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Integrated strategic and tactical planning in a supply chain network design with a heuristic solution method

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ABSTRACT

In the current competitive business world, viable companies are those that have flexible strategies and long-term plans, by which they can appropriately respond to a dynamic environment. These strategies are used to find the optimum allocation of company income to the main sources of development, for the expansion of company activities and for service expansions.

This paper presents a new mathematical model for multiple echelon, multiple commodity Supply Chain Network Design (SCND) and considers different time resolutions for tactical and strategic decisions. Expansions of the supply chain in the proposed model are planned according to cumulative net profits and fund supplied by external sources. Furthermore, some features, such as the minimum and maximum utilisation rates of facilities, public warehouses and potential sites for the establishment of private warehouses, are considered. To solve the model, an approach based on a Lagrangian Relaxation (LR) method has been developed, and some numerical analyses have been conducted to evaluate the performance of the designed approach.

1. Introduction

A supply chain is defined as the chain that links each entity of a manufacturing and supply process, from the raw materials to the end user. A supply chain comprises many systems, including various procurement, manufacturing, storage, transportation and retail systems [1].

The terminology Supply Chain Network Design (SCND) is sometimes employed as a synonym for strategic supply chain planning (see [2–5]). In the current competitive world, a supply chain network is expected to be viable for a considerable time, during which many parameters can change. It may be important to consider the possibility of making future adjustments in the network configuration to allow gradual changes in the supply chain structure and/or in the capacities of the facilities. In this case, a planning horizon that is divided into several time periods is typically considered, and strategic decisions are to be planned for each period. Such a situation occurs, for example, when large facility investments are limited by the budget that is available during each period [6].

In Supply Chain Management (SCM), three planning levels are usually distinguished, depending on the time horizons: strategic, tactical and operational [7,8]. The strategic level addresses decisions that have a long-lasting effect on the firm, such as decisions about the number, location and capacities of warehouses and manufacturing plants or the flow of material through the logistics network [5].

In strategic decisions that involve large investments, facilities that are currently in operation are expected to operate for a long-term horizon. Moreover, changes of various types during the facility's lifetime could make a location that is good today become a bad location in the future [6].

There are several models that have been developed to help managers when designing and planning their supply chain. Arntzen et al. [9] developed a mixed integer linear programming model for production and distribution planning with multiple products and a network of sellers. Amiri [10] proposed a mixed integer linear model to select the optimum numbers, locations and capacities of plants and warehouses to open so that all of the customer demands are satisfied at a minimum total cost for the distribution network in three echelons for a single period and a single product. In this paper, an efficient heuristic solution procedure for this supply chain system problem is provided.

Wouda et al. [11] developed a mixed integer linear programming model for the optimisation of the supply network of Nutricia Hungary. Their model focused on consolidation and

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product specialisation of plants, and the objective was to find the optimal number of plants, their locations and the allocation of the product portfolio of these plants, while minimising the sum of the production and transportation costs.

Noorul Haq and Kannan [1] developed an integrated supplier selection and multi-echelon distribution inventory model in a built-to-order supply chain that involved a single selected supplier, multiple plants, multiple distributors, wholesalers, and retailers. Dias et al. [12] worked on the re-engineering of a two-echelon network (facilities and customers). The authors assumed that facilities could be opened, closed, and reopened more than once during the planning horizon. They studied these conditions within three scenarios: with maximum capacity restrictions, with both maximum and minimum capacity restrictions, and with a maximum capacity that decreases. All of these problems are solved by primal-dual heuristics. In this paper, three linear formulations correspond to the three previous scenarios, and their linear dual formulations are presented.

Melo et al. [13] aimed at relocating a network with expansion/reduction capacity scenarios. The capacity can be exchanged between an existing facility and a new facility, or between two existing facilities under certain conditions. Each change of capacity is penalised by a cost. In this model, closed facilities cannot be reopened, and new facilities will remain active until the end of the planning horizon. Thanh et al. [14] propose a dynamic mixed integer linear programming model for a four echelon supply chain that includes suppliers, manufacturing firms, distribution centres, and customers. The bill of materials and multiple products have been taken into consideration. This paper aims to help strategic and tactical decisions, which include the following: opening, closing or enlargement of facilities, supplier selection, and flows along the supply chain. They make a distinction between a private warehouse (owned by the company) and a public warehouse (hired by the company). The status of a public warehouse can change more than once during the planning horizon. Park et al. [15] proposed a mathematical model for single-sourcing a network design problem with a three-level supply chain that consists of multiple suppliers, distribution centres and retailers. The proposed integer nonlinear programming model is solved using a two-phase heuristic solution algorithm based on the Lagrangian relaxation approach.

Some scholars in the field of supply chain modelling have considered the location problem in their networks [16–20]. Thanh et al. [14] presented four echelons for a multiple period supply chain with dynamic demands in which they suggest adding budget constraints to their model.

