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Reconfiguration of Supply Chain: A Two Stage Stochastic Programming

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ABSTRACT

In this paper, we propose an extended relocation model for warehouses configuration in a supply chain network, in which uncertainty is associated to operational costs, production capacity and demands whereas, existing researches in this area are often restricted to deterministic environments. In real cases, we usually deal with stochastic parameters and this point justifies why the relocation model under uncertainty should be evaluated. Albeit the random parameterscan be replaced by their expectations for solving the problem, but sometimes, some methodologies such as two-stage stochastic programming works more capable. Thus, in this paper, for implementation of two stage stochastic approach, the sample average approximation (SAA) technique is integrated with the Bender's decomposition approach to improve the proposed model results. Moreover, this approach leads to approximate the fitted objective function of the problem comparison with the real stochastic problem especially for numerous scenarios. The proposed approach has been evaluated by some hypothetical numerical examples and the results show that the proposed approach can find better strategic solution in an uncertain environment comparing to the mean-value procedure (MVP) during the time horizon.

1. Introduction

Any kind of industries needs to use an efficient and flexible supply chain network. A supply chain network comprises echelons such as suppliers, plants, warehouses, distribution centers and customers. This network after producing the goods, flows them from plants to customers to achieve customer satisfaction with an optimum cost [1].

Usually, top managers in supply chain networks face a sever challenge of trying to relocate their current facilities for more productivity and efficiency. As real evidence, according to Ballou and Master's investigation [2] of 200 logistics managers, 65% of them decided to evaluate their current warehouse network and have considered relocating it in the near future. This survey shows the importance of relocation models. On the other hand, due to parameter variation during the considered horizon time, if we do not apply an appropriate approach to overcome uncertainties, solving the problem leads to make wrong strategic decisions with considerable costs.

Some conceptual questions, which are discussed for redesigning the facilities in each supply chain echelon, are given as: "Which facilities should be retained, established, eliminated or consolidated?"

All of the above questions and related concepts are expanded and aggregated in the novel research area named relocation models. This paper proposed a mathematical model of warehouse relocation in a

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supply chain. Moreover, we integrate uncertainty in some parameters of the proposed model such as operational costs, production capacity and demands for the proposed relocation problem. Generally, there is some stochastic programming to overcome the uncertain situation such as chance programming, mean value procedure, deterministic equivalent and two-stage stochastic programming.

Usually, deterministic equivalent finds better solution in uncertain environment but for large number of scenarios, deterministic equivalent cannot work capable due to the large dimensions of the problem[3]. In this regard, using a heuristic approach such as two-stage stochastic programming will help to approximate objective function with huge number of scenarios derived from probability distribution or gathered from probabilistic data. It is worth to noting that in two-stage stochastic programming which is applied in this paper, the objective function is constructed from strategic decisions' costs and expectation of operational costs resulting from the same strategic decisions, therefore proposing a well-defined and closed form function are needed. Two-stage stochastic programming involves Bender's decomposition [4] and SAA (sample average approximation) [5]. In this paper, Bender's decomposition is used to solve the mixed integer linear model iteratively. Moreover, stochastic scenarios are joined to Bender's decomposition in each iteration through SAA.

The remainder of this paper is organized as follows: The literature of supply chain location and relocation models is reviewed in section 2, then, the proposed relocation model and its uncertain problem description are discussed in section 3. In section 4, after clarifying the Bender's decomposition and its integration with SAA, the proposed heuristic approach for considered model are presented. Then in section 5, a computational results based on some hypothetical numerical examples are analyzed to illustrate how the proposed approach works on the relocation model with stochastic parameters. Finally, some concluding remarks are suggested in section 6.

2. Literature Survey

The recent review for facility location and SCND (supply chain network design) demonstrated that most of the literature deals with deterministic models versus stochastic ones (approximately 82% against 18%) [6], while uncertainty is more applicable in real cases. On the other hand, as mentioned before, the managers want to analyze their supply chain's efficacy and productivity.

Consequently, relocation models are more capable and suitable approaches for proposing the best configuration of the SCND at each time horizon. The advantage of relocation models is in considering the relocation costs, which have been ignored in the location models. In this regard, addition to traditional SCND costs, relocation models consider income of eliminating the redundant facilities, consolidation and capacities extending costs, etc. Surveying the published works with uncertain parametersshows that researchers have received significant attention to stochastic programming in the last decade. For example, demand has been considered as an uncertain parameter in some researches [7-8].

By reviewing the literature of stochastic programming on supply chain network, it can be understood that Santoso et al. [3] have had significant role in extending the two stage stochastic programming

g. They have done a research on SCND problem with uncertainty in production capacity, demand, space capacity for facilities and transportation costs. Their method integrates an accelerated decomposition scheme along with the SAA method. Their proposed method's results have confirmed efficiency of the two-stage approach respected to MVP in terms of improving the solutions and its deviations.

MirHassani et al. [9] have studied capacity planning problem in the stochastic situation. In addition, Tsiakis et al. [10] have presented stochastic programming for locating the warehouses and distribution centers for uncertain demands. MohammadiBidhandi and Yusuff [11] have utilized the surrogate constraints method in a simple supply chain modelto accelerate the decomposition method so that their numerical example shows an improvement in computational results. Addition to SCND problems, stochastic optimization have received attention in some other areas such as location-allocation and hub location problems. In this regard, Wang et al. [12] have applied genetic algorithm (GA) to find the strategic decision of location-allocation in stochastic environment. Their solution algorithm can find near optimal solution while consuming less computational time for large-sized problems. Contreras et al. [13] have applied the two-stage stochastic programming for uncapacitated hub location problem where demand and transportation costs are probabilistic.

As mentioned earlier, we want to propose stochastic form for relocation of warehouses in a supply chain network. In this regard, Min and Melachrinoudis [14] have defined some criteria such as cost, traffic access, quality of living and etc to relocate the current situation of supply chain using analytic hierarchy process (AHP). In addition, Melachrinoudis and Min [15] have considered a relocation model, in which warehouse location can be changed in each period.

