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Multi-objective genetic algorithm for economic statistical design of \bar{X} control chart

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Abstract Control charts are widely used for monitoring the quality of a product or a process. Their implementation cost motivates researchers to design them with the lowest cost and most desirable statistical properties. Usually, the cost function is optimized subject to statistical properties. However, the cost function also depends on statistical properties, and minimizing it as the only objective is not an efficient method of economic statistical design of control charts. In this paper, cost function, as well as statistical properties, including probability of Type I error, power of \bar{X} control chart, and Average Time to Signal (ATS), are considered as objectives; the corresponding constraints are also used. Then, a Multi-Objective Genetic Algorithm for Economic Statistical Design (MOGAESD) is proposed for identifying the Pareto optimal solutions of control chart design. The preferred solution is selected by the designer. The performance of the proposed method is compared through some numerical examples reported in the literature. The results show that the proposed approach is effective.

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1. Introduction

Control charts are well known tools for monitoring various processes and detecting assignable causes and potential improvements in the processes. Since Shewhart [1] first proposed the control chart, many types of control chart have been developed by researchers (for example, see [2]). The design of a control chart includes determining the sample size (n), the sampling frequency or time interval between samples (h), and the coefficient of control limits (k). These parameters should be determined so as to satisfy economic considerations. Thus, proper design of the control chart, including optimization of the related parameters according to economic issues, is necessary.

To this end, some cost functions have been proposed in the literature (see for example [3,4]). Subsequently, some authors

focused on minimizing the cost function, an approach referred to as economic design (for example, see [5]). According to Woodall [6], however, economic design of a control chart leads to poor statistical properties. Hence, design of a control chart is now performed with the goal of minimizing the cost function subject to statistical properties; this is known as economic statistical design of a control chart (for examples of this, see [7]). A constraint approach is not an efficient method for economic statistical design of control charts because statistical properties are of the same importance as economic properties and should be optimized simultaneously.

Multi-objective economic statistical design was first introduced by Evans and Emberton [8] for joint \bar{X} and R control charts. In their approach, multiple objectives, including cost function and statistical properties, are optimized simultaneously. Therefore, optimal design of a control chart is represented as a Multiple Criteria Decision-Making (MCDM) problem.

There are two methods in multi-objective problems: aggregative and non-aggregative. In aggregative methods, all objectives are considered in one weighted function. This method has two main drawbacks: firstly, only one solution is provided, and thus, it is not a reasonable method to be used in a multi-objective approach; secondly, Fleming [9] showed that this method is unable to deal with a concave Pareto front in multi-objective problems.

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Using this procedure, Celano and Fichera [10] proposed a weighted function of all economic and statistical objectives for the \bar{X} control chart and optimized it using a Genetic Algorithm (GA). Zarandi et al. [11] modeled the multi-objective economic statistical design of the \bar{X} control chart with fuzzy parameters and used Adaptive Neuro-Fuzzy Inference System (ANFIS) to make the complex model fuzzy rule based, then, an aggregative multi-objective function was proposed and optimized by a GA.

In non-aggregative methods, all objectives are optimized simultaneously. Unfortunately, in most cases, there is not a unique solution that optimizes all objectives simultaneously. Therefore, Pareto optimal solutions, which have a better value for at least one objective, are obtained, forming a Pareto optimal set.

With this procedure, Del Castillo et al. [12] obtained the optimal design of the \bar{X} control chart using Mackin's [13] proposed multi-objective optimization method. Chen and Liao [14] applied a non-aggregative approach to the \bar{X} control chart; this produced a Pareto optimal set in a discrete space, from which, they then selected the efficient solution(s) using Data Envelopment Analysis (DEA). A similar approach was implemented by Asadzadeh and Khoshalhan [15] in the case of multiple assignable causes. Recently, Safaei et al. [16] proposed an economic statistical multi-objective model to design the \bar{X} control chart considering Taguchi loss function. They obtained the Pareto frontier of the objectives using NSGA-II.

These approaches to the design of control charts, including their advantages and disadvantages, are summarized in Table 1.

In this paper, we consider a non-aggregative, multi-objective model to obtain the economic statistical design of the \bar{X} control chart considering multiple assignable causes. The main contribution of this paper is adding ATS as an objective, as well as considering the expected cost in constraints compared to the traditional mathematical models. Note that considering

ATS as an objective can lead to improvement in this statistical property. However, this may surely affect the value of cost because there is a trade-off between the ATS and the expected cost. In other words, decreasing the ATS leads to increasing the expected cost. However, we control the increasing in the expected cost with a constraint in the model. It means we allow the increase of the expected cost, up to the values desired, for the decision maker. Another contribution of the paper is proposing a new multi-objective optimization tool (MOGAESD), based on the GA to obtain the Pareto optimal set. Besides using the great advantages of meta-heuristics optimization tools, we proposed considering a continuous region for design parameters, which led to achieving better results. To evaluate the performance of the proposed method, some numerical examples reported in the literature are solved. In Table 2, the scope of this paper, compared to that of other papers using the multi-objective approach, is shown.

The structure of the paper is organized as follows: the multi-objective model is illustrated in Section 2, and an MOGAESD that uses an interactive non-aggregative procedure as the optimization method is proposed in Section 3. Section 4 includes some numerical examples solved by the proposed method, as well as comparisons between the proposed method and other existing methods. Concluding remarks and some future researches are given in the final section.

2. Multi-objective model

In our multi-objective economic statistical design of the \bar{X} control chart, we are going to obtain the sample size (n), sampling frequency (h), and control limit width (k), such that the cost function (L), probability of Type I error (α), power of the control chart (p), and ATS are simultaneously optimized. Specifically, we aim to achieve the minimum values of L , α , and ATS,

Table 1: Different approaches for design of control charts.

