

An Efficient Timing and Frequency Synchronization Algorithm for OFDMA/TDD Mode in Downlink of IEEE 802.16-2004 and its Analysis

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Abstract: Timing and frequency synchronizations are the important tasks in receiver. In this paper, an efficient timing and carrier frequency offset (CFO) estimation algorithm for orthogonal frequency division multiple access/ time division duplexing (OFDMA/TDD) mode in IEEE 802.16-2004 standard (WiMAX) is represented. This method has high performance and jointly estimates timing error and CFO. The performance is evaluated in AWGN channel. This algorithm can be easily used for other OFDM based systems which receiver knows preamble by itself.

Key words: timing synchronization, carrier frequency offset estimation, IEEE 802.16, AWGN channel.

INTRODUCTION

Orthogonal frequency division multiplexing has so applications in broadband communication systems such as wireless metropolitan area network (WiMAX) (IEEE std. 802.16-2004; IEEE std. 802.16e-2005). OFDM systems are very sensitive to timing and frequency errors as all the tasks in receiver will be feckless with imperfect synchronization (Pollet *et al.*, 1995). Thus timing and frequency synchronization in OFDM systems are very important task. Timing error and CFO lead to inter-symbol interference (ISI) and inter-carrier interference (ICI) respectively. Some methods depend on preamble (Wu and Zhu, 2005; Schmidl and Cox, 1997; Kishore and Reddy, 2006), while the other methods are based on cyclic prefix (CP) (VandeBeek *et al.*, 1997; Zhou *et al.*, 2001). Synchronization algorithms for OFDMA-based systems in downlink are the same as synchronization algorithms for OFDM-based systems. We can't use previous methods in OFDMA mode in WiMAX, because of preamble features. We should modify these methods for WiMAX preamble to use them in downlink transmission. The first symbol of downlink transmission is preamble. The preamble of OFDMA mode in WiMAX has size of 2048 in time domain, which makes use of boosted BPSK modulation with specific pseudo-noise (PN) code (IEEE std. 802.16-2004). Accordingly, there are 114 different preamble signals. Each of them is served at a particular segment of a cell. Each cell is divided to three segments: 0, 1, and 2. The preamble carrier-sets are defined using equation (1) (IEEE std. 802.16-2004):

$$\text{Preamble carrier set}_n = n + 3k \quad (1)$$

n and k are the number of preamble carrier set indexed 0, 1, 2 and a running index 0, 1, ..., 567 respectively. Each segment uses a preamble composed of a carrier set out of the three available carrier set in the following manner (IEEE std. 802.16-2004):

- Segment 0 uses preamble carrier set 0.
- Segment 1 uses preamble carrier set 1.
- Segment 2 uses preamble carrier set 2.

Therefore each segment eventually modulates each third subcarrier. Thus we have 3 blocks (A, B, and C) with length of 682 in time domain that they are not same to each other, but they have good cross correlation. This matter is because of 2048 is not divisible to 3. Blocks A, B, and C begin from 0 to 681, 682 to 1363, and 1366 to 2047 respectively. In (Pushpa *et al.*, 2008) some methods are modified for this preamble. In WiMAX, the user situated in a cell is known by IDcell and number of segment and has its own preamble.

In section 2, system model is presented in AWGN channel. In section 3, Reddy algorithm is presented. In section 4, the proposed timing metric is presented. In section 5, analysis of proposed algorithm is driven. In section 6, simulation results are demonstrated and finally the conclusion is given in section 7.

System Model:

The transmitted samples in baseband OFDM modulation can be written as:

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} C_k e^{j \frac{2\pi n k}{N}}; \quad -N_g \leq n \leq N - 1 \quad (2)$$

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Where N is IFFT length. N_g , C_k , and n are cyclic prefix length, preamble data in subcarrier k , and timing sample index respectively. We concentrate on preamble symbol. n th sample of received signal in AWGN channel is given by:

$$r(n) = e^{\frac{j2\pi\epsilon n}{N}} s(n) + v(n); -N_g \leq n \leq N - 1 \tag{3}$$

Where ϵ and $v[n]$ are carrier frequency offset normalized to the subcarrier spacing and zero mean complex AWGN with variance σ_v^2 respectively.