Melachrinoudis and Min [16] have presented a relocation model on redesigning the warehouse for reducing the network's costs in three echelons of a supply chain. Melo et al.[17] have proposed a relocation model in a supply chain network. They have considered opening or closing decision for the facilities in each period but their model does not contain the consolidation decisions. Some other researches such as Lowe et al. [18], carlsson and Ronnqvist [19] have focused on assessing the current situation of supply chain network. Moreover, Melachrinoudis et al. [20] have applied goal programming to solve a relocation model with deterministic parameters and multiple objectives such as cost and customer coverage (%). By surveying the published works, whichare cited in this paper, we can categorize the case studies of relocation models to manufacturing the chain link fence, chemical materials, pulp production and plastic film. Moreover, considering the stochastic assumption in relocation model have been suggested as future research in investigation of Melo et al. [17]. Moreover, a review paper on facility location and supply chain have demonstrated that the stochastic and fuzziness parameters in relocation models and using an adapted solving approach can be considered as appropriate future researches [6].

All of the mentioned reasons emphasize on applicability of proposed approach in reality. Table 1 shows some existing studies related to therelocation problem specially in redesigning warehouse.

Tab. 1. The characteristics of some existing researches related to the relocation problem

References	Parameters		Solution approach	Multi periods	Multi products	Covering	Inventory	Capacity
	Stochastic	Deterministic						
Ref [14]		✓	AHP					
Ref [15]		✓	Min/Cost	✓	✓			✓
Ref [16]		✓	Min/Cost					✓
Ref [17]		✓	Min/Cost	✓	✓			✓
Ref [18]		✓	-					
Ref [19]		✓	Min/Cost					
Ref [20]		✓	G. P			✓		✓
Proposed model and its solving approach	✓		Two-Stage Stochastic programming	✓	✓	✓	✓	✓

In this paper, we extend the model that has been presented previously by Melachrinoudis and Min [16], then, the proposed model is constructed in stochastic environment. In this regard, combination of bender's decomposition and SAA are applied to overcome the uncertainty.

3. Problem Description

Consider a supply chain network consists of suppliers, plants, warehouses and customers. The plant manufactures products from raw materials and sends them to capacitated warehouses according to requested demands. In the current system that isactive now, the manager wants to evaluate productivity and efficacy of his/her system. The main relocation costs in this system include supply, manufacturing, shipment, moving, relocating and consolidation of the facilities costs. In this research, it has been supposed that production cost, production capacity and demands are stochastic.

Moreover, according to gathered information from historical data, there are some fitted probabilistic distributions for uncertain parameters. As an instance, customer demand in node *k* has a lognormal probability distribution (because of non-negativity demands) with known mean and variance. In this section, the proposed relocation model can be expressed in a general probabilistic form as follows:

Nomenclature

Sets and Indices

S	Set of suppliers, indexed by s
P	Set of manufacturing plants, indexed by p
E	Set of existing warehouses, indexed by j
F	Set of new candidate site for warehouse, indexed by f
A	Set of all warehouses, indexed by i, ($E \cup F = A$)
K	Set of customers, indexed by k
O	Set of product, indexed by o
R	Set of raw materials, indexed by r
N	Set of scenarios, indexed by n
T	Set of periods, indexed by t

Parameters

pr_n	Probability of scenario n to occur
c_i^{UF}	Cost per unit for creating capacity in warehouse i (without considering consolidated capacities from other warehouses)

c_{sprt}^{SP}	Unit cost of supplying and moving raw material *r* to plant *p* from supplier *s* at time period *t*
c_{piton}^{PI}	Manufacturing and shipment cost between plant *p* and warehouse *i* for product *o* at time period *t* under scenario *n*
c_{ikto}^{IK}	Transportation cost from warehouse *i* to customer *k* for product *o* at time period *t*
f_i^V	Cost per unit for accommodation of moved capacity and its equipment in destination warehouse *i*
c_{kto}^{SH}	Shortfall cost of customer *k* for one unit of product *o* at time period *t*
c_{ito}^{I}	unit handling cost of product *o* at warehouse *i* during time period *t*
cr_{ji}	Fixed cost of moving and relocating the capacity of warehouse *j* to warehouse *i* ($j \neq i$), (considering saved cost achieved from closure of existing warehouse *j*),
f_{it}^{C}	Fixed cost of retaining warehouse *i* excluding capacity cost at time period *t*
f_f^{CF}	Fixed cost of establishing new warehouse *f*
f_j^{S}	Saved cost achieved from complete closure of existing warehouse *j*
f_{st}^{SU}	Fixed cost of selecting the supplier *s* during time period *t*
f_{spt}^{SUP}	Fixed cost of providing raw materials to plant *p* by supplier *s* at time period *t*
d_{kton}	Demand of customer *k* for product *o* during time period *t* under scenario *n*
u_j	Throughput capacity of existing warehouse *j* (available for consolidation)
q_{pton}	Production capacity of plant *p* for product *o* at time period *t* under scenario *n*
q_{sr}^{SU}	Capacity of supplier *s* for raw material *r*
q_{spr}^{SP}	Transportation capacity of the product *o* from supplier *s* to plant *p*
γ_{pro}	Rate of needed raw material *r* for producing the product *o* at plant *p*
η_o	Required space volume of product *o* in the warehouse
τ_{srp}	Transportation capacity requirement of raw material *r* between supplier *s* and plant *p*
q_i^{UF}	Maximum capacity of warehouse *i*
NU	Number of desirable warehouses
b_{ik}	Covering matrix of customer *k* by warehouse *i* (according to desirable coverage radius)
Continuous variables *(Operational decision variables)*	
x_{sprtn}^{SP}	Amount of raw material *r* provided by supplier *s* to plant *p* at time period *t* under scenario *n*
x_{piton}^{PI}	Amount of product *o* provided by plant *p* to warehouse *i* at time period *t* under scenario *n*
x_{ikton}^{IK}	Amount of product *o* provided by warehouse *i* to customer *k* at time period *t* under scenario *n*
I_{iton}	Inventory level of product *o* being held at warehouse *i* at the end of time period *t* under scenario *n*
x_{kton}^{SH}	Shortfall of customer *k* for product *o* during time period *t* under scenario *n*
uf_{in}	Capacity of warehouse *i* (excluding consolidated capacity from other warehouses) under scenario *n*
Binary variables *(Investment decision variables)*	
z_{ji}	Relocation decision of warehouse j to warehouse i (for $i = j$ warehouse j remains open)
z_{ff}	Opening decision of the new warehouse *f*(restatement: z_{ii} for $i = f \in F$)
su_s	Selection decision of supplier *s*
sp_{sp}	Allocation decision of supplier *s* to plant *p*