Economic design		Economic statistical design		
		Constraint approach	Multi objective approach	
			Aggregative method	Non-aggregative
Advantages	Lowest cost	Minimum cost subject to desired statistical properties	Optimal cost and statistical properties based on one desired function	Set of Pareto designs
Disadvantages	Poor statistical properties	Statistical properties isn't optimized	Two main weaknesses ^a	–
Sample paper	Ho and Case [5]	Celano [7]	Zarandi et al. [11]	Asadzadeh and Khoshalhan [15]

^a Firstly, only one solution can be provided. Secondly, Fleming [9] showed its inability in concave Pareto front problems.

Table 2: Multi objective economic statistical design of \bar{X} control chart.

Research	Assumption	Objectives	Constraints	Optimizer	Approach
Del Castillo et al. [12]	Single assignable cause, crisp parameters	Sampling cost, average false alarm, ATS	Probability of Type I error, detection power	Mackin [13]	Non aggregative
Celano and Fichera [13]	Single assignable cause, crisp parameters	Cost, probability of Type I error, detection power	No constraint	GA	Aggregative
Chen and Liao [14]	Single assignable cause, crisp parameters	Cost, in control average run length, detection power	Probability of Type I error, detection power	Heuristic	Non aggregative
Zarandi et al. [11]	Single assignable cause, fuzzy parameters	Cost, in control average run length, detection power	Probability of Type I error, detection power	GA	Aggregative
Asadzadeh and Khoshalhan [15]	Multi assignable cause, crisp parameters	Cost, in control average run length, detection power	Probability of Type I error, detection power, ATS	Heuristic	Non aggregative
Safaei et al. [16]	Single assignable cause, crisp parameters	Cost, out of control average run length	In control average run length	NSGA-II	Non aggregative
This research	Multi assignable cause, crisp parameters	Cost, probability of Type I error, detection power, ATS	Cost, probability of Type I error, detection power, ATS	MOGAESD	Non aggregative

along with the maximum value of p . The proposed model is given in Eq. (1):

$$\begin{aligned}
 \min F_1 &= L \\
 \min F_2 &= \alpha \\
 \max F_3 &= p \\
 \min F_4 &= \text{ATS} \\
 \text{subject to:} \\
 L &\leq U_1 \\
 \alpha &\leq U_2 \\
 p_{\min} &\geq U_3 \\
 \text{ATS}_{\max} &\leq U_4 \\
 n &\text{ is a positive integer} \\
 h \text{ and } k &> 0.
 \end{aligned} \quad (1)$$

Note that considering the constraints for all objectives is necessary, for the following reasons:

- As mentioned by Woodall [6], statistical properties should be subjected to those satisfactory limits. Similar to the statistical properties, increasing expected cost could be unreasonable for managers. Therefore, subjecting the objectives with constraint could avoid exceeding the limits desired.
- In the multi-objective model, some objectives are incompatible. For example, optimizing one objective leads to the worst result in the others (more explanations are given at the beginning of Section 3). To sustain the solutions in the feasible region, subjecting all objectives is inevitable.
- In the MCDM area, the Pareto frontier may be unbounded. In this problem, some objectives such as Type I error and power of control chart has some constraints in its nature (between 0 and 1). But some others such as expected cost, ATS and average run length (ARL) may be unbounded at least from one side. Therefore, considering constraints for them may let us some feasible region for Pareto frontier.

Since, in real practice, multiple assignable causes occur in the process, the cost function (L) proposed by Duncan [17] is used in this paper (Eq. (2)). In this function, s number of assignable causes randomly occur, which are indicated by j index ($j = 1, 2, \dots, s$):

$$L = \frac{b + cn}{h} + \frac{\sum_{j=1}^s M_j \lambda_j \left(\frac{h}{p_j} - \tau_j + gn + D_j \right) + \sum_{j=1}^s W_j \lambda_j}{\lambda} + \frac{T \alpha e^{-\lambda h}}{1 - e^{-\lambda h}}, \quad (2)$$

where b and c are the constant costs of sampling, b is independent of the sample size (n), and c is dependent on it; W_j and T are the costs of detecting the j th assignable cause and false alarm recognition, respectively; M_j is the lost benefit due to the out-of-control state when the j th assignable cause occurs; g is the time required to take a sample and interpret the results; D_j is the time required to find the j th assignable cause after a point is plotted outside the control limits; λ_j is the average number of assignable causes j occurs per hour; $1/\lambda$ is the average time of minimum of s expected random variables with parameters of λ_j ; τ_j is the expected time of the j th cause occurrence in the interval between two samples and is given in Eq. (3):

$$\tau_j = \frac{1 - (1 + \lambda_j h) e^{-\lambda_j h}}{\lambda_j (1 - e^{-\lambda_j h})}, \quad (3)$$

and the probability of Type I error (α) and power of control chart (p_j) are computed by Eqs. (4) and (5), respectively:

$$\alpha = 2 \int_{-\infty}^{-k} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz, \quad \text{and} \quad (4)$$

$$p_j = \int_{-\infty}^{-k - \delta_j \sqrt{n}} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz + \int_{k - \delta_j \sqrt{n}}^{\infty} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz, \quad (5)$$

where δ_j is the magnitude of the shift in units of σ in the mean (μ) of the quality characteristics, due to the occurrence of an assignable cause, j .