In many papers, the expansion of facilities is restricted to a predetermined fund or a fixed number of maximum facilities allowed to be established in each period. In real situations, a company's expansion budget is supplied mostly by their net profit after tax and stakeholders' share deduction.

In this paper, a supply chain network design problem with multiple commodities is considered in which the main objective is to make strategic and tactical decisions. This model is a mixed integer linear programming (MILP) model for network design and expansion planning of a four echelon multiple commodity supply chain. This approach also considers different time resolutions for strategic and tactical decisions. Furthermore, this model makes decisions about supplier selection, production facility and warehouse location as well as production, distribution, and expansion planning in a long-term horizon. Expansion of the supply chain in the proposed model is restricted to cumulative net profit and funds supplied from external sources.

This model can be applied in firms with the capability of producing a family of products. For example, firms in the food industry design their market strategies to promote their brand by producing a variety of products. In such markets, customers' demands are affected mainly by population changes and, consequently, can be predicted for a strategic time horizon with a negligible deviation.

The remainder of this paper is organised as follows: in Section 2, the MILP model for the SCND problem will be presented. To evaluate this model, some numerical analyses are conducted using the CPLEX solver and are presented in Section 3. In Section 4, a solution procedure based on the Lagrangian relaxation method will be presented. To evaluate the performance of the proposed approach, some computational experiments will be performed and described in Section 5, and finally, conclusions are drawn in Section 6.

2. MILP model for the SCND problem

In this section, a MILP model for the supply chain network design, expansion planning and production–distribution planning is presented based on the model proposed by Bashiri et al [51]. This model helps managers to make their strategic and tactical decisions. Because in a real-world situation many parameters are dynamic, it is essential to take this dynamic nature into consideration when using multiple period models. In contrast, one of the most important features of strategic decisions is that they should be maintained the same for a long time. The facility location problem is a type of strategic problem, but there are some papers in the literature in which the locations are determined in a single period and with static parameters. There are a few papers in the literature that consider a facility location and the production–distribution problem in a dynamic model [14].

The proposed model in this paper makes some strategic decisions, such as facility locations and capacity expansions. From the tactical viewpoint, this model determines the production and distribution planning during the decision horizon. Strategic decisions are those in which a long-term horizon is considered during the decision making, but many papers make strategic decisions in a midterm horizon in addition to tactical decisions. In the proposed model, the strategic and tactical decisions are made in different periods and time resolutions, which are high resolution for tactical decisions and low resolution for strategic decisions. Furthermore, the interest rate is considered to take time into consideration in monetary calculations.

Thanh et al. [14] suggest that budget constraints be added for the establishment of new facilities in each period. Melo et al. [13] consider a predetermined budget for investment in each period. In many firms, an expansion budget is supplied by the cumulative net profit after tax and stakeholder share reductions. Because costs, incomes and, thus, net profit are unknown parameters before the supply chain is designed, managers are not able to determine the expansion budget to use in a budget constraint. In the proposed model it is supposed that the main financial resource for expansion is the net profit of the chain. Therefore, the proposed model in this paper uses cumulative net profits and a budget limited to a predetermined maximum amount which could be supplied from external sources. Obviously, the difference between maximum (potential) external budget and the amount actually supplied for each period has not been assigned and therefore, cannot be transferred to the next periods. In [13] it is assumed that the predetermined budget has been assigned completely, and unused amount of this budget is transferred to the next periods. A similar approach in both models is that in Melo et al. [13] budget not invested in each period is accumulated for the next periods and in our proposed model remaining net profits are an accumulating capital.

Some of the most important decisions in the proposed model are as follows:

- Location and establishment time of facilities (production plant, warehouse) during the planning horizon.
- Decisions about establishing a new facility or adding capacity to one or more of the established facilities.
- Supplier selection and the raw material quantity to be supplied from them.
- Product quantity to be produced in each production plant.
- Product quantity to be transported from each production plant to each warehouse.
- Product quantity to be transported from each warehouse to each customer zone.
  Moreover, some of the most important assumptions in the proposed model are as follows:
- The objective is to maximise the supply chain net profit.
- The customer demands are dynamic and deterministic during the time periods.
- Each potential node has an initial capacity and a maximum installable capacity.
- Facilities should operate between minimum and maximum utilisation rates.
- Established production plants and private warehouses cannot be closed.
- Closing public warehouses is permitted.
- Costs associated to facility construction and expansion are supplied by cumulated net profit and external funds (in the first strategic period these costs are covered by only the external fund).
- Cost associated to supply, production and distribution is supported by net incomes over the related strategic period.