3.1. Mathematical Model

The objective function and the constraints of the proposed model in a deterministic equivalent form arepresented as follows:

$$Min \sum_{t\in T}\Bigg[\overset{(1)}{\sum_{s\in S} f_{st}^{SU} su_s} + \overset{(2)}{\sum_{s\in S}\sum_{p\in P} f_{spt}^{SUP} sp_{sp}} + \overset{(3)}{\sum_{i\in A} f_{it}^{C} z_{ii}}$$

$$+ \sum_{n\in N} pr_n \Bigg(\overset{(4)}{\sum_{s\in S}\sum_{p\in P}\sum_{r\in R} c_{sprt}^{SP} x_{sprtn}^{SP}} + \overset{(5)}{\sum_{p\in P}\sum_{i\in A}\sum_{o\in O} c_{piton}^{PI} x_{piton}^{PI}}$$

$$+ \overset{(6)}{\sum_{i\in A}\sum_{k\in K}\sum_{o\in O} c_{ikto}^{IK} x_{ikton}^{IK}} + \overset{(7)}{\sum_{k\in K}\sum_{o\in O} c_{kto}^{SH} x_{kton}^{SH}} \qquad (1)$$

$$\overset{(8)}{\sum_{i\in A}\sum_{o\in O} c_{ito}^{I} \frac{I_{iton} + I_{i(t-1)on}}{2}}\Bigg] + \overset{(9)}{\sum_{i} c_i^{UF} uf_{in}}\Bigg) + \overset{(10)}{\sum_{i\in A} f_i^V \sum_{j\in E} u_j z_{ji}}$$

$$+ \overset{(11)}{\sum_{j\in E,(j\neq i)}\sum_{i\in A} cr_{ji} z_{ji}} + \overset{(12)}{\sum_{(i=f)\in F} f_i^{CF} z_{ii}} - \overset{(13)}{\sum_{j\in E}\left[f_j^S \left(1 - \sum_{i\in A} z_{ji}\right)\right]}\Bigg]$$

s.t.

$$\sum_{s\in S} x_{sprtn}^{SP} = \sum_{i\in A}\sum_{o\in O} x_{piton}^{PI} \gamma_{pro}, \quad \forall p\in P, r\in R, t\in T, n\in N \qquad (2)$$

$$\sum_{i\in A} x_{piton}^{PI} \le q_{pton} \quad \forall p\in P, t\in T, o\in O, n\in N \qquad (3)$$

$$I_{i(t-1)on} + \sum_{p\in P} x_{piton}^{PI} = I_{iton} + \sum_{k\in K} x_{ikton}^{IK} \quad \forall i\in A, t\in T, o\in O, n\in N \qquad (4)$$

$$\sum_{k\in K} x_{ikton}^{IK} \eta_o \le \sum_{j\in E} u_j z_{ji} + uf_{in} \quad \forall i\in A, t\in T, o\in O, n\in N \qquad (5)$$

$$\sum_{i\in A} b_{ik} x_{ikton}^{IK} + x_{kton}^{SH} \ge d_{kton} \quad \forall k\in K, t\in T, o\in O, n\in N \qquad (6)$$

$$\sum_{j\in E} u_j z_{ji} + uf_{in} \le q_i^{UF} z_{ii}, \forall i\in A, n\in N \qquad (7)$$

$$\sum_{p\in P} x_{sprtn}^{SP} \le q_{sr}^{SU} su_s, \forall s\in S, r\in R, t\in T, n\in N \qquad (8)$$

$$\sum_{r\in R} x_{sprtn}^{SP} \tau_{srp} \le q_{sp}^{SP} sp_{sp}, \forall s\in S, p\in P, t\in T, n\in N \qquad (9)$$

$$sp_{sp} \le su_s, \forall s\in S, p\in P \qquad (10)$$

$$\sum_{j\in E} z_{ji} \le |E| z_{ii}, \ \forall i\in E \qquad (11)$$

$$\sum_{j\in E} z_{ji} \le |E| z_{ii}, \ \forall i\in F \qquad (12)$$

$$\sum_{i\in A} z_{ii} \le NU, \ \forall i\in A \qquad (13)$$

$$\sum_{i\in A} z_{ji} \le 1, \ \forall j\in E \qquad (14)$$

$$x_{sprtn}^{SP}, x_{ikton}^{IK}, x_{piton}^{PI}, I_{iton}, x_{kton}^{SH}, uf_{in} \ge 0 \qquad (15)$$

$$z_{ji}, z_{ff}, su_s, sp_{sp} = \{0,1\} \qquad (16)$$

The objective function (1) is composed of thirteen terms. The first term of objective function is indicated by (1-1). Term (1-1) and (1-2) present supplier selection's costs and fixed cost of linking between each supplier and related plants. Term (1-3) including maintaining the warehouses. Terms (1-4)-(1-6) show the cost of supplying, manufacturing and transmission the goods from the supplier to customers. Moreover, (1-7) emphasizes on cost resulting from shortfall in destination demand nodes. The cost of warehousing the inventory costs is considered in term (1-8). Terms (1-9)-(1-11) introduce the cost of needed capacities in warehouses, accommodation cost in destination warehouse for consolidated capacity and fixed cost/income resulting from closure of existing warehouse and consolidation of its equipment and capacities in destination warehouse. Term (1-12) shows the cost of establishing the new warehouse and (1-13) expressed the revenue resulting from completely closure of redundant warehouses.