The overall power in the objective, F_3 , is computed based on Eq. (6). Note that the p_{\min} in the third constraint is the minimum power among the set of p_j 's.

$$\begin{aligned}
 p &= \sum_{j=1}^s P(\text{control chart alarms} | j\text{th assignable cause occurs}) \\
 &\times P(j\text{th assignable cause occurs}) = \frac{1}{\sum_{j=1}^s \lambda_j} \sum_{j=1}^s \lambda_j p_j. \quad (6)
 \end{aligned}$$

In the model presented in Eq. (1), the overall ATS is calculated by dividing the sampling frequencies (h) by overall power (p). The ATS_{\max} mentioned in the fourth constraint is the maximum ATS between the set of ATS_j 's, whose elements are obtained as follows:

$$\text{ATS}_j = \frac{h}{p_j}. \quad (7)$$

3. Optimization method

In the multi-objective model, which was explained in the previous section, some objectives are incompatible. For example, Type I error is decreased by increasing the control limit (k), while such an increase in the control limit (k) also leads to a decrease in the power of the control chart. To sustain or increase the power of the control chart while still decreasing Type I error, the sample size (n) must be increased. Furthermore, the control limit (k) and sample size (n) have nonlinear relations with the cost function, and increasing them leads to cost increase. Consequently, the use of a powerful multi-objective optimizer is necessary to determine the sample size (n), control limit (k), and sampling frequency (h), in such a way that all objectives are optimized simultaneously. Thus, in this section, an MOGAESD based on a non-aggregative approach is proposed as an optimization tool. To clarify the proposed method, some basic concepts are given as follows:

3.1. Genetic algorithm

GA is a global search and optimization tool that simulates the evolution of an initial population based on the rule of survival of the fittest. Karr and Gentry [18] showed that a GA is different from other optimization tools for the following reasons: it considers many points in a search space simultaneously; it works directly with a set of parameters characterized as strings of chromosomes instead of the parameters themselves, and, in guiding the population, a GA applies probabilistic, rather than deterministic, rules.

GAs are used in many economic and economic statistical designs of control charts (for examples, see [19–22]).

The population of the GA used in this article includes 50 chromosomes. Each possible solution of the control chart design problem consists of three parameters: n , h , and k . This is considered the decimal coding of three genes in each chromosome. A pair of chromosomes is selected without replacement as parents, and the selection operator is the elitist. The crossover operator, shown in Table 3, is applied to each parent. Meanwhile, the mutation operator is applied with a probability $P_m = 0.01$ on each offspring; this changes one randomly selected gene to a new random value. In each iteration, all parents and offspring are kept in a pool to select 50 chromosomes as a new population using the selection operator. This procedure is iterated 500 times ($N_{it} = 500$).

Table 3: Crossover point.

Parent 1	n_1	h_1	k_1
Parent 2	n_2	h_2	k_2
Offspring 1	n_1	h_2	k_1
Offspring 2	n_2	h_1	k_2

3.2. Penalty function

Yeniay [23] classified the proposed approaches for optimizing the constraint problem using a GA into four categories, among them the penalty function is the most common. In addition, the penalty function can also be classified into two categories, including additive or multiplicative and interior or exterior (for more information, see [24]).

In this paper, the penalty function is implemented based on the combined additive and exterior method, which has received much more attention than the other methods. Note that our model has four constraints. The general form of the fitness function is: $Fitness_i = F_i + F'_i$, where F_i is the value of the i th objective function, and Eq. (8) shows the value of the penalty function (F'_i) for the i th objective.

$$F'_i = \sum_{j=1}^r w_{ij} * (\max(0, R_j))^{\beta}, \quad (8)$$

where w_{ij} and β are penalty parameters and r is the number of constraints; note that the constraints should be changed to $R_j \leq 0$ form. In this paper, we propose that $\beta = 1$. For simplicity, a ratio of U_i 's is considered as w_{ij} , although other values can be used, too. We studied different structures of U_i 's and proposed the penalty function in Eq. (9). In determining the ratio of U_i 's, some kind of normalizing approach is considered. It means that when the i th objective is optimized, the weight of penalty function for the related constraint is equal to 1. Also, values of penalty for other constraints are approximately in the same scale, when they exceeded their limits.

$$F'_i = \frac{U_i}{U_1} * \max(0, L - U_1) + \frac{U_i}{U_2} * \max(0, \alpha - U_2) + \frac{U_i}{U_3} * \max(0, U_3 - p_{\min}) + \frac{U_i}{U_4} * \max(0, ATS_{\max} - U_4). \quad (9)$$

The penalty function prevents the search from entering into the infeasible region. Yeh et al. [25] showed that if the penalty values are high, the algorithm usually remains in a local optimum. However, if the penalty values are low, the feasible optimal solution is not always identified.

3.3. Interactive method

There are three types of method used in asking the decision maker's opinion in MCDM problems: prior approach, progressive or interactive approach, and posterior approach. In the second approach (interactive method), the decision maker cooperates in making the decision during the whole process [26]. In the remainder of this paper, we consider the Quality Control (QC) manager as the decision maker.

In this paper, four objectives are considered; the ideal is to obtain zero for the cost, Type I error, and ATS and 1 for the power of the control chart. These objectives may not be achievable; improving one objective can often make the others worse. Therefore, first, the value of the achievable goals is obtained by optimizing each objective separately. Note that in each optimization model, all of the constraints in Eq. (1) are considered. In addition, since we have no limit for the cost in advance, we optimize the cost function only based on the initial statistical constraints. The upper initial limit for a Type I error is 0.01, the lower initial limit for the power of the control chart is 0.9, and the upper limit for ATS is 8 based on Saniga [27]. Eq. (10) shows the fitness function used:

$$F_1^{\min} = F_1 + \left(\frac{L}{0.01} * \max(0, \alpha - 0.01) + \frac{L}{0.9} * \max(0, 0.9 - p_{\min}) + \frac{L}{8} * \max(0, ATS_{\max} - 8) \right). \quad (10)$$

Next, F_1^{\min} is reported to the QC manager and he/she determines the limit ($U_1 = F_1^{\max}$) for the cost function. The same procedure could be used for statistical properties, which means that the statistical properties are optimized subject to the cost limit, which was obtained in the previous optimization, and the initial limits for statistical properties. This is performed to determine the achievable goals for the statistical properties. Eq. (11) is the fitness function to obtain F_2^{\min} and determine $U_2 = F_2^{\max}$.

$$F_2^{\min} = F_2 + \left(\frac{0.01}{U_1} * \max(0, L - U_1) + \max(0, \alpha - 0.01) + \frac{0.01}{0.9} * \max(0, 0.9 - p_{\min}) + \frac{0.01}{8} * \max(0, ATS_{\max} - 8) \right). \quad (11)$$