2.1. Notations

Sets

$\mathcal{K}(k \in \mathcal{K},\ k = 1,\ldots,K)$ set of strategic periods
$\mathcal{T}(t \in \mathcal{T},\ t = 1,\ldots,T)$ set of tactical periods
$\mathcal{S}(s \in \mathcal{S})$ set of suppliers
$\mathcal{M}(i \in \mathcal{M})$ set of production plants
$\mathcal{W}(j \in \mathcal{W})$ set of warehouses
$\mathcal{W}p$ set of private (permanent) warehouses
$\mathcal{WH}$ set of public (hired) warehouses
$\mathcal{O}(o \in \mathcal{O})$ set of capacity options for expansion
$\mathcal{C}(c \in \mathcal{C})$ set of customers
$p(p \in p)$ set of products (raw material and finished product)
$p_r(p_r \subset p)$ set of raw materials
$p_f(p_f \subset p)$ set of finished products

Parameters

INV^k maximum amount of funds could be supplied by external sources such as loans for investment in period k
Ir interest rate
TR tax rate
SH stakeholders' share (in %)
$BigM$ a large number
$R^{k,t}_{s,p}$ available capacity of supplier s for p at each tactical period
MK_i initial capacity at i
NK_i maximal installable capacity at i
MU_i minimal utilization rate of facility i
NUi maximal utilization rate of facility i
CK_o capacity of option o
$D^{k,t}_{c,p}$ demand of customer c for product p at each tth tactical period of the kth strategic period
$B_{p',p}$ quantity of raw material p' necessary to manufacture a unit of p
$WL_{p,i}$ work load to produce a unit p at plant i
V_p work load of a unit p at warehouses
$MO_{s,p}$ minimal allowable order of a unit p to supplier s
$A_{i,j}$ number of deliveries from plant i to warehouse j in one tactical period
$\mathcal{PR}_p$ selling price of a unit p to customers
$\mathcal{PS}_{p,s}$ price of raw material p supplied by supplier s
Co_i fixed cost for opening a facility at a potential location i
$CA_{i,o}$ fixed cost for adding capacity option o to facility i
CU_i fixed cost for operating facility i
$Cop_{i,o}$ fixed cost for operating capacity option o at facility i
$CP_{p,i}$ variable cost of production of a unit p at plant i
$CS_{p,j}$ storage cost of a unit of p at warehouse j
$CT_{p,i,j}$ transportation cost of a unit of p from plant i to warehouse j
$CD_{p,s,i}$ transportation cost of a unit of p from supplier s to plant i
$CF_{p,j,c}$ transportation cost of a unit of p from warehouse j to customer c

Variables

Inc^k net income in period k, ($Inc^k = 0,\ \forall k = 0$)
$\mathcal{F}^k$ cumulative net profit from the first period to period $k-1$,
DL^k cumulative net profit after tax and stakeholders' share reduction from the first period to period $k-$
$\mathcal{F}$ total profit
x^k_i 1 if the facility i is active at k; 0 otherwise ($x^k_i = 0,\ \forall k = 0$)
$y^k_{i,o}$ 1 if the capacity option o is added to i; 0 otherwise ($y^k_{i,o} = 0,\ \forall k = 0$)
$z^{k,t}_{s,p}$ 1 if the supplier s is selected for the raw material p; 0 otherwise
$f^{k,t}_{p,i,j}$ quantity of item p transferred from location $i9j$
$q^{k,t}_{p,i}$ quantity of product p produced in plant i
$h^{k,t}_{p,j}$ quantity of product p held in warehouse j at the end of t ($h^{k,t}_{p,j} = 0,\ \forall k = 0$)

2.2. Objective function

The objective function is to maximise the total net income over the time periods, which is computed by subtracting the total cost from the total revenue. The total cost includes the fixed costs of opening facilities, adding facility options, operating facilities and the variable costs of raw materials, production, inventory and transportation. Eq. (1) shows the objective function in which the net present value of the total net income is maximised.

$\mathcal{K}(k \in \mathcal{K},\ k = 1,\ldots,K)$

$\mathcal{T}(t \in \mathcal{T},\ t = 1,\ldots,T)$

$$\text{Maximize } \mathcal{F} = \sum_{k \in k} \frac{Inc^k}{(1+Ir)^{k-1}} \tag{1}$$

2.3. Constraints

$$\sum_{j \in \mathcal{W}} f^{k,t}_{p,j,c} \le D^{k,t}_{c,p} \quad \forall c \in \mathcal{C},\ \forall p \in k_f,\ \forall k \in k,\ \forall t \in \mathcal{T} \tag{2}$$

Constraint (2) states that all of the products that are transferred to customers should not be more than their demands in each period. We should note that, in this model, it is not necessary to satisfy all of the customer demands:

$$h_{p,j}^{(k-1),T} + \sum_{i\in\mathcal{M}} f_{p,i,j}^{k,t} = \sum_{c\in\mathcal{C}} f_{p,j,c}^{k,t} + h_{p,j}^{k,t} \ \forall j\in\mathcal{W},\ \forall p\in p_f,\ \forall k\epsilon\mathcal{K},\ \ t=1 \quad (3)$$

$$h_{p,j}^{k,(t-1)} + \sum_{i\in\mathcal{M}} f_{p,i,j}^{k,t} = \sum_{c\in\mathcal{C}} f_{p,j,c}^{k,t} + h_{p,j}^{k,t} \ \forall j\in\mathcal{W},\ \forall p\in p_f,\ \forall k\,\epsilon\mathcal{K},\ \ t\neq 1 \quad (4)$$

