\tfrac{1}{2}\lambda_{min}(Q)\|\boldsymbol{e}\|^2 + \tfrac{1}{2}\|\boldsymbol{e}\|^2 + \tfrac{1}{2}\rho, \\ \rho &= \sigma\boldsymbol{\theta}^{*T} C\boldsymbol{\theta}^* + \|p\boldsymbol{b}_c\|^2\|\varepsilon^*\|^2 \\ &\Rightarrow \dot{V} \leq -\tfrac{1}{2}(\lambda_{min}(Q)-1)\|\boldsymbol{e}\|^2 + \tfrac{1}{2}\rho.\end{aligned} \tag{24}$$

As $\lambda_{min}(Q)>1$, (24) shows that $\dot{V}$is negative for $(\lambda_{min}(Q)-1)\|\boldsymbol{e}\|^2>\rho$. This means that the system is ultimately upper bound and $\boldsymbol{e}(t)$ converges to the compact set Ω_e in a finite time:

$$\Omega_e = \left\{ e(t) \,|\, \|e(t)\| \leq \sqrt{\frac{\rho}{\lambda_{min}(Q)-1}} \right\} \tag{25}$$

Remark. It is noted that Theorem 1 is a result of semi-global stability. Since the function approximation property of the adaptive fuzzy system is only guaranteed within a compact set, the stability result proposed in this paper is semi-global in the sense that, for any compact set, there exists a controller with a sufficient number of rules and proper adaptive learning rules that all closed loop signals are bounded when initial states are within this compact set [16, 26]. □

IV. FUZZY SYSTEM TUNING

Due to their fixed structures, classic fuzzy systems are not flexible, especially when it comes to control problems. Therefore, tuning them may be very helpful for having fuzzy systems with good tracking properties. In our proposed method, the number of membership functions in defined fuzzy sets for each state is fixed, resulting in a fixed number of rules. Tuning of the system is done by adjusting the span and domain intervals of the membership functions. x is supposed to be the state signal and the input of the fuzzy system, and r is the reference signal that has to be tracked by the state signal. For example, consider the membership functions and reference signal shown in Fig. 1. When $x=x_1$ the fuzzy rules have a satisfactory effect on the convergence of x to r. When $x=x_2$, however, the fuzzy rules are not very effective because the ratio of the span of membership functions to the tracking error is very large. Therefore, the span should be reduced and the membership functions will need to be updated afterwards.

As shown in Fig. 2, n triangular membership functions are described as $(r+\beta_{i1}\Delta, r+\beta_{i2}\Delta, r+\beta_{i3}\Delta)$, $i=1,2,\ldots,n$, which specify lower, center, and upper bounds of each triangle, respectively. So, when the reference signal r varies, the interval and span of the membership functions can be improved by adjusting Δ. For convenience, the above membership functions can be shown in a matrix as:

$$z=\begin{bmatrix} r+\beta_{11}\Delta & r+\beta_{12}\Delta & r+\beta_{13}\Delta \\ r+\beta_{21}\Delta & r+\beta_{22}\Delta & r+\beta_{23}\Delta \\ \vdots & \vdots & \vdots \\ r+\beta_{n1}\Delta & r+\beta_{n2}\Delta & r+\beta_{n3}\Delta \end{bmatrix} \tag{26}$$

Updating of the membership function is done by varying Δ for each updating time t_u. The updating time

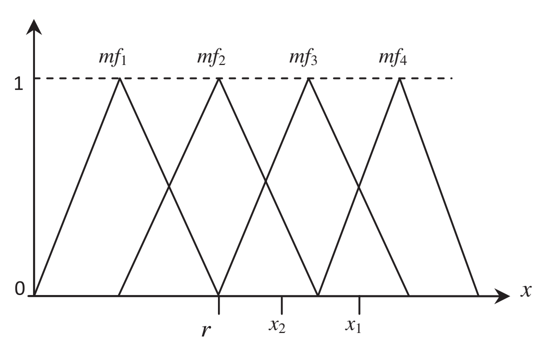

Fig. 1. Membership functions, input, and reference.