Similarly, the maximizing fitness function in Eq. (12) is used to obtain F_3^{\max} and to determine $U_3 = F_3^{\min}$, and the minimizing fitness function in Eq. (13) is applied to obtain F_4^{\min} and to determine $U_4 = F_4^{\max}$.

$$F_3^{\max} = F_3 - \left(\frac{0.9}{U_1} * \max(0, L - U_1) + \frac{0.9}{0.01} * \max(0, \alpha - 0.01) + \max(0, 0.9 - p_{\min}) + \frac{0.9}{8} * \max(0, ATS_{\max} - 8) \right), \quad (12)$$

$$F_4^{\min} = F_4 + \left(\frac{8}{U_1} \max(0, L - U_1) + \frac{8}{0.01} \right. \\ \left. * \max(0, \alpha - 0.01) + \frac{8}{0.9} * \max(0, 0.9 - p_{\min}) \right. \\ \left. + \max(0, ATS_{\max} - 8) \right). \quad (13)$$

Therefore, in the proposed optimization method, the optimizer and the QC manager cooperate with each other to determine the parameters. Hence, this procedure is an interactive approach.

3.4. MOAMP framework

By including a population of solutions, Multi-Objective Evolutionary Algorithms (MOEAs) are able to obtain the Pareto optimal set in a single run. A large number of researchers have developed MOEAs throughout the last decade; Zhou et al. [28] have conducted a survey of this area.

The design of the framework of the algorithm is the first step in the MOEA (see the review paper by Zhou et al. [28] for more information on the different frameworks of the MOEA).

In this paper, the framework of our proposed method (MO-GAESD) is “multi-objective meta heuristic using an adaptive memory procedure (MOAMP)”, which was proposed by Caballero et al. [29]. This framework has been developed for the Location Routing Problem (LRP) [30].

In this framework, an initial population is first produced randomly. Then, the Pareto optimal set is obtained by $u + 1 + v$ iterations in two phases, where u is equal to the number of objectives, and v is determined based on a stopping rule.

In phase I, u integrated optimizations are performed by considering the i th objective as a fitness function in each iteration. Meanwhile, all constraints are taken into account simultaneously in each iteration. The input population of each iteration is the output population produced by the previous optimization. Because the first objective is optimized by a randomly generated population, we optimize this objective once more, and the input population is generated from the optimization of the last objective. This step is done in the $u + 1$ th iteration of the framework.

In phase II, the v later optimizations are performed by minimizing the Q_i function, which is shown in Eq. (14), until the stopping rule is invoked. In Eq. (14), $\theta_i = (\theta_{i1} \dots \theta_{iu})$ is a randomly generated weighting vector. Again, the input population of each replication is the output population produced by the previous optimization. Pareto solutions are obtained after each optimization iteration and put in the Pareto optimal set. The Q_i function in the original framework is introduced in Eq. (14), when all objectives must be maximized. However, in our proposed model, there are 3 objectives which should be minimized, and one objective which should be maximized. Therefore, Q_i in Eq. (14) is changed to Q_i in Eq. (16) based on the proposed model.

$$Q_i = \max \left\{ \theta_{ij} \left(\frac{F_j^{\max} - F_j}{F_j^{\max} - F_j^{\min}} \right); j = 1, \dots, u \right\}. \quad (14)$$

This procedure of optimization allows us to obtain some of the possible Pareto optimal solutions. In this paper, u is set to 4,

and v is set to 3 to obtain better solutions. We perform the framework as a loop until the stopping rule is invoked. This means that after $4 + 1 + 3$ iterations, we use the last output population as the input population and repeat the framework. Figure 1 shows one repeat of this framework. The first objective (F_1) is optimized on arc no. 1, from which we move to optimize the three other objectives (F_2 on arc no. 2, F_3 on arc no. 3, and F_4 on arc no. 4); then, we move back to the first objective (F_1) on arc no. 5. Finally, from there, we continue three more searches by minimizing Q_6 , Q_7 , and Q_8 on arcs 6, 7, and 8, respectively. These optimizations can continue as a loop until the stopping rule is invoked. The stopping rule considered in this paper is the number of replications. Pareto optimal solutions are obtained after each iteration, and then are put in the Pareto optimal set.

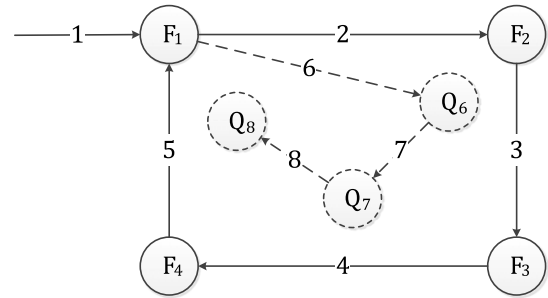


Figure 1: MOAMP framework.

3.5. Selection

After obtaining the Pareto optimal set, one of the MCDM tools can be used to find the preferred solution. DEA is one of these methods and is used in this area by authors such as Chen and Liao [14] and Asadzadeh and Khoshalhan [15] (for more information about DEA see [31]). This method calculates the utility function for each Pareto optimal solution, allowing the optimizer to select the most efficient Pareto optimal solution. The values of expected cost, probability of Type I error and ATS, are set as inputs and the power of the control chart is set as output in the DEA model. Although there is some DEA software, such as DEAP, in this paper, we have optimized mathematical programming with the linear optimizer of MATLAB software. Some other MCDM tools can be found in [32].