Constraints (3–4) are related to the equilibrium of flows at warehouses. The quantity of a product that is stored at the end of the previous tactical period plus the total quantity of the product that is delivered to the warehouse during the current tactical period should be equal to the quantity of that product that is transported to customer zones plus the quantity that is stored at the end of the current tactical period:

$$\sum_{s\epsilon\,\mathcal{S}} f_{p,s,i}^{k,t} = \sum_{p\in p_f} B_{p',p}\cdot q_{p,i}^{k,t} \ \ \forall i\in\mathcal{M},\forall p'\in p_r,\ \forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T} \quad (5)$$

Constraint (5) ensures that plants receive sufficient raw materials to produce the required quantity of finished products:

$$q_{p,i}^{k,t} = \sum_{j\in\mathcal{W}} f_{p,i,j}^{k,t} \ \ \forall i\in\mathcal{M},\forall p\in p_f,\ \forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T} \quad (6)$$

Constraint (6) states that the quantity of the manufactured products at a plant should be equal to the quantity that is delivered to warehouses.

$$\sum_{p\in p_f} WL_{p,i}.q_{p,i}^{k,t} \le NU_i.\left(MK_i.x_i^k + \sum_{o\in\mathcal{O}} CK_o y_{i,o}^k\right) \ \ \forall i\in\mathcal{M},\ \forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T} \quad (7)$$

$$\sum_{p\in p_f} WL_{p,i}.q_{p,i}^{k,t} \ge MU_i.\left(MK_i.x_i^k + \sum_{o\in\mathcal{O}} CK_o y_{i,o}^k\right) \ \ \forall i\in\mathcal{M},\ \forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T} \quad (8)$$

Constraints (7) and (8) are related to the capacity of production plants. These constraints prevent a plant from functioning under its minimum rate of utilisation and from exceeding its maximum rate of utilisation of its installed capacity. The installed capacity is the sum of the initial capacity and the capacity of the added options:

$$\sum_{p\in p_f} V_p.\left(h_{p,j}^{k,t} + \sum_{i\in\mathcal{M}} \frac{1}{\text{A}_{\text{i,j}}}.f_{p,i,j}^{k,t}\right) \le MK_j x_j^k + \sum_{0\in\mathcal{O}} CK_o.y_{j,o}^k \ \ \forall j\in\mathcal{W}p,\ \forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T} \quad (9)$$

$$MK_i.x_i^k + \sum_{0\in\mathcal{O}} CK_o.y_{i,o}^k \le NK_i \ \ \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall k\epsilon\mathcal{K} \quad (10)$$

Warehouses must not store more than their storage capacity (9). In addition, the installed capacity at any plant and any warehouse must not exceed its maximal installable capacity (10):

$$\sum_{i\in\mathcal{M}} f_{p,s,i}^{k,t} \le z_{s,p}^{k,t}.R_{s,p}^{k,t} \ \ \forall s\in\mathcal{S},\forall p\in p_r,\ \forall k\epsilon\ k,\forall t\ \epsilon\ \mathcal{T} \quad (11)$$

$$\sum_{i\in\mathcal{M}} f_{p,s,i}^{k,t} \ge MO_{s,p}.z_{s,p}^{k,t} \ \ \forall s\in\mathcal{S},\forall p\in p_r,\forall k\epsilon\ k,\forall t\epsilon\ \mathcal{T} \quad (12)$$

Suppliers deliver a raw material if and only if they are selected for this raw material (11), and their delivery cannot exceed their available capacity. Constraint (12) is to avoid purchasing each raw material less than a predetermined minimal amount of the deliverable quantity of each supplier:

$$Inc^k = \sum_{t\epsilon\mathcal{T}}\sum_{j\epsilon\mathcal{W}}\sum_{p\in p_f}\sum_{c\epsilon\mathcal{C}} \mathcal{PR}_p.f_{p,j,c}^{k,t} \quad (13)$$

$$-\sum_{i\in\mathcal{M}\cup\mathcal{W}p} Co_i.\left(x_i^{k+1} - x_i^k\right) \quad (14)$$

$$-\sum_{i\in\mathcal{M}\cup\mathcal{W}}\ \sum_{o\epsilon\ \mathcal{O}} CA_{i,o}.\left(y_{i,o}^{k+1} - y_{i,o}^k\right) \quad (15)$$

$$-\sum_{i\in\mathcal{M}\cup\mathcal{W}p}\left(CU_i.x_i^k + \sum_{o\epsilon\ \mathcal{O}} Cop_{i,o}.y_{i,o}^k\right) \quad (16)$$