Constraint (2) assures tradeoff between supplied raw materials and produced products in each plant. Inequality (3) shows production capacity in each plant. Constraint (4) indicates flow tradeoff between transmitted product to each warehouse and saved inventories in each period (Inventory equilibrium). Constraints (5) insure that the total volume of products shipped to customers after consolidation cannot surpass the throughput capacity of the serving warehouse. Constraint (6) emphasizes on demand satisfaction considering requested demands that should be satisfied by at least one active warehouse (after consolidation) inside coverage radius. Constraint (7) ensures that for a destination warehouse, consolidated capacity from other warehouses and capacity of destination warehouse should be less than the maximum limit. Constraints (8) and (9) state the limitations of suppliers for providing the raw materials and sending them to plants through transport routes. Constraint (10) insures that if a supplier is inactive,the same supplier and plants cannot be related. Constraint (11) assures that an existing warehouse cannot be consolidated into another existing one, unless such consolidated warehouse remains open. In addition, $|E|$ is the cardinality of set E resulted from aggregation of constraints over set E with the equal right hand side (RHS). Similarly, constraints (12) have the same concept of previous constraint but for consolidation of existing warehouses into new warehouses. Constraints (13) denote that each warehouse can merge with only one of the destination warehouses. And finally, inequality (14) lets to decide about the maximum number of active warehouses. For more understanding, we describe the whole possibilities for z_{ii}. For $j \in E$, $z_{jj} = 1$ if the existing warehouse j remains open. Also, for $i \in A$ and $z_{ji} = 1 (i \neq j)$, existing warehouse j is consolidated into warehouse i. Note that for $(i = f) \in F$ and $z_{ii} = 1$, the new warehouse is established in the f^{th} candidate site. Also, $\sum_{i \in A} z_{ji} = 0$ demonstrates that warehouse j is redundant and should be eliminated from the supply chain network.

Constraints (15) assure decision variables positivity. Constraints (16) states that variables are binary type.

3.2. Uncertain Parameters

In this section, we introduce uncertain parameters and how the mentioned model (relations (1)-(16)) can be solved through a heuristic.

It issupposed that operational cost (production cost and transmitting the goods from plants), production capacity and demands are stochastic with known distribution. $\boldsymbol{\xi} = (\boldsymbol{c}, \boldsymbol{q}, \boldsymbol{d})$ represents the random data vector while $\xi^n = (c^n, q^n, d^n)$ stands for n^{th} generated scenario. The scenarios may have a specific probability but in this paper because of generating the random scenarios derived from probability distribution with known mean and variance, we suppose equal probability for each scenario. Demands and production capacity scenarios are generated based on lognormal distribution and the distribution of production cost is uniform.

4. Solution Methodology

In this paper, two-stage stochastic programming is used to find supply chain reconfiguration. Hence, we need to separate the problem into two sections in which, the first one labeled master problem (MP) is an integer programming problem and the second one named sub problem (SP) involves mixed integer linear programming problem (for more understanding about details see [4],[5] and [9]). In this regard, we consider investment decisions (which is mentioned before in nomenclature) in the master problem. Also, operational decisions involving the volume of production, shipment and outsourcing (resulted from shortfall in demands) areconsidered in the sub problem.

The Solving approach for the proposed model in an uncertain environment is explain as follows:

Definitions:
i': iteration number
lb: lower bound
ub: upper bound
$BS^{i'}$: optimal solutions of master problem in iteration i' (including z_{ji} and etc.)

Step0: Set lower bound, upper bound and iteration number equal to $-\infty$, ∞ and 0 respectively.

Step 1: Decompose the mathematical model in to MP and SP.

Master Problem (First Stage)

$$Min \sum_{t \in T} \Bigg[\overset{(1)}{\sum_{s \in S} f_{st}^{SU} su_s} + \overset{(2)}{\sum_{s \in S} \sum_{p \in P} f_{spt}^{SUP} sp_{sp}} + \overset{(3)}{\sum_{i \in A} f_{it}^{C} z_{ii}}$$

$$+ \overset{(10)}{\sum_{i \in A} f_i^V \sum_{j \in E} u_j z_{ji}} + \overset{(11)}{\sum_{j \in E, (j \neq i)} \sum_{i \in A} cr_{ji} z_{ji}} + \overset{(12)}{\sum_{(i=f) \in F} f_i^{CF} z_{ii}}$$

$$\overset{13}{- \sum_{j \in E} f_j^S \left(1 - \sum_{i \in A} z_{ji} \right)} \Bigg] + E[Q(\xi, BS)]$$

s.t.

$$sp_{sp} \le su_s, \forall s \in S, p \in P$$

$$\sum_{j \in E} z_{ji} \le |E| z_{ii} \quad \forall i \in E$$

$$\sum_{j \in E} z_{ji} \le |E| z_{ii} \quad \forall i \in F$$

$$\sum_{i \in A} z_{ii} \le NU \quad \forall i \in A$$

$$\sum_{i \in A} z_{ji} \le 1 \quad \forall j \in E$$

Where, BS indicates binary solution of MP substituting in sub problems and $Q(\xi, BS)$is sub problem's objective function, which is calculated for a specific random vector $\xi^n = (t^n, q^n, d^n)$in second stage as follows:

Sub problem (Second Stage)

$$Min \sum_{t \in T} \Bigg[\sum_{n \in N} pr_n \Bigg(\overset{(4)}{\sum_{s \in S} \sum_{p \in P} \sum_{r \in R} c_{sprt}^{SP} x_{sprtn}^{SP}} +$$

$$\overset{(5)}{\sum_{p \in P} \sum_{i \in A} \sum_{o \in O} c_{piton}^{PI} x_{piton}^{PI}} + \overset{(6)}{\sum_{i \in A} \sum_{k \in K} \sum_{o \in O} c_{ikto}^{IK} x_{ikton}^{IK}} +$$

$$\overset{(7)}{\sum_{k \in K} \sum_{o \in O} c_{kto}^{SH} x_{kton}^{SH}} + \overset{(8)}{\sum_{i \in A} \sum_{o \in O} c_{ito}^{I} \frac{I_{iton} + I_{i(t-1)on}}{2}} \Bigg] + \overset{(9)}{\sum_{i \in A} c_i^{UF} uf_{in}} \Bigg)$$

s.t.