3.6. Optimization steps

The proposed algorithm is illustrated as follows:

- Step 1. Single optimization of cost function:
Optimize the cost function using the GA by applying the fitness function in Eq. (10). Then, based on the F_1^{\min} , determine the limit ($U_1 = F_1^{\max}$) for the cost function.
- Step 2. Single optimization of statistical objectives:
Using the GA, optimize each statistical objective separately based on the fitness function in Eqs. (11)–(13). Then, obtain F_2^{\min} , F_3^{\max} , and F_4^{\min} and determine the limits ($U_2 = F_2^{\max}$, $U_3 = F_3^{\min}$, and $U_4 = F_4^{\max}$) for the statistical properties.
- Step 3. Fitness functions of MOAMP framework:
Form the $4 + 1 + 3$ fitness functions given in Eqs. (15) and (16), respectively, and use them in the

MOAMP framework.

$$\begin{aligned}
 Fitness_1 &= F_1 + \left(\max(0, L - U_1) + \frac{U_1}{U_2} \right. \\
 &\quad \left. * \max(0, \alpha - U_2) + \frac{U_1}{U_3} * \max(0, U_3 - p_{\min}) \right. \\
 &\quad \left. + \frac{U_1}{U_4} * \max(0, ATS_{\max} - U_4) \right) \\
 Fitness_2 &= F_2 + \left(\frac{U_2}{U_1} * \max(0, L - U_1) \right. \\
 &\quad \left. + \max(0, \alpha - U_2) + \frac{U_2}{U_3} * \max(0, U_3 - p_{\min}) \right. \\
 &\quad \left. + \frac{U_2}{U_4} * \max(0, ATS_{\max} - U_4) \right) \\
 Fitness_3 &= F_3 + \left(\frac{U_3}{U_1} * \max(0, L - U_1) + \frac{U_3}{U_2} \right. \\
 &\quad \left. * \max(0, \alpha - U_2) + \max(0, U_3 - p_{\min}) \right. \\
 &\quad \left. + \frac{U_3}{U_4} * \max(0, ATS_{\max} - U_4) \right) \quad (15) \\
 Fitness_4 &= F_4 + \left(\frac{U_4}{U_1} * \max(0, L - U_1) + \frac{U_4}{U_2} \right. \\
 &\quad \left. * \max(0, \alpha - U_2) + \frac{U_4}{U_3} \right. \\
 &\quad \left. * \max(0, U_3 - p_{\min}) + \max(0, ATS_{\max} - U_4) \right) \\
 Fitness_5 &= F_1 + \left(\max(0, L - U_1) + \frac{U_1}{U_2} \right. \\
 &\quad \left. * \max(0, \alpha - U_2) + \frac{U_1}{U_3} * \max(0, U_3 - p_{\min}) \right. \\
 &\quad \left. + \frac{U_1}{U_4} * \max(0, ATS_{\max} - U_4) \right),
 \end{aligned}$$

$$\begin{aligned}
 Fitness_i &= Q_i = \max \left\{ \theta_{i1} \left(\frac{F_1 - F_1^{\min}}{F_1^{\max} - F_1^{\min}} \right), \right. \\
 &\quad \theta_{i2} \left(\frac{F_2 - F_2^{\min}}{F_2^{\max} - F_2^{\min}} \right), \theta_{i3} \left(\frac{F_3^{\max} - F_3}{F_3^{\max} - F_3^{\min}} \right), \\
 &\quad \left. \theta_{i4} \left(\frac{F_4 - F_4^{\min}}{F_4^{\max} - F_4^{\min}} \right) \right\}; \quad i = 6, 7, 8. \quad (16)
 \end{aligned}$$

In Eq. (16), $\theta_i = (\theta_{i1} \ \theta_{i2} \ \theta_{i3} \ \theta_{i4})$ is a randomly generated weighting vector.

Step 4. Initial population:

Produce the first random population.

Step 5. Identification of the Pareto optimal set:

Until the stopping condition has been invoked, perform the following steps (the stopping condition is the number of replications):

- For $i = 1$ to $4 + 1 + 3$

- Optimize the *Fitness* i .
- Consider the output population of the previous iteration to be the input population for the current optimization.
- Obtain the Pareto optimal solutions and put them in the Pareto optimal set.

Step 6. Selection of the preferred solution:

Report the Pareto optimal set, and select the preferred solution(s) using MCDM tools.

To clarify the optimization method more clearly, Figure 2 illustrates the summarized steps of the proposed approach.

4. Numerical examples

In this section, the performance of the proposed method is evaluated using some numerical examples reported in the literature. In the first subsection, the steps of the proposed method are explained through design of the control chart for the case introduced by Duncan [17]. Also, the obtained results are compared with those reported by Asadzadeh and Khoshalhan [15]. Finally, the sensitivity analysis is performed for evaluating the robustness of the chart designed. For more explanations, in the second subsection, the optimal design is obtained for a case which was reported by Chen and Liao [14]. Note that in the proposed general model in Eq. (1), there are not any constraints on the design parameters. However, to compare the proposed method with the method by Asadzadeh and Khoshalhan [15] and Chen and Liao [14], we added the same constraints on the design parameters (n , h , and k).

4.1. Case study I and sensitivity analysis

In this subsection, the performance of the proposed method is evaluated through a case study reported by Duncan [17]. In this case, 12 assignable causes with the average of λ_j for j th ($j = 1, 2, \dots, 12$) assignable cause may be occurred, which lead to a shift in the mean by size δ_j for the j th assignable cause. The cost of sampling is 1\$ in addition to 0.1\$ per sample. The time used for analysis of the samples is 0.05 h and the cost of false alarm is 25\$. Other parameters of the cost function, which depend on the assignable causes are reported in Table 4.

Table 4: Input parameters for 12 assignable causes introduced by Duncan [17].

#	δ_j	λ_j	D_j	M_j	W_j
1	0.75	0.001098	4.17	7.22	19.68
2	1.25	0.000855	3.08	27.6	14.57
3	1.75	0.000666	2.50	76.14	11.81
4	2.25	0.000519	2.08	165.69	9.84
5	2.75	0.000404	1.92	302.36	9.06
6	3.25	0.000314	1.84	433.64	8.66
7	3.75	0.000245	1.77	570.32	8.37
8	4.25	0.000191	1.72	659.86	8.17
9	4.75	0.000148	1.70	708.4	8.05
10	5.25	0.000115	1.68	728.97	7.93
11	5.75	0.000090	1.66	735.78	7.83
12	6.25	0.000070	1.64	737.56	7.73

First of all, based on our proposed approach, the cost function is optimized based on the fitness function in Eq. (10) using GA. The results are summarized in Table 5.