$$-\sum_{t\epsilon\ \mathcal{T}}\sum_{p\in p_f}\sum_{i\in\mathcal{M}} CP_{p,i}.q_{p,i}^{k,t} \quad (17)$$

$$-\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{j\in\mathcal{W}} CS_{p,j}.\left(h_{p,j}^{k,t} + \sum_{i\in\mathcal{M}} \frac{f_{p,i,j}^{k,t}}{2\text{A}_{\text{i,j}}}\right) \quad (18)$$

$$-\sum_{t\epsilon\ \mathcal{T}}\sum_{s\epsilon\mathcal{S}}\sum_{p\in p_r}\sum_{i\in\mathcal{M}} CD_{p,s,i}.f_{p,s,i}^{k,t} \quad (19)$$

$$-\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{W}} CT_{p,i,j}.f_{p,i,j}^{k,t} \quad (20)$$

$$-\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{j\in\mathcal{W}}\sum_{c\in\mathcal{C}} CF_{p,j,c}.f_{p,j,c}^{k,t} \quad (21)$$

$$-\sum_{t\epsilon\mathcal{T}}\sum_{s\epsilon\mathcal{S}}\sum_{p\in p_r}\sum_{i\in\mathcal{M}} \mathcal{P}S_{p,s}.f_{p,s,i}^{k,t} \quad (22)$$

Constraints (13)–(22) are related to the net income of the supply chain during each period, which is obtained by subtracting the total cost from the total revenue. The total revenue is calculated by (13), and the total cost includes the fixed costs of opening a facility (14), adding capacity options to plants and warehouses (15), operating facility fixed costs (16), production variable costs (17), storage variable costs (18), transportation costs from the suppliers to plants (19), from plants to warehouses (20) and from warehouses to customers (21), and finally, the raw material supply costs (22):

$$\mathcal{F}^k = \sum_{l=1}^{k} Inc^{l-1} \ \ \forall k\epsilon\mathcal{K} \quad (23)$$

$$DL^k = (1-TR).(1-SH).\mathcal{F}^k \ \ \forall k\epsilon\mathcal{K} \quad (24)$$

$$\sum_{i\in\mathcal{M}\cup\mathcal{W}p} Co_i.\left(x_i^k - x_i^{k-1}\right) + \sum_{i\in\mathcal{M}\cup\mathcal{W}p}\sum_{o\epsilon\mathcal{O}} CA_{i,o}.\left(y_{i,o}^k - y_{i,o}^{k-1}\right) \le DL^k + INV^k \ \ \forall k\epsilon\ \mathcal{K} \quad (25)$$

Constraint (23) calculates the cumulative net income from the first period to period $k-1$. Constraint (24) calculates the net profit after taxes and stakeholder share reduction. Constraint (25) prevents excess costs for opening a facility and adding an option to some opened facilities to be more than the expansion budget in each period. This constraint states that the expansion budget is limited to the net profit of the supply chain in the previous strategic periods plus the funds that are could be supplied from external sources such as loans:

$$y_{i,o}^k \le x_i^k \quad \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall o\in\mathcal{O},\ \forall k\epsilon\mathcal{K} \quad (26)$$

$$x_i^{k-1} \le x_i^k \ \ \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall k\epsilon\ \ \mathcal{K} \quad (27)$$

$$y_{i,o}^{k-1} \le y_{i,o}^k \quad \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall o\in\mathcal{O},\ \ \forall k\epsilon\mathcal{K} \quad (28)$$

Constraint (26) states that only the opened facility can extend its capacity. Constraint (27) prevents the opened facilities from being closed during the next period. Constraint (28) states that

Table 1
Structure of test problems and investment strategies.

Class	Problem	Strategic period	Tactical period	Supplier	Production plant	Warehouse		Customer	Raw material	Product	Capacity option	Investment in each strategic period				
						Private	Public					1	2	3	4	5
S	P1	5	4	5	5	3	2	10	4	5	3	8000	7000	–	–	–
	P2	5	4	12	7	4	3	25	8	6	3	8000	7000	–	–	–
	P3	5	4	15	8	5	3	35	10	7	4	8000	7000	–	–	–
	P4	5	4	18	9	6	4	50	12	8	4	8000	7000	–	–	–
	P5	5	4	20	10	7	5	60	15	8	5	8000	7000	–	–	–
M	P6	5	4	23	11	8	6	70	18	9	5	10000	8000	–	–	–
	P7	5	4	25	12	8	6	80	20	10	5	10000	8000	–	–	–
	P8	5	4	27	13	9	7	85	23	11	5	10000	8000	–	–	–
L	P9	5	4	30	17	10	8	100	27	13	6	10000	8000	–	–	–
	P10	5	4	35	20	12	10	120	32	15	7	10000	8000	–	–	–

the new capacity options can be added but cannot be removed:

$$\sum_{c\in C}\sum_{p\in p_f} f^{k,t}_{p,j,c} \le x^k_j.BigM \quad \forall j\in\mathcal{W},\ \forall k\in \mathcal{K},\ \forall t\in\mathcal{T} \tag{29}$$

$$\sum_{o\in\mathcal{O}} y^k_{i,o} \le 1 \quad \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall k\in\mathcal{K} \tag{30}$$

$$y^k_{i,o} \le 1-\left(x^k_i - x^{k-1}_i\right) \quad \forall i\in\mathcal{M}\cup\mathcal{W}p,\ \forall o\in\mathcal{O},\ \forall k\in\mathcal{K},\ \forall t\in\mathcal{T} \tag{31}$$