$$\sum_{s} x_{sprtn}^{SP} = \sum_{i} \sum_{o} x_{piton}^{PI} \gamma_{pro}, \forall p, r, t, n$$

$$\sum_{i \in A} x_{piton}^{PI} \le q_{pton} \quad \forall p, t, o, n \qquad (\psi_{i'n}^{1})$$

$$I_{i(t-1)on} + \sum_{p \in P} x_{piton}^{PI} = I_{iton} + \sum_{k \in K} x_{ikton}^{IK} \quad \forall i, t, o, n$$

$$\sum_{k\in K} x_{ikton}^{IK}\eta_o \le \sum_{j\in E} u_j\, z_{ji}^{i'} + uf_{in} \quad \forall i,\, t, o, n \qquad (\psi_{i'n}^{2})$$

$$\sum_{i\in A} b_{ik}\, x_{ikton}^{IK} + x_{kton}^{SH} \ge d_{kton} \quad \forall k,t,o,n \qquad (\psi_{i'n}^{3})$$

$$\sum_{j\in E} u_j\, z_{ji}^{i'} + uf_{in} \le q_i^{UF} z_i^{i'}, \forall i, n \qquad (\psi_{i'n}^{4})$$

$$\sum_{p} x_{sprtn}^{SP} \le q_{sr}^{SU} su_s^{i'}, \forall s, r, t, n \qquad (\psi_{i'n}^{5})$$

$$\sum_{r} x_{sprtn}^{SP}\tau_{srp} \le q_{sp}^{SP} sp_{sp}^{i'}, \forall s, p, t, n \qquad (\psi_{i'n}^{6})$$

$$\forall s \in S, p \in P, r \in R, t \in T, n \in N, o \in O, k \in K$$

Where $\psi_{i'n}^1, \psi_{i'n}^2, \psi_{i'n}^3, \psi_{i'n}^4, \psi_{i'n}^5, \psi_{i'n}^6$ symbolize the optimal dual solutions for the sub problem (constraints (3),(5),(6),(7),(8),(9)) corresponding to iteration i', $BS^{i'}$ and ξ^n.

Step 2: Solve the master problem and set the lower bound equal to:

$$lb = min_{BS,\theta}\; fc^T BS_{i'} + \theta$$
$$s.t. \quad BS_{i'} \in Z$$
$$\theta \ge a_k^T BS_{i'} + b_k, k = 1,\ldots,i'$$

Where Z is feasibility space for investment decision variables in master problem. $BS_{i'}$ is optimal solution achieved in iteration i'. Moreover, θ is a free variable in master problem's objective function.

Step 3: Solve N sub problems substituting given $BS_{i'}$ in the related sub problem (for example $z_{ji}^{i'}$ in sub problem)and corresponding to $\xi^n = \left(c^n, q^n, d^n\right)$ for $n=1, \ldots, N$. Then, set $ub = \hat{f}_N\left(BS^{i'}\right)$ if ub is greater than $\hat{f}_N\left(BS^{i'}\right)$. Also, save $BS^{i'}$ in BS^* (optimum solution up to now).

$$\hat{f}_N\left(BS^{i}\right) = fc^T BS^{i'} + \frac{1}{N}\left[\sum_{n=1}^{N} Q\left(BS^{i'}, \xi^n\right)\right] \qquad (17)$$

Where, fc^T is cost coefficients of each binary solution obtained in master problem such as f_{st}^{su}, f_{it}^{c} and etc.

Step 4: Check the convergence test for attained solution. If $ub - lb \le \varepsilon$ (ε is desired gap for accepting the solutions) stop and return BS^* as optimal reconfiguration decisions and upper bound as optimal objective function value, otherwise, go to step 5.

Step 5: For each generated scenario ($n=1,\ldots,N$), $\left(\psi_{i'n}^1, \psi_{i'n}^2, \psi_{i'n}^3, \psi_{i'n}^4, \psi_{i'n}^5, \psi_{i'n}^6\right)$ denote the optimal dual solutions for the sub problem (constraints (3),(5),(6),(7),(8),(9)) corresponding to iteration i', $BS^{i'}$ and ξ^n computed in step 2 and 3. Therefore, cut constant term and coefficient term for adding the new optimality cut to master problem are presented as follows:

Cut constant for iteration ($i'+1$):

$$b_{i'+1} = \frac{1}{N}\left[\sum_{n=1}^{N}\left(\left(\psi_{i'n}^1\right)^T q_n + \left(\psi_{i'n}^3\right)^T d_n\right)\right] \qquad (18)$$

Cut coefficient for iteration ($i'+1$):

$$\begin{aligned} a_{i'+1} = &\left(\left(\frac{1}{N}\sum_n \psi_{i'n}^2\right)^T\left(\sum_{j\in E} u_j z_{ji}\right)\right) + \left(\left(\frac{1}{N}\sum_n \left(\psi_{i'n}^4\right)^T q_i^{UF}\right) z_{ii}\right) \\ &- \left(\left(\frac{1}{N}\sum_n \psi_{i'n}^4\right)^T\left(\sum_{j\in E} u_j z_{ji}\right)\right) + \left(\left(\frac{1}{N}\sum_n \left(\psi_{i'n}^5\right) q_{sr}^{SU}\right) su_s\right) \\ &+ \left(\left(\frac{1}{N}\sum_n \left(\psi_{i'n}^6\right) q_{sp}^{SP}\right) sp_{sp}\right) \end{aligned} \qquad (19)$$

Update iteration number $i'=i'+1$ and go to step 2.