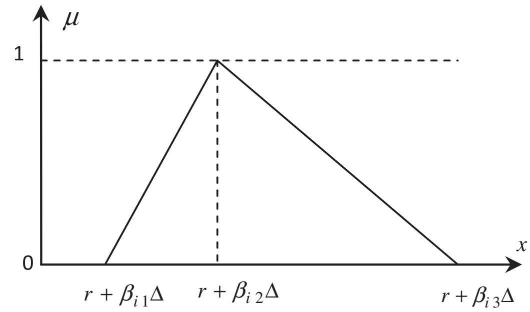

Fig. 2. Triangular membership functions.

depends on the complexity of the system, the required minimum tracking error, and the convergence rate. A recommended matrix z may be:

$$z=\begin{bmatrix} r-2\Delta & r-\Delta & r \\ r-\Delta & r & r+\Delta \\ r & r+\Delta & r+2\Delta \\ r+\Delta & r+2\Delta & r+3\Delta \end{bmatrix}$$

It should be noted that, according to the problem statement in Section II, there are reference signals r, $r,\ldots,r^{(n-1)}$, for state variables x_1, $x_2,\ldots,x_n$, respectively. It is obvious that if Δ is chosen to be a factor of absolute error value (*e.g.* $\Delta=10|e|$), the span of the introduced membership functions is tuned automatically and a good tracking possibility is achieved (see Fig. 3). Defining parameters L_1, L_2, b_1 and b_2 as $1\le L_1, 0<L_2\le 1$, $1\le b_1$ and $0<b_2\le 1$, Δ can be adjusted by the following law:

$$\Delta_{new}=\begin{cases} L_1\Delta_{old} & \text{if } |e|>b_1 \text{ and } \Delta_{old}<\Delta_{max} \\ L_2\Delta_{old} & \text{if } |e|<b_2 \text{ and } \Delta_{old}>\Delta_{min} \\ \Delta_{old} & \text{else} \end{cases} \tag{27}$$

Two new constant parameters Δ_{max} and Δ_{min} are defined to prevent Δ from becoming very small or very large. Therefore, for very small or very large errors, Δ is constant all the time.

V. SIMULATION RESULTS

Two applications are introduced for the proposed method. The first is an inverted pendulum problem, and the second is a magnetic levitation system.

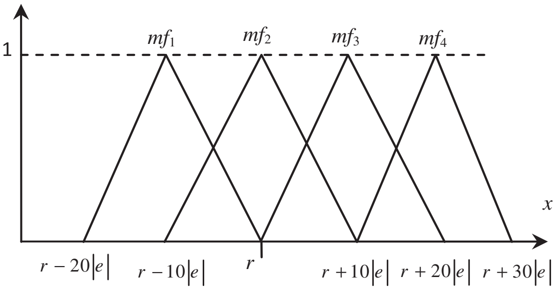

Fig. 3. Membership function for $\Delta = 10|e|$.

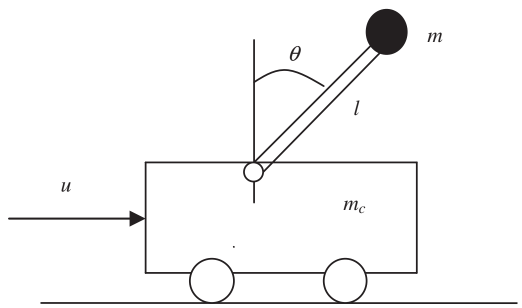

Fig. 4. Inverted pendulum.

5.1 Inverted pendulum problem

The dynamics of the inverted pendulum are shown in Fig. 4 and can be described as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \left[g \sin x_1 - \frac{m.l.x_2^2.\cos x_1 .\sin x_1}{m_c + m} \right] \frac{1}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)} + \frac{\frac{\cos^2 x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)} u = f(\boldsymbol{x}) + g(\boldsymbol{x})u$$
$$y = x_1$$

where x_1 is the angular position of the pendulum, x_2 is its angular velocity, m_c is the cart mass, m is the pendulum mass, and l is the half-length of the pendulum. The following data are used in performing the simulations:

$$m_c = 1\,\text{kg}, \quad m = 0.1\,\text{kg}, \quad l = 0.5\,\text{m}$$
$$b_1 = 1.5, \quad b_2 = 0.3, \quad L_1 = 1.75,$$
$$L_2 = 0.75, \quad \boldsymbol{k} = [1, 1]^T$$

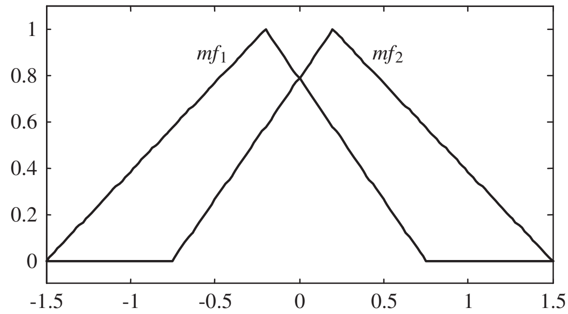

Fig. 5. Initial membership functions for x_1 and x_2.