Based on the QC manager's opinion, the cost function could be increased to 6.62 ($U_1 = F_1^{\max} = 6.62$). In the second step, the

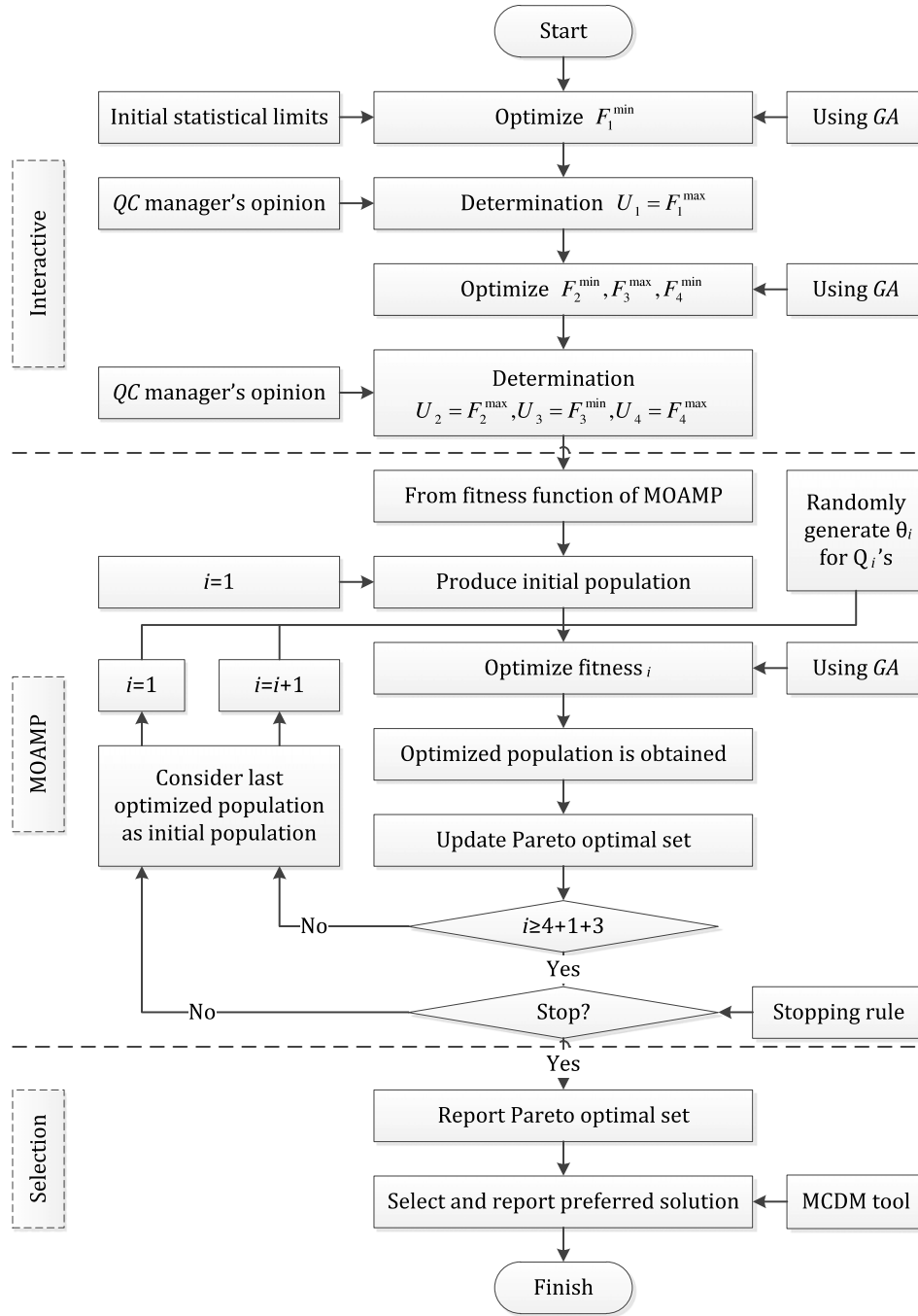


Figure 2: Flowchart of the proposed MOGAESD.

Table 5: Optimization results for the cost function.

n	h	k	L	α	p	ATS
20	2.8238	2.0632	5.64	0.0391	0.9771	2.8902

statistical objectives are optimized with the fitness functions from Eq. (11) to (13), using the GA. Table 6 shows the results of optimization of these objectives.

Then, F_2^{\min} , F_3^{\max} , and F_4^{\max} are reported to the QC manager, according to the interactive method. He/she then defines the limits as $U_2 = F_2^{\max} = 0.01$, $U_3 = F_3^{\min} = 0.9$, and $U_4 = F_4^{\max}$

$= 4$. In step 3, the fitness functions in Eqs. (15) and (16) are formed and used in the MOAMP framework. The fitness functions used in the MOAMP framework are illustrated in Eqs. (15) and (17).

$$Fitness_i = \max \left\{ \theta_{i1} \left(\frac{F_1 - 5.64}{6.62 - 5.64} \right), \theta_{i2} \left(\frac{F_2 - 0.0025}{0.01 - 0.0025} \right), \theta_{i3} \left(\frac{0.9966 - F_3}{0.9966 - 0.95} \right), \theta_{i4} \left(\frac{F_4 - 1.0465}{4 - 1.0465} \right) \right\};$$

$$i = 6, 7, 8. \quad (17)$$

Based on the fitness functions defined in Eqs. (15) and (17), as well as the framework explained in Section 3.4, 43 NDS's are

Table 6: Pay-off matrix for case study I.

	n	h	k	L	α	p	ATS
α	33	3.1014	3.0266	6.40	0.0025	0.9767	3.1754
p	26	3.4640	1.6449	6.57	0.1	0.9966	3.4759
ATS	6	0.5072	4.6332	6.62	3.60E-6	0.4847	1.0465

obtained in the Pareto optimal set. For instance, Table 7 shows 5 extracted Pareto optimal solutions.