Constraint (29) ensures that only opened warehouses can send products to customers. Eq. (30) states that we cannot add more than one capacity option to a facility in one period, and constraint (31) prevents adding any facility option during the first period of opening a facility:

$$x^k_i \in\{0,1\} \quad \forall k\in\mathcal{K} \tag{32}$$

$$y^k_{i,o} \in\{0,1\} \quad \forall k\in\mathcal{K} \tag{33}$$

$$z^{k,t}_{s,p} \in\{0,1\} \quad \forall s\in\mathcal{S},\ \forall p\in p_r,\ \forall k\in\mathcal{K},\ \forall t\in\mathcal{T} \tag{34}$$

$$f^{k,t}_{p,i,i'} \ge 0 \quad \forall p\in p,\ \forall i\in\mathcal{S}\cup\mathcal{M}\cup\mathcal{W},\ \forall i'\in\mathcal{M}\cup\mathcal{W},\ \forall k\in\mathcal{K},\ \forall t\in\mathcal{T} \tag{35}$$

$$q^{k,t}_{p,i} \ge 0 \quad \forall p\in p_f,\ \forall i\in\mathcal{M},\ \forall k\in\mathcal{K},\ \forall t\in\mathcal{T} \tag{36}$$

$$h^{k,t}_{p,j} \ge 0 \quad \forall p\in p_f, \forall j\in\mathcal{W}, \forall k\in\mathcal{K}, \forall t\in\mathcal{T} \tag{37}$$

Constraints (32)–(34) require that these variables are binary. Constraints (35)–(37) restrict these variables from taking non-negative values.

3. Numerical analysis

The computational experiments that are described in this section were designed to evaluate the proposed model. Different test problems were designed in three classes, with five test problems (P1–P5) in the small class (S), three test problems (P6–P8) in the medium class (M) and two test problems (P9 and P10) in the large class (L). These instances have been solved using the CPLEX MIP solver. The CPLEX MIP solver was run on a Dual core 2.26 GHz processor with 2 GHz of RAM. The planning horizon was fixed during five strategic periods, and each strategic period includes four tactical periods. In each class, we generate five instances with the same characteristics to reduce the impact of using specific data sets. Table 1 shows the structure of the test problems and the investment strategies.

After solving the instances, the total number of variables, the total number of binary variables, and the total number of constraints and CPU times to reach the optimal solution are reported. We observe that, in the small class, the total number of variables varies from 11,113 to 141,128, the total number of binary variables varies from 600 to 6660, the total number of constraints varies from 4138 to 30,748, and the CPU time varies from 3.56 s to 208.1 s. In the medium class of instances, the increasing trend in the number of variables, the number of binary variables and the number of constraints causes the CPU time to be increased significantly. An average of the CPU time in instance P6 is less than 7 min, and this amount in instance P8 reaches 20 min.

In a large group of instances, the total number of variables exceeds 1,400,000, and the total number of binary variables exceeds 100,000. The CPU time for this group is significantly different from the small and medium classes. The CPU time in instance P9 exceeds 2 h, and in instance P10, it exceeds 3 h. The computational results of the instances are given in Table 2.

Table 2
Computational results of instances.

Class	Problem	Total variables	Binary variable	Constraint	CPU(s)		
					Min	Ave.	Max
S	P1	11,113	600	4138	2.8	3.56	4.1
	P2	44,213	2200	10,938	13.8	16.52	17.7
	P3	77,813	3400	16,463	33.8	39.76	46.2
	P4	141,128	4795	23,863	88.5	97.04	108.9
	P5	204,593	6660	30,748	191.1	208.1	228.4
M	P6	308,743	9030	40,588	320.8	408.64	469.1
	P7	393,593	10,780	49,098	675.6	732.52	788
	P8	526,103	13,290	59,338	1020.8	1220.48	1305.3
L	P9	849,498	17,425	81,333	7335	7991.6	8469
	P10	1,408,693	24,080	112,338	> 3 h	> 3 h	> 3 h

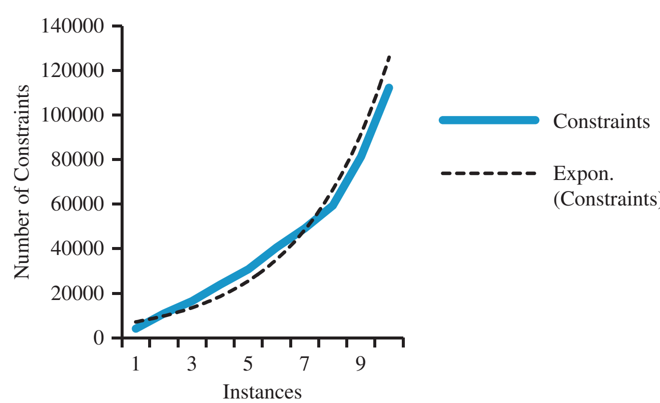

Fig. 1. Number of constraints in each instance.