$$\Delta_{initial} = 0.5, \quad u_L = -15, \quad u_U = 15$$
$$\text{Sampling Time} = 0.01s$$

Initial membership functions for x_1 and x_2 are shown in Fig. 5. The proposed fuzzy system contains only four rules and the structure of membership functions is updated according to the following z matrix.

$$z = \begin{bmatrix} r-3 & r-0.4\Delta & r+1.5\Delta \\ r-1.5\Delta & r+0.4\Delta & r+3\Delta \end{bmatrix}$$

First, the regulation problem is investigated, *i.e.* from an arbitrary initial condition, the angular position should become zero. The proposed controller is applied to the system with the initial conditions $x_{10} = 20\,\text{deg}$, $x_{20} = 0$ and updating time $t_s = 0.03\,\text{s}$. The controller parameters are chosen as:

$$min(\Delta) = 0.3, \quad P = \begin{bmatrix} 100 & 20 \\ 20 & 10 \end{bmatrix},$$
$$Q = \begin{bmatrix} 40 & 0 \\ 0 & 80 \end{bmatrix}, \quad \gamma = 500$$

The system response and the control signal are shown in Figs 6 and 7. It is obvious that the performance is satisfactory.

To show the effectiveness of our proposed controller in the tracking problem, a sinusoidal reference signal $r = 0.5\sin(t)$ is applied to the system. The control signal, system output, and tracking error are shown in Figs 8 to 10.

5.2 Magnetic levitation system

The objective of this example is to control the position of a magnet suspended above an electromagnet. The magnet is constrained so that it can only move in the vertical direction (Fig. 11). The dynamics of the

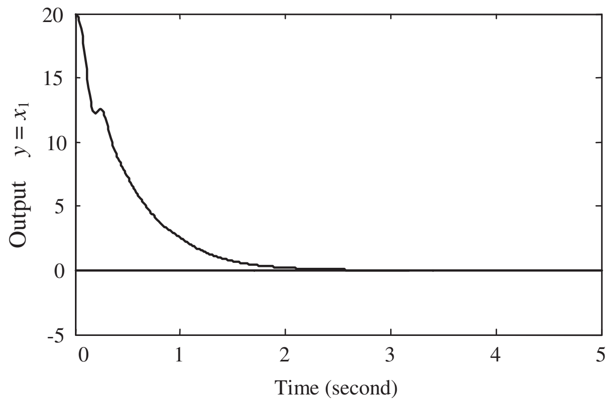

Fig. 6. System response to an arbitrary initial condition.

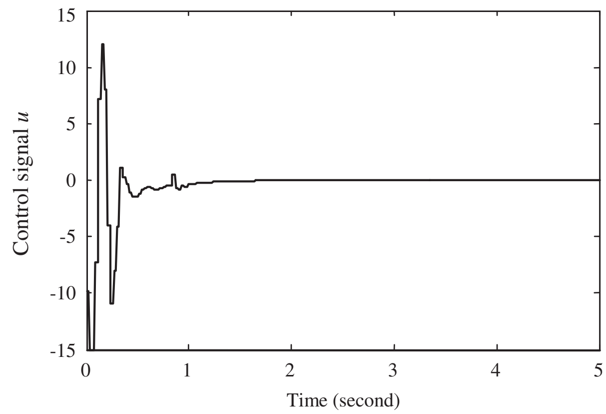

Fig. 7. Control signal for regulation problem.

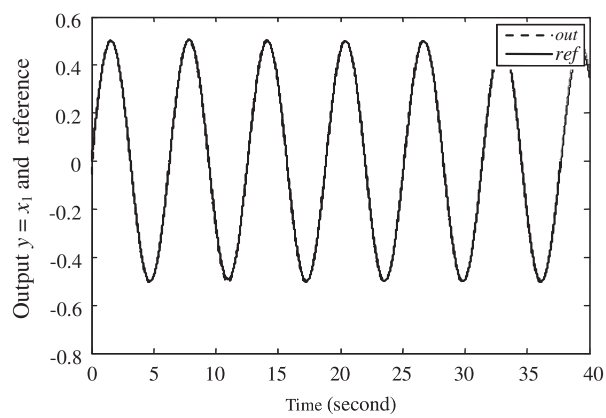

Fig. 8. System response to sinusoidal reference.