Table 7: Extracted Pareto optimal solutions for the case study I.

n	h	k	L	α	p	ATS
32	3.0318	2.6016	6.3718	0.0093	0.9883	3.0678
33	3.0761	2.6022	6.4515	0.0093	0.9898	3.1079
33	3.0471	2.7018	6.4331	0.0069	0.9874	3.0860
33	3.0367	2.8013	6.4189	0.0051	0.9847	3.0840
35	3.1489	2.6007	6.6106	0.0093	0.9923	3.1734

Next, based on the proposed DEA method the preferred solutions are selected (Table 8).

Table 8: The most efficient economic statistical designs for the case study I.

n	h	k	L	α	p	ATS
33	2.7049	3.0008	6.4188	0.0027	0.9778	2.7665
27	2.8427	2.6009	5.9660	0.0093	0.9773	2.9087

The analysis of the obtained Pareto optimal set compared to those reported by Asadzadeh and Khoshalhan [15] (NDS's before applying DEA) shows that there are solutions which dominate some of the solutions reported by Asadzadeh and Khoshalhan [15]. This improvement is due to considering a continuous region for design parameters in the proposed method. Although this enhancement may be small, in the long run, this improvement could lead to more cost saving. On the other hand, designing a control chart is done once and used for a long horizon. Meanwhile, reducing the cost of the process and decreasing the false alarm increases the reliability of management and personnel to the control chart. Note that a continuous region is considered only for sampling frequency (h) and control limit (k), while sample size (n) is still defined in a discrete region (see the model in Eq. (1)).

Table 9 shows the comparison between the preferred solutions obtained from the proposed method and Asadzadeh and Khoshalhan [15] after applying the DEA. As shown in Table 8, considering ATS as an objective leads to a better performance of this objective as well.

Referring to almost all cited paper dealing with design of control charts, the sensitivity analysis is performed to investigate the effect of model parameters on the solution. Therefore, in this paper we extract the sensitivity analysis from Asadzadeh and Koshalhan [15] for this case study. The obtained results are reported in Table 10. Note that the steps of the optimization method for the case study are explained in detail. However, only the results are reported from Duncan's case study [17] in Table 10 for sensitivity analysis. The investigated parameters are fixed and variable sampling cost (b and c), cost of false alarm (T) and required time for analysis of sampling (g), respectively.

Some notes are concluded from the results of sensitivity analysis, which are listed below:

- Reducing the sampling cost (fixed or variable) can lead to increasing the sampling frequency (h). Hence, ATS and the expected cost are decreased.
- The cost of a false alarm has no critical effect on the optimal design and overall cost.
- The optimal design is robust to the required time for analysis of sampling (g), but an increase in g can lead to higher overall cost.

4.2. Case study II

In this subsection, to analyze our method further, another problem introduced by Chen and Liao [14], is considered. In this case, they assumed $\lambda = 0.25$, $b = 1\$$, $c = 0.1\$$, $g = 0.01$ h, $W = 50\$$, $T = 50\$$, $D = 2$, $\delta = 1$, and $M = 200\$$. Note that the cost function in this example is modeling one assignable cause, which is the special case of a multiple assignable causes model. The analysis of the results for the proposed approach illustrates the effectiveness of the proposed method.

According to our proposed method, the cost function is first minimized with the fitness function in Eq. (10), subject to the initial statistical limits. F_1^{\min} is reported in Table 11.

Based on the QC manager's opinion, the cost function could be increased to 99 ($U_1 = F_1^{\max} = 99$). In the second step, the statistical objectives are optimized with the fitness functions in Eqs. (11)–(13), using the GA. Table 12 shows the results of the optimization of these objectives.

Next, F_2^{\min} , F_3^{\max} and F_4^{\min} are reported to the QC manager, according to the interactive method. He/she then defines the limits as $U_2 = F_2^{\max} = 0.005$, $U_3 = F_3^{\min} = 0.95$ and $U_4 = F_4^{\max} = 4$. In step 3, the fitness functions in Eqs. (15) and (16) are formed and used in the MOAMP framework.

The 4 + 1 fitness functions are the same as those reported in Eq. (15). In addition, the later 3 fitness functions are formed in Eq. (18) by replacing the results in Eq. (16). Now, we can start performing the optimization framework.

$$\text{Fitness}_i = \max \left\{ \theta_{i1} \left(\frac{F_1 - 98.47}{99 - 98.47} \right), \theta_{i2} \left(\frac{F_2 - 0}{0.005 - 0} \right), \theta_{i3} \left(\frac{1 - F_3}{1 - 0.95} \right), \theta_{i4} \left(\frac{F_4 - 0.1646}{4 - 0.1646} \right) \right\};$$

$$i = 6, 7, 8. \quad (18)$$

After applying the MOAMP framework, 76 NDS's are obtained in the Pareto optimal set by using the multi objective framework. For example, 5 Pareto optimal solutions are reported in Table 13.

Finally, the DEA is used to obtain more efficient solutions. In this case, two Pareto optimal solutions are efficient and reported in Table 14.

Since the results reported by Chen and Liao [14] are not accurate (see [15]), we use the correct optimum values reported in [15] for design parameters for the sake of comparison. Note that the value of ARL_0 is reported instead of the Type I error, where $ARL_0 = 1/\alpha$. The analysis of the resulting Pareto optimal set show that there are Pareto optimal solutions better than those reported in [15].

Moreover, the most efficient economic statistical designs (after applying DEA) are compared with the results of the method proposed by Asadzadeh and Khoshalhan (A & K) [15], as well as the results of the method by Chen and Liao (C & L) [14] which is corrected by Asadzadeh and Khoshalhan [15].