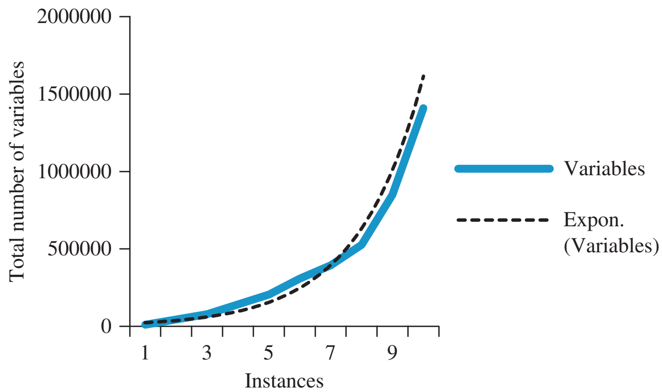

Fig. 2. Total number of variables in each instance.

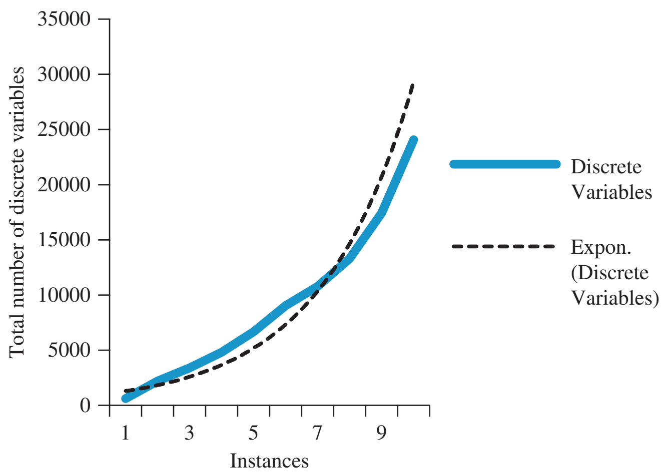

Fig. 3. Number of binary variables in each instance.

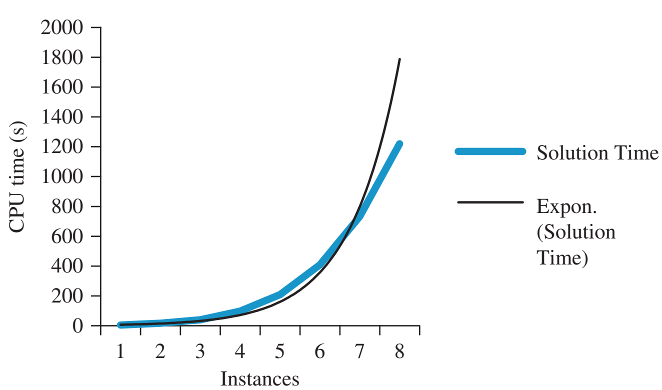

Fig. 4. CPU time for instances of the small and medium classes.

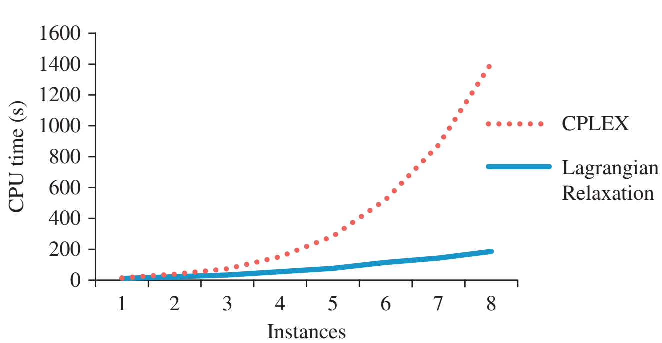

Fig. 5. Solution time of CPLEX and the Lagrangian relaxation approach.

Although capacitated facility location problems are known to be NP-hard [21], complexity analysis of mathematical models can help to realise a model′s features as well as to understand the importance and necessity of designing some new solution approaches. The most important feature of NP-hard problems is that, in this type of problem, the solution time follows an exponential function [22]. In this paper, we observe that, by increasing the number of variables and constraints, the CPU time is increased exponentially. Figs. 1–5 show trends that involve the number of constraints, the total number of variables, the number of binary variables and the CPU time.