system and control action are:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -g - \frac{\beta}{m}x_2 + \frac{\alpha}{mx_1}u$$
$$y = x_1$$
$$u = sgn(i)i(t)^2$$

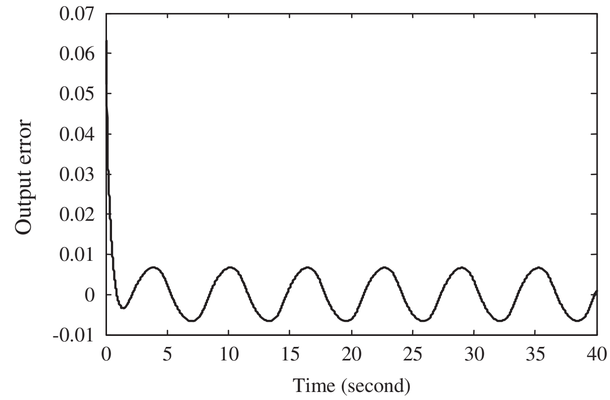

Fig. 9. Tracking error for the sinusoidal reference.

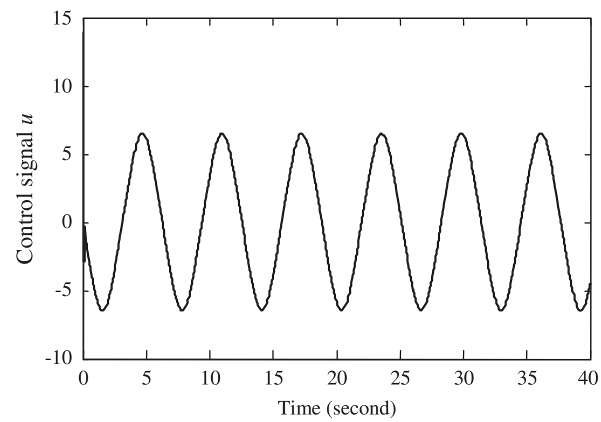

Fig. 10. Control signal for the sinusoidal reference.

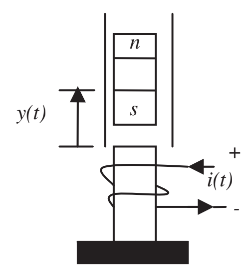

Fig. 11. Magnetic levitation system.

in which y is the distance from the top of the electromagnet, $i(t)$ is the current flowing in the electromagnet, m is the magnet mass, g is the gravitational constant, β is a viscous friction coefficient determined by the material in which the magnet moves, and α is the field strength constant determined by the number of wire turns around the electromagnet and the magnet strength.

The data is assumed to be as in [13], *i.e.* $m = 3$ kg, $\alpha = 15$ and $\beta = 12$. The desired position $y_d(t)$ is set randomly in the [0.5 cm, 4 cm] range, and the current

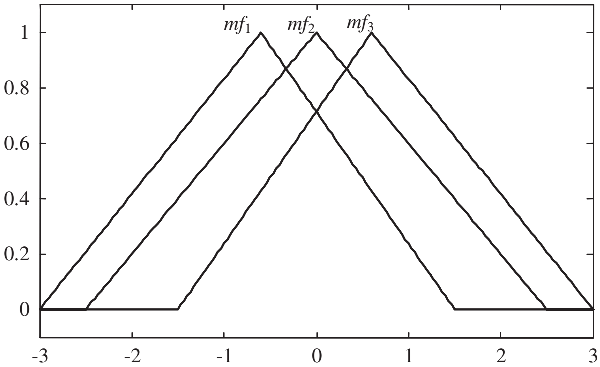

Fig. 12. Initial membership functions.

flowing in the electromagnet is constrained to the $[-5A, 5A]$ range. The initial membership functions, shown in Fig. 12, are in accordance with the following z matrix:

$$z = \begin{bmatrix} r-3\Delta & r-0.4\Delta & r+1.5\Delta \\ r-2.5\Delta & r & r+2.5\Delta \\ r-1.5\Delta & r+0.4\Delta & r+3\Delta \end{bmatrix}$$