Table 9: Comparison between our obtained preferred solutions with those reported by Asadzadeh and Khoshalhan (A & K) [15].

	n	h	k	L	Improvement %	ARL_0	p	ATS	Improvement %
This research	33	2.7049	3.0008	6.4188	-0.288	370.3704	0.9778	2.7665	9.99
A & K	33	3	3	6.4003		370.3704	0.9777	3.0736	
This research	27	2.8427	2.6009	5.9660	0.003	170.5269	0.9773	2.9087	2.16
A & K	27	2.9	2.6	5.9662		170.5269	0.9773	2.9728	

Table 10: Evaluating robustness of control chart design through sensitivity analysis.

Parameter	Value	n	h	k	L	α	p	ATS
b	0.1	33	2.5976	3.0022	6.0879	0.0027	0.9777	2.6568
		27	2.4736	2.6014	5.6322	0.0093	0.9773	2.5311
	1	33	2.7049	3.0008	6.4188	0.0027	0.9778	2.7665
		27	2.8427	2.6009	5.9660	0.0093	0.9773	2.9087
	5	33	3.4194	3.0005	7.5905	0.0027	0.9778	3.4971
		27	3.4551	2.6022	7.1700	0.0093	0.9772	3.5356
c	0.01	33	1.6086	3.0022	5.1497	0.0027	0.9777	1.6453
		27	1.7032	2.6013	4.9191	0.0093	0.9773	1.7428
	0.1	33	2.7049	3.0008	6.4188	0.0027	0.9778	2.7665
		27	2.8427	2.6009	5.9660	0.0093	0.9773	2.9087
	0.5	33	3.5199	3.0020	10.1819	0.0027	0.9777	3.6001
		27	3.5117	2.6002	9.9066	0.0093	0.9773	3.5932
T	10	33	2.7023	3.0021	6.4046	0.0027	0.9777	2.7639
		27	2.7999	2.6017	5.9148	0.0093	0.9773	2.8650
	25	33	2.7049	3.0008	6.4188	0.0027	0.9778	2.7665
		27	2.8427	2.6009	5.9660	0.0093	0.9773	2.9087
	50	33	2.7096	3.0023	6.4420	0.0027	0.9777	2.7714
		27	2.8892	2.6019	6.0433	0.0093	0.9773	2.9564
g	0.01	33	2.7034	3.0022	5.1570	0.0027	0.9777	2.7651
		27	2.8451	2.6018	4.9326	0.0093	0.9773	2.9113
	0.05	33	2.7049	3.0008	6.4188	0.0027	0.9778	2.7665
		27	2.8427	2.6009	5.9660	0.0093	0.9773	2.9087
	0.25	33	2.7115	3.0022	12.5044	0.0027	0.9777	2.7733
		27	2.8476	2.6016	10.9816	0.0093	0.9773	2.9138

Table 11: Optimization results for the case introduced by Chen and Liao [14].

n	h	k	L	α	p	ATS
15	0.5005	2.5762	89.466	0.01	0.9026	0.5545

Table 12: Pay-off matrix for case II.

	n	h	k	L	α	p	ATS
α	45	0.7380	5.4178	98.93	0.0000	0.9015	0.8187
p	49	0.8607	2.5999	99	0.0093	1	0.8607
ATS	16	0.1493	2.6779	98.98	0.0074	0.9069	0.1646

Table 13: Extracted Pareto optimal solutions for the case study II.

n	h	k	L	α	p	ATS
21	0.5698	2.9005	90.7602	0.0037	0.9537	0.5974
22	0.5793	2.9003	91.0055	0.0037	0.9633	0.6014
22	0.5728	3.0001	91.0510	0.0027	0.9545	0.6001
23	0.5812	3.0997	91.3555	0.0019	0.9551	0.6085
23	0.5812	3.1002	91.3559	0.0019	0.9550	0.6086

Table 14: The most efficient economic statistical designs for the case study II.

n	h	k	L	ARL_0	p	ATS
25	0.3804	2.9022	93.2128	269.8677	0.9820	0.3874
30	0.3743	3.7995	95.4624	6897.107	0.9533	0.3926

Table 15 shows the comparison between the preferred solutions. The obtained results show that, although the values of expected costs in the preferred solutions are worse, the ATS is significantly better than those reported by previous approaches. These results indicate the effectiveness of the proposed approach.

5. Conclusions and future researches

In this paper, a multi-objective economic statistical model for the \bar{X} control chart was developed under the assumption of multiple assignable causes. In the proposed model, ATS and the expected cost were added as an objective and a constraint, respectively, to the previous multi-objective models considered in the literature. To solve the model, a non-aggregative MOGAESD optimization method was proposed. Components of the optimization method included an MOEA, a GA, an MOAMP framework, a penalty function, an interactive method, and selection. The performance of the proposed method was evaluated through a case study reported by Duncan [17] under multiple assignable causes. Also, a sensitivity analysis was performed to investigate the robustness of the control chart designed. In addition, a case study reported by Chen and Liao [14] was considered to evaluate the performance of the proposed method under a single assignable cause. Comparison between the obtained results and some results reported in the literature showed that considering ATS beside the other objectives can lead to significant improvements in this property, while the others may become a little

Table 15: Comparison between preferred solutions.

	n	h	k	L	Improvement (%)	ARL_0	p	ATS	Improvement
This research	25	0.3804	2.9022	93.2128		269.8677	0.9820	0.3874	
A & K	25	0.4	2.9	92.9364	−0.30	267.9797	0.9821	0.41	5.51
C & L	26	0.6	2.9	92.1238	−1.17	267.9797	0.986	0.61	36.49
This research	30	0.3743	3.7995	95.4624		6897.107	0.9533	0.3926	
A & K	30	0.4	3.8	95.0319	−0.45	6911.037	0.9532	0.42	6.52
C & L	30	0.6	3.8	93.6692	−1.91	6911.037	0.9532	0.63	37.68

worse. Generally, based on analyses of the paper, considering a continuous region for the design parameters could lead to better results. This method can be applied to other cost functions, such as Lorenzen and Vance [4], or under different assumptions, such as Weibull shock. In addition, other control charts, such as EWMA and CUSUM, can be optimized using the method proposed in this paper.

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