Reddy algorithm:

The Reddy algorithm is one of the most important algorithms that the receiver knows the preamble. The modified Reddy's timing metric is given by (Pushpa et al., 2008):

$$M(d) = \frac{|P(d)|^2}{(R(d))^2} \tag{4}$$

where

$$P(d) = \sum_{n=0}^{G-1} (r(n+d)s(n+N_g))^* (r(n+d+M)s(n+N_g)) \tag{5}$$

$$R(d) = \sum_{n=0}^{G-1} |r(n+d+M)|^2 \tag{6}$$

A^* means conjugate A . $G = \lfloor \frac{N}{3} \rfloor$, where $\lfloor a \rfloor$, means floor(a). If $M = G$, cross-correlation between blocks A and B is considered. If $M = 2G + 2$ (or $M = N - G$), cross-correlation between blocks A and C is considered.

Proposed Algorithm (PA):

By knowing the preamble in receiver, we propose a timing metric that has good performance even in low SNR and use it to estimate the CFO. The proposed timing metric is given by:

$$M(d) = \frac{|P(d)|}{R(d)} \tag{7}$$

Where

$$P(d) = (\sum_{n=0}^{G-1} r^*(n+d)s(n+N_g)) (\sum_{n=0}^{G-1} r(n+d+M)s^*(n+M+N_g)), \tag{8}$$

$$R(d) = (\sum_{n=0}^{G-1} |r(n+d)|^2) (\sum_{n=0}^{G-1} |r(n+d+M)|^2), \tag{9}$$

If $M = G$, cross-correlation between blocks A by A and B by B from received preamble and save preamble is considered. if $M = 2.G + 2$, cross-correlation between blocks A by A and C by C from received preamble and save preamble is considered. The index of maximum of metric shows the start of symbol. We use this index (d_p) to estimate CFO. CFO estimator is given by:

$$\hat{\epsilon} = \frac{N}{2\pi M} \angle(P(d_p)). \tag{10}$$

Sign \angle , means measured angle. We have CFO because of the change in environment and mismatching between transmitter and receiver oscillators. CFO consist of fractional and integer parts. In fixed-WiMAX, CFO is smaller than subcarrier spacing (fractional). In this paper fractional part of CFO will be considered.

Analysis of proposed Timing Metric:

In this section Expectation timing metric is derived and Variance of it in appendix is calculated. $P(d)$ in AWGN channel is:

$$P(d) = \sum_{i=0}^{G-1} \left[e^{-\frac{j2\pi\epsilon(i+d)}{N}} s^*(i+d) + v^*(i+d) \right] s(i+N_g) \sum_{i=0}^{G-1} \left[e^{\frac{j2\pi\epsilon(i+d+M)}{N}} s(i+d+M) + v(i+d+M) \right] s^*(i+M+N_g) \tag{11}$$

At $d = d_p$, d_p means index d at optimum place that is equal N_g . $s(i+d_p) = s(i+N_g)$ and $(i+d_p+M) = s(i+M+N_g)$. $|s(i+d_p)|^2 |s(k+d_p+M)|^2$ has a phase $\varphi = \frac{2\pi\epsilon M}{N}$, then $P(d)$ given in (11) can be broken into parts that are inphase and quadrature phase to $|s(i+d_p)|^2 |s(k+d_p+M)|^2$, similar to that given

in (Schmidl and Cox, 1997). For moderate values of SNR, the magnitude of the quadrature part is small compared to that of inPhase part and can be neglected (Schmidl and Cox, 1997). Then, upper bound of $|P(d_p)|$ is:

$$|P(d_p)| \leq e^{j\varphi} \sum_{i=0}^{G-1} \sum_{k=0}^{G-1} |s(i+d_p)|^2 |s(k+d_p+M)|^2 + \text{inphase}_\varphi \left\{ v^*(i+d_p)s(i) |s(k+d_p+M)|^2 + v(k+d_p+M)s^*(k+M) |s(i+d_p)|^2 + s(i)s^*(k+M)v^*(i+d_p)v(k+d_p+M) \right\} \quad (12)$$

And estimate of $R(d)$ is (Kishore and Reddy, 2006):

$$R(d) = (\sum_{i=0}^{G-1} |s(i+d)|^2 + |v(i+d)|^2 + 2\text{Re}\{s^*(i+d)v(i+d)\}) (\sum_{i=0}^{G-1} |s(i+d+M)|^2 + |v(i+d+M)|^2 + 2\text{Re}\{s^*(i+d+M)v(i+d+M)\}) \quad (13)$$

From *central limit theorem*, both $|P(d_p)|$ and $R(d_p)$ are Gaussian distributed. Expectations of them are according by:

$$E[|P(d_p)|] = \mu_{|P(d_p)|} \leq \sum_{i=0}^{G-1} \sum_{k=0}^{G-1} |s(i+d_p)|^2 |s(k+d_p+M)|^2 = E_b^2 \quad (14)$$