The reference trajectory is generated using a reference model with the following transfer function:

$$\frac{y_{ref}(s)}{y_d(s)} = \frac{4}{(s+2)^2}$$

In this simulation, the variable ranges are considered as:

$$x_1 \in [0, 5], \quad x_2 \in [-5, 10]$$

and the initial conditions are determined as:

$$[x_{10}, x_{20}] = [0.5cm, 0],$$

The updating time is also considered to be $t_s = 0.03s$

and the controller parameters are:

$$[k_1, k_2] = [1, 1]; \quad \Delta_{min} = 1; \quad P = \begin{bmatrix} 10 & 7 \\ 7 & 9 \end{bmatrix};$$

$$Q = \begin{bmatrix} 10 & 1.5 \\ 1.5 & 4 \end{bmatrix} \quad b_1 = 1.5; \quad b_2 = 0.3;$$

$$L_1 = 1.75; \quad L_2 = 0.75; \quad \gamma = 200; \quad \Delta_{initial} = 2.$$

Figures 13 to 15 show the simulation results. It is obvious that the actual output closely follows the reference trajectory. Therefore, the proposed controller successfully controls the position of the magnet. When the set-point is changed, the tracking error converges quickly to zero. The control signal is always in the desired $[-5A, 5A]$ range with a smooth deviation and without any undesired oscillation, in comparison with simulation results in [13].

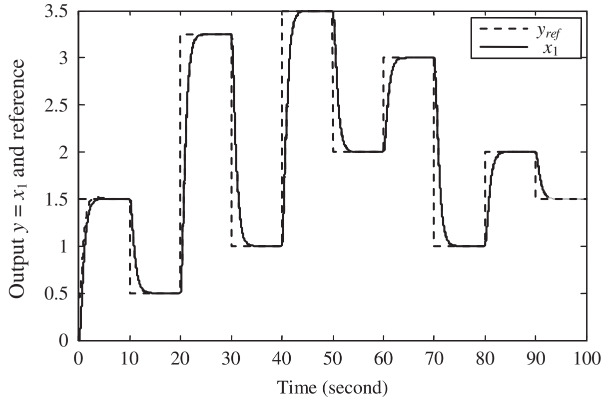

Fig. 13. System output and reference signal.

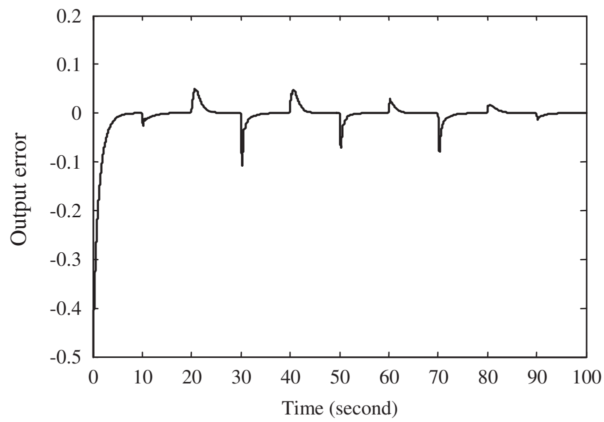

Fig. 14. Output error.

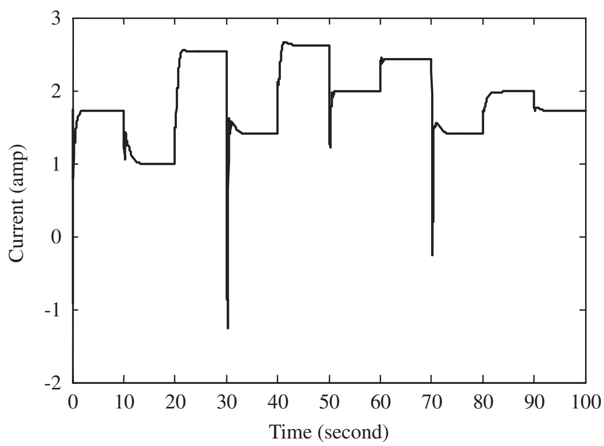

Fig. 15. Control signal (amp).

VI. CONCLUSION

A direct adaptive fuzzy control scheme for affine nonlinear systems was proposed in this paper. The only restriction for the control gain was the controllability condition $g(\boldsymbol{x})>0$. Control was accomplished by an adaptive fuzzy system with a fixed number of rules and adaptive membership functions. The reference signal and the state errors were used to tune the membership functions and update them instantaneously. The Lyapunov synthesis method was used to guarantee the stability of the closed loop system. The proposed control scheme was applied to an inverted pendulum and a magnetic levitation system, and its effective properties were shown by performing some simulations. Using a lower number of rules in comparison with [13], our proposed controller has a more suitable response and a very smooth control action.

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