For obtaining the solution gap, we can apply statistical relations derived from SAA (for more realization see [3], [5], [21]). For this purpose, let us to introduce the calculation procedure of optimality gap and its variance as follows:

Step 0: Determine N and M value so that N is the number of samples and M is the number of independent samples each of size N.

Step 1: Generate M independent samples: $\xi_j^1, \xi_j^2, \ldots, \xi_j^N$ for $j=1,\ldots,M$.

For each j compute:

$$\min_{BS\in Z}\left\{f(BS) := fc^T BS + \frac{1}{N}\sum_{s=1}^{N} Q\left(BS, \xi_j^N\right)\right\} \qquad (20)$$

Let V_N^j and $\hat{BS}_N^j$ be the corresponding optimal objective value and an optimal solution for $j=1,\ldots,M$, respectively.

Step 2: After calculation of objective functions for $j=1,\ldots,M$ compute:

$$\overline{V}_N^M = \frac{1}{M}\sum_{j=1}^{M} V_N^j \qquad (21)$$

$$\sigma_{V_N^M}^2 = \frac{1}{M(M-1)}\sum_{j=1}^{M}\left(V_N^j - \overline{V}_N^M\right)^2 \qquad (22)$$

We can say that $\overline{V}_S^M$ is a lower bound for optimal V_N^j (which is named V^*)[22].

Step 3: Estimate true objective function for one of the $\hat{BS}_N^j$ vector that is obtained in *jth* problem as follows (for example l^{th} problem and its solution vector):

$$\min_{z\in Z} \left\{ \hat{f}_{S'}\left(BS^l\right) := fc^T BS^l + \frac{1}{N'}\sum_{n=1}^{N'} Q\left(BS^l, \xi_j^n\right) \right\} \quad (23)$$

Note that the number of scenarios (N') based on considered probability distribution is huge and much bigger than *N*. Thus we can have an appropriate estimation for $f(BS^l)$ so that this approximation gives us a upper bound for problem. Moreover, if random sample $\xi_j^1, \xi_j^2, ..., \xi_j^{N'}$ would be iid,

(independent identically distributed), based on mentioned concepts, compute the variance of $\hat{f}_{N'}\left(BS^l\right)$ as follows:

$$\sigma^2_{N'}\left(BS^l\right) = \frac{1}{N'\left(N'-1\right)} \sum_{n=1}^{N'} \left[fc^T BS^l + Q\left(BS^l, \xi_j^n\right) - \hat{f}_{N'}\left(BS^l\right) \right]^2 \quad (24)$$

All of the relations (21)-(24) lead to compute optimality gap and its variance. Hence, consider equations (25)-(26) for evaluating the quality of solution as follows:

$$gap_{N,M,N'}\left(BS^l\right) = \hat{f}_{N'}\left(BS^l\right) - \bar{V}_N^M \quad (25)$$

$$\sigma^2_{gap}\left(BS^l\right) = \sigma^2_{N'}\left(BS^l\right) + \sigma^2_{\bar{V}_N^M} \quad (26)$$

As mentioned before, the heuristic algorithmis summarized in figure 1.

5. Computational Results

In this section, we describe two hypothetical examples in which the model parameters are stochastic. At the first, the characteristics of problem are explained then, we continue the example considering three assumptions: the first is no change in the supply chain configurations that like the former, active warehouses and other facilities will continue to work. In the second assumption, we consider a relocation model with stochastic parameters in which to find and solve the relocation model, obtained decision variables resulting MVP are considered. Finally, proposed solution method considering two-stage stochastic optimization is presented for stochastic model.To solve the problem, the iterative algorithm has been implemented in GAMS software monolithicallyusing the CPLEX solver (2 GHz CPU).