$E[]$ and E_b , mean Expectation and energy of each block of preamble respectively. the energy of each block is equal to each other approximately.

$$E[R(d_p)] = \mu_{R(d_p)} = \sum_{i=0}^{G-1} \sum_{k=0}^{G-1} (|s(i+d_p)|^2 + \sigma_v^2) (|s(k+d_p+M)|^2 + \sigma_v^2) = (E_b + G\sigma_v^2)^2 \quad (15)$$

From (Van Kempen and Van Vliet, 2000), expectation and variance of $\frac{|P(d)|}{R(d)}$ are obtained:

$$\mu_{M(d_p)} = E \left[\frac{|P(d)|}{R(d)} \right] \approx \frac{\mu_{|P(d)|}}{\mu_{R(d_p)}} + \frac{\sigma_{R(d_p)}^2 \mu_{|P(d)|}}{\mu_{R(d_p)}^3} + \frac{\text{cov}(|P(d_p)|, R(d_p))}{\mu_{R(d_p)}^2} \quad (16)$$

$$\sigma_{M(d_p)}^2 = \frac{\sigma_{|P(d_p)|}^2}{\mu_{R(d_p)}^2} + \frac{\sigma_{R(d_p)}^2 \mu_{|P(d_p)|}^2}{\mu_{R(d_p)}^4} - \frac{2\text{cov}(|P(d_p)|, R(d_p)) \mu_{|P(d)|}}{\mu_{R(d_p)}^3} \quad (17)$$

By definition $= \frac{E_b}{G\sigma_v^2}$, it can be seen that by increasing SNR, $\mu_{M(d_p)}$ goes to one. But for Reddy algorithm by increasing SNR, $\mu_{M(d_p)}$ goes to $\frac{\sum_{i=0}^{G-1} |s(i)|^4}{E_b^2}$ that in WiMAX preamble is 0.17 approximately and smaller than one. But for variance in d_p simulation results indicate, both of algorithms have variances near to zero.

Simulation Results:

OFDM system with 2048 subcarriers and 256 samples for CP and AWGN channel is considered. The performance of proposed algorithm is evaluated by computer simulations. Simulation results demonstrate that this timing metric can achieve more accurate timing offset and CFO estimation than Reddy algorithm. Fig. 1, indicates timing metric for theory and simulation with $2\pi\epsilon=3$ and SNR=0 dB for proposed algorithm with $M=G$ and $M=N-G$ can be seen theory and simulation coincide to each other. Fig. 2, also runs for Reddy algorithm in these qualifications. The largest peak is the first of symbol. We can see that proposed timing metric has better performance for symbol detecting. Fig. 3, indicates comparison estimation of CFO ($2\pi\epsilon=3$) in various SNR between two algorithms by mean square error (MSE) criterion. Based on Fig. 3, the proposed algorithm has the better performance than Reddy algorithm.

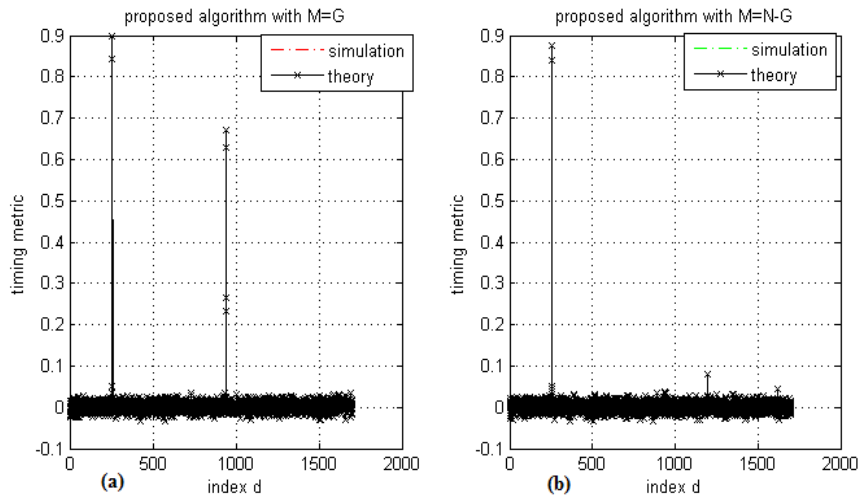


Fig. 1: Proposed timing metric, theory and simulation. (a) M=G and (b) M=N-G.