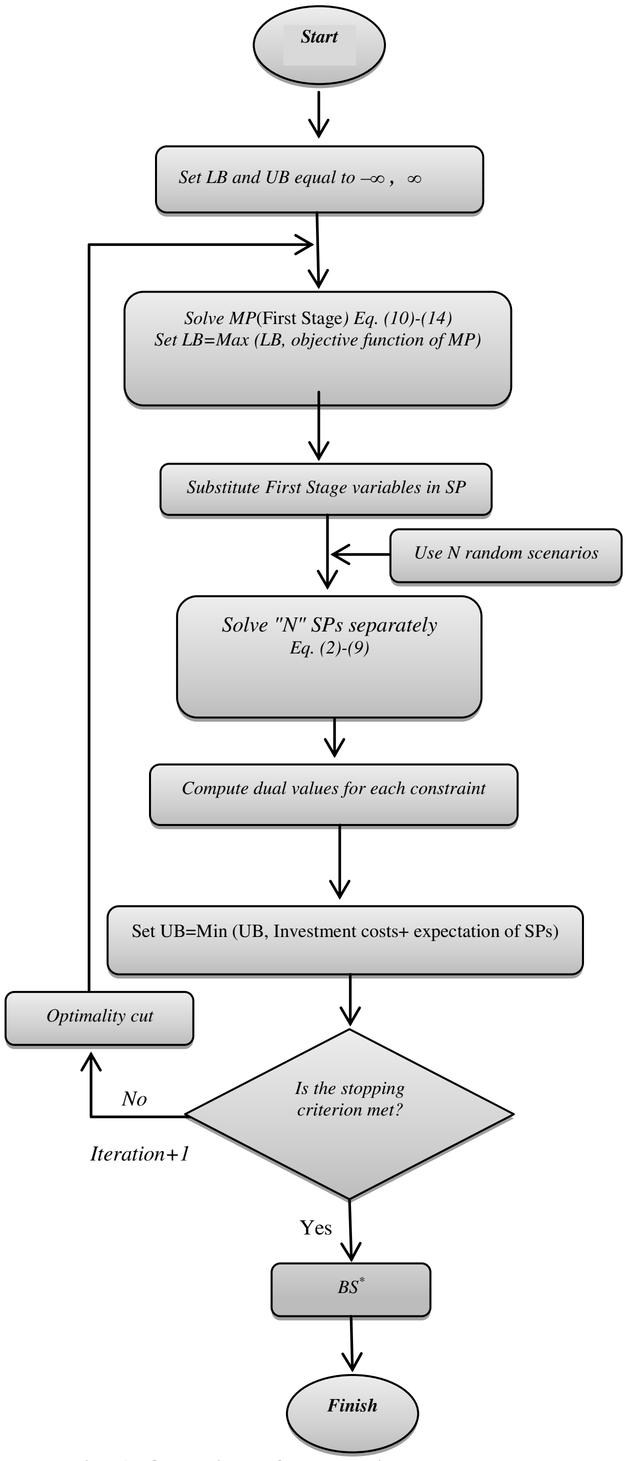

Fig. 1. Overview of the solving procedure

5.1.Supply Chain Network Characteristics

In this section, we use two models forexamples to illustrate that how the two-stage stochastic programming works on a relocation model. The first modelis consideredbased on Melachrinoudis and Min[16] (labeled P1), and the second one is based on the proposed mathematical model in section 3.1(labeled P2). For highlighting the reality of dimensions of numerical examples, the characteristics of P_{11} and $P_{21}$are summarized in Table 2.

Tab. 2. Characteristics of two numerical examples

	P_{11}	P_{21}
Total facilities	21	24
Number of suppliers	-	3
Number of plants	3	3
Number of existing warehouses	4	4
Number of New candidates warehouses	4	4
Number of customers	10	10
Sample Size	N=30	N=35
N'-value	N'=1000	N'=1000
M-value	M=20	M=10
Constraints-Equality	240	1750
Constraints-Inequality	642	3626
Variables-Binary	36	48
Variable- Continuous	3360	19390
Production and shipment cost	Uncertainty (Uniform)	Uncertainty (Uniform)
Production capacity	Uncertainty (Lognormal)	Uncertainty (Lognormal)
Demand	Uncertainty (Lognormal)	Uncertainty (Lognormal)

It is worth to noting that the number of constraints and continues variables have been presented based on deterministic equivalent approach. The main motivation for presenting table 2 is determination of problem's dimensions.As you know, deterministic equivalent can work similar to two stage stochastic programming but this approach cannot be implemented in GAMS software in large numbers of scenarios. In second example that is categorized in medium kind of problems, deterministic equivalent cannot find the solutions for $N > 24$.

Consequently, using the deterministic equivalent for problems with large scale is impossible. However, Bender's decomposition and SAA solve each problem separately in each iteration and add the optimality cut derived from duality concepts to the master problem. For more confirmation, we solved10 numerical examples (M=10, model of P2) and in all examples, the proposed method could solve the problem with sample size that are greater than 24. Accordingly, if we want to solve the problem using huge scenarios, the proposed approach can work suitable. In the next section, the quality of solution obtained by proposed approach is evaluated.

5.2. Performance of Two-Stage Stochastic Programming

In this section, The results of two-stage stochastic programming are compared with the MVP.

Table 3 reveals that the solutions based on two-stage stochastic programming are not only dominant to the MVP solutions in terms of optimality gap, but proposed solution also leads to comparatively smaller variability of cost which is denoted by σ_{gap} for both P_{12} and P_{21}. This table demonstrates that integration of Bender's decomposition and SAA proposed reliable and robust solutions under uncertainty and simulation results based on SAA (*N'=1000*) show that $\hat{f}_{N'}\left(BS^*\right)$ has the less average cost. Moreover, we calculate the cost of current situation (the configuration is selected randomly) in which the facilities that were active before reconfiguration, continue their activities without change. Based upon this, the total cost resulting from current situation can be compared with relocation results. Hence, we can state that relocation model is capable for reducing the cost in both of models (P1 and P2) and based on both stochastic programming approaches (MVP and proposed solving methodology). Note that in the P_{11}, after solving the numerical example with the proposed methodology, due to the thirteen term of the objective function, the cost is negative.

It can be interpreted that saved cost achieved by redundant warehouses is considerable. Moreover, three examples with different scenarios (N) were solved for each model (P1 and P2), based upon this, we observed that by increasing the scenarios (*N*), the proposed method works more effective in creating tight and precise statistical bounds.

As an instance, we have showed the criteria results for P1and P2 with different scenarios (*N*) and the results involving optimality gaps and its deviations are reported in Table 4.

Also, as an instance, convergence procedures for P2 with *N*=35 and P1 with *N*=20 are illustrated in figure 2 and 3 respectively. This figures show the values of upper and lower bounds during the iterations and convergence procedure.

Moreover, for more evaluation about verification of the proposed model and its solving method, two other examples were investigated for P2 model addition to P_{21} that was surveyed before (P_{22} and P_{23}). The results demonstrate that the proposed method works capable considering the pre-determined criteria such as gap, σ_{gap} ,etc. Table 5 shows the details of complementary sample problems. It's worth to nothing that all of the reported results in Table 5 have been analyzed based on N=35, M=10 for P2. This sample size's dimension for P_2 leads to create a reasonable data set according to computed dimensions in Table 2 and it can be compared with published works in this scope such as investigation of Mohammadi Bidhandi and Yusuff [11].

Table 5 shows that the proposed model and its solving method improves the current situation's costs, gapand σ_{gap}. Moreover, to check the model validation, we generated 20 problems with pre-determined parameters' values based on twenty specific decisions, which have been defined in advance. In all of them, the proposed model can find the decision variables' values correctly. For example, five sample problems' results and their consideration are given in Table 6.

Tab. 3. Costs statistics for obtained solution in P_{11} and P_{21}

Criteria	MVP solutions		Two-stage stochastic programming		Current situation	
	P_{11}	P_{21}	P_{11}	P_{21}	P_{11}	P_{21}
$\hat{f}_{N'}(BS^*)$	33770	3.5966E+09	-1470679.964	3.4658E+09	2426325	9.111787E+09
Gap	1.58E+06	1.74E+07	7.22E+04	6.36E+06	-	-
Gap (%)	>100%	0.4%	4.9%	0.18%	-	-
σ_{gap}	47710	1.25E+10	39604.65	2.2E+07	-	-

Tab. 4. Variability of costs in $P_{11}$and P_{21} for different sample size (Criteria versus generated sample size)

Problem	*N*	*Gap*	*Gap (%)*	σ_{gap}
	15	9.34E+04	6.35%	65194.44
P1	30	7.22E+04	4.91%	39604.65
	50	6.83E+04	4.64%	32325.12
	15	5.58E+07	1.55%	1.126E+08
P2	25	1.3E+07	0.36%	5.19E+07
	35	6.36E+06	0.18%	2.2E+07

Tab. 5. Costs statistics for obtained solution in complementary numerical examples from the P2 model (P_{22} and P_{23})

Criteria	MVP solutions		Two-stage stochastic programming		Current situation	
	P_{22}	P_{23}	P_{22}	P_{23}	P_{22}	P_{23}
$\hat{f}_{N'}(BS^*)$	412341	1.4359E+08	308762	5.98267E+07	549875	6.871057E+09
Gap	45678	4.29064E+06	22196	4.01934E+05	-	-
Gap (%)	11%	2%	7%	0.6%	-	-
σ_{gap}	52103	3.245E+8	29349.2	2.1937E+07	-	-

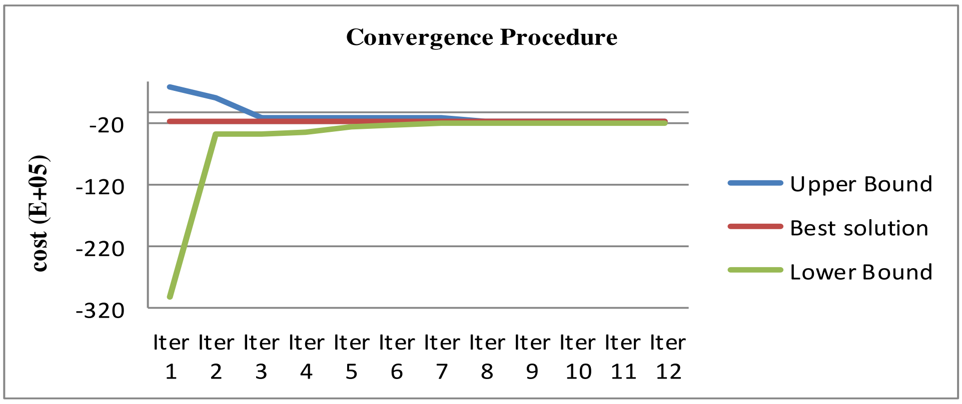

Fig. 2 . Iterative procedure for the convergence (P_{11})

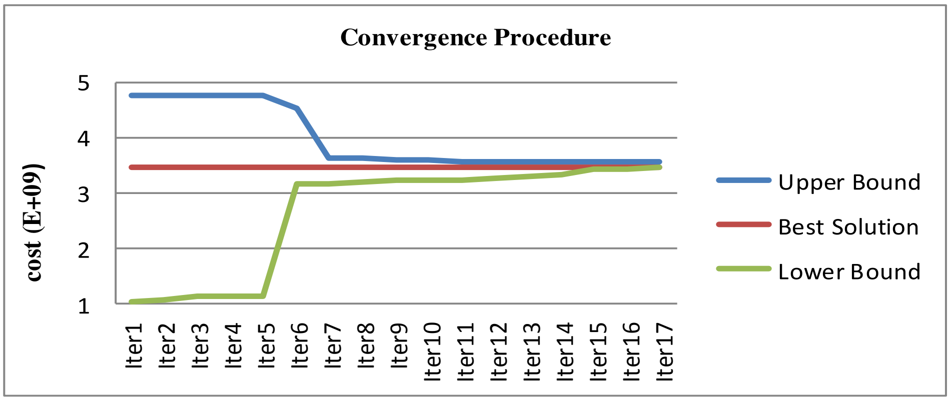

Fig. 3. Iterative procedure for the convergence (P_{21})

Tab. 6. Model validation for pre-determined solutions

Sample Problems	Expected Solutions	parameters' values	Validity
1	z_{11}=1 will be added to other variables	f_{11}^{C}, f_{12}^{C}: 86000→8600 q_{1}^{UF} :3000→30000	✓
Reducing the fixed cost and increasing the capacity lead to opening the warehouse 1			
2	sp_{1p}will be eliminated (for at least one of the plants):sp_{12}=0	q_{12}^{SU} :80000→8000	✓
Reducing the capacity of supplier 1 for raw material leads to elimination of link between supplier 1 with other plants due to reduction in capacity			
3	Warehouse 3 will be eliminated from warehouses(z_{33}=0)	$b_{3k} = 0, \forall k$	✓
Warehouse 3 will not cover the customers, so, covering equation leads to closing this warehouse			
4	The number of Warehouses will be less than two warehouses	$NU \le 2$	✓
5	sp_{11}=0	$q_{11r}^{SP} = 0, \forall r$	✓
Reducing the capacity of transportation link between supplier and plant leads to closing the link			

6. Conclusion

In this paper, redesigning the warehouse in a supply chain were investigated in which parameters such as production capacity, demands and transportation costs were analyzed in stochastic environment. Integration of SAA scheme and Bender's decomposition method were applied to show two-stage stochastic program improve the quality of solutions. Moreover, the total costs obtained by proposed approach not only were superior to solutions of the MVP, but proposed solution also has the more desirable statistics criteria such as optimality gap and its deviation in solving procedure. As a conclusion, we can state that the proposed methodology has more applicability in case of more variability in the uncertain environment with numerous scenarios so that confiding to MVP solutions may lead to decision with high risk and consequently facing to unpredicted events and costs during the time horizon. As a future research, developing the proposed mathematical model with closed loops supply chain network is suggested. Moreover, multi objective decision making in mentioned model with stochastic parameters is another suggestion.

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