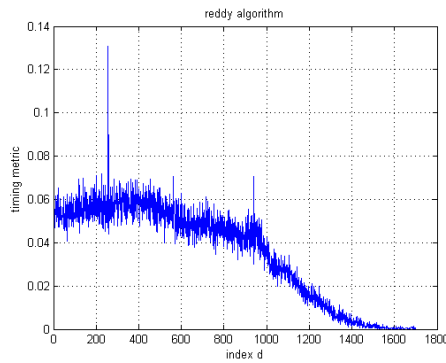


Fig. 2: Timing metric for reddy algorithm.

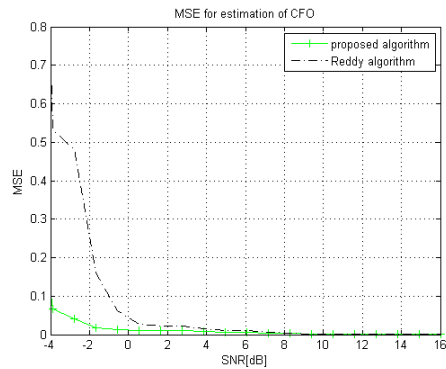


Fig. 3: MSE for estimation of CFO ($2\pi\epsilon=3$).

Conclusion:

In this paper, we proposed an algorithm for timing metric that can be used for CFO estimation. This algorithm can be used for downlink in OFDM(A)-based systems that preamble is known by receiver. We compare the performance of the proposed algorithm with Reddy algorithm. Its analysis and Simulation results indicate that the proposed algorithm has high performance even in low SNR in AWGN channel.

Appendix:

In this appendix, variances of $|P(d_p)|$ and $R(d_p)$ using formulas (12) and (13) are derived.

$$\sigma_{|P(d_p)|}^2 = E \left[\left(|P(d_p)| - \mu_{|P(d_p)|} \right)^2 \right] = E \left[\left(\sum_{i=0}^{G-1} \sum_{k=0}^{G-1} \text{inphase}_\varphi \left\{ v^*(i+d_p) s(i) |s(k+d_p+M)|^2 + v(k+d_p+M) s^*(k+M) |s(i+d_p)|^2 + s(i) s^*(k+M) v^*(i+d_p) v(k+d_p+M) \right\} \right)^2 \right] = E_b^3 \sigma_v^2 + \frac{E_b^2 \sigma_v^4}{4} \quad (\text{A.1})$$

From (13) variance of $R(d_p)$ can be calculated as:

$$\sigma_{R(d_p)}^2 = E \left[\left(R(d_p) - \mu_{R(d_p)} \right)^2 \right] = E \left[\left(|s(i+d_p)|^2 |v(k+d_p+M)|^2 + |s(k+d_p+M)|^2 |v(i+d_p)|^2 + |v(i+d_p)|^2 |v(k+d_p+M)|^2 + 2 |s(i+d_p)|^2 \text{Re} \{ s^*(k+d_p+M) v(k+d_p+M) \} + 2 |v(i+d_p)|^2 \text{Re} \{ s^*(k+d_p+M) v(k+d_p+M) \} + 2 |s(k+d_p+M)|^2 \text{Re} \{ s^*(i+d_p) v(i+d_p) \} + 2 |v(k+d_p+M)|^2 \text{Re} \{ s^*(i+d_p) v(i+d_p) \} + 4 \text{Re} \{ s^*(i+d_p) v(i+d_p) \} \text{Re} \{ s^*(k+d_p+M) v(k+d_p+M) \} - 2 E_b G \sigma_v^2 - G^2 \sigma_v^4 \right)^2 \right] = \sigma_v^2 (4 E_b^3) + \sigma_v^4 (4 E_b^2 + 12 E_b^2 G) + \sigma_v^6 (12 E_b G^2 + 8 E_b G + 6 G^3 - 6 G^2) + \sigma_v^8 (10 G^2 - 2 G^3) \quad (\text{A.2})$$

From (12) and (13) covariance between $|P(d_p)|$ and $R(d_p)$ is derived as:

$$\text{cov} \left(|P(d_p)|, R(d_p) \right) = E \left[\left(|P(d_p)| - \mu_{|P(d_p)|} \right) \left(R(d_p) - \mu_{R(d_p)} \right) \right] = \sigma_v^2 (2 E_b^3) + \sigma_v^4 (E_b^2 + 2 E_b^2 G) \quad (\text{A.3})$$

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