In addition, the existence of some special complex constraints, such as a budget constraint, and using two types of decision periods that are related to each other, increase the complexity of the proposed model. Furthermore, because of the mentioned descriptions, a heuristic solution approach that is based on Lagrangian Relaxation is proposed in this study, which is illustrated in the next section.

4. Solution procedure

In facility location problems, when the number of discrete variables is large (which often occurs when the strategic location decisions refer to more than one facility layer in the supply chain network), the resulting models are comparatively more complex, and realistically sized problems can be solved only with a heuristic method. Lagrangian relaxation, linear programming based heuristics and metaheuristics are among the most popular techniques [6].

In this regard, Rezapour et al. [23] applied the Simulated Annealing (SA) metaheuristic method to solve sequential two-stage models of SCND with duopolistic competitors. They developed bi-level optimisation models, which consider von Stackelberg and minimum regret strategies. The upper level model has been solved by SA, and the Branch & Bound method has been used to solve the inner model.

Ross and Jayaraman [24] developed a new heuristic approach for finding the optimal location of cross-docks and distribution centres in a supply chain network. Their model is characterised by different families of products, a central manufacturing site, and multiple cross-docking and distribution centres. Altiparmak et al. [25] presented an approach based on Genetic Algorithms (GAs) for designing a single-source, multi-product, multi-stage supply chain network. The proposed GA has a new encoding structure, and the results have been compared with those obtained by CPLEX, Lagrangean heuristics, and hybrid GAs and SAs. Another study on this topic was conducted by Ko and Evans [26] in which a mixed integer nonlinear programme was developed to optimise a dynamic integrated distribution network to account for the integrated aspect of optimising the forward and return network simultaneously. As a solution procedure, they applied GAs to finding the best design of the multi-period, two-echelon, multi-commodity capacitated supply chain network.

Montoya-Torres et al. [27] proposed a Greedy Randomised Adaptive Search Procedure (GRASP) to solve the three-echelon uncapacitated facility location problem with multiple items. Furthermore, a production–distribution system that consists of defining the flow of produced products from manufacturing plants to clients (markets) via a set of warehouses was considered. Zegordi et al. [28] considered a two-stage supply chain in which a production and transportation plan is optimised using a GA. Table 3 reviews some major studies on SCND, which focus on the modelling aspects as well as the solution methods.

- *A Lagrangian relaxation of the proposed model*

  The proposed model is a mixed-integer programming model, which includes, as a special case, the classical

Table 3
Major studies on SCND and related characteristics.

Study	Modelling aspects	Solution method
[23]	Game against nature Two stage SCND models Multi-tiered Single-product Price-dependant demands	Simulated annealing for upper level model Branch and Bound algorithm for inner model
[24]	Location selection of cross-docks in supply chain Different products Multiple cross-docking Multiple distribution centres	Integration of SA and Tabu search algorithms
[25]	Single-source Multi-product Multi-stage supply chain networks	Genetic algorithm
[26]	Multi-period Two-echelon Multi-commodity Capacitated supply chain network Forward and return network	Genetic algorithm
[29]	Single-source Multi-product Multi-stage	Lagrangian based heuristic
[30]	Multi-source Multi-product	A heuristic approach based on Lagrangian relaxation and SA
[31]	Multi-source Single-product Multi-stage	Memetic algorithm (combination of GA, greedy heuristic and local search methods)
[15]	Single-source Three-echelon DC-to-supplier dependent lead times	Two phased heuristic based on Lagrangian relaxation (construction and improvement phases) Using TS in the improvement phase
This paper	Multi-product Single stage Four-echelon Multi-period Budget constraint	Lagrangian relaxation Subgradient optimization

Table 4
Main characteristics of models with LR solution method.

Paper	Levels	Periods	Location	Budget constraint	Decision levels
[42]	P–D–C	Single	✓	–	S–T
[45]	P–C	Multiple	–	–	T
[43]	P–D–C	Single	✓	–	S–T
[44]	P–D–C	Multiple	–	–	T
[10]	P–D–C	Single	✓		S–T
[29]	S–P–W–C	Single	✓	-	S–T
This paper	S–P–W–C	Multiple	✓	✓	S-T

S, supplier; P, production unit; D, distribution centre; W, warehouse; C, customer; S, strategic; T, tactical.

capacitated facility location problem, which is well known to be NP-hard [21].

Many algorithms have been developed based on Lagrangian relaxation to solve facility location problems [32–41,15]. Performance of these algorithms is dependent on the structure and characteristics of the model. There are some studies that use a single period model in which an algorithm could easily be developed based on the Lagrangian relaxation method [10,29,42,43,15]. In multiple period models, applying Lagrangian relaxation becomes more complex, although some scholars have adopted this approach in their dynamic models [44,45]. When such models include budget constraints, applying Lagrangian relaxation presents many problems because of the interdependency of the periods. Table 4 shows the most important characteristics of some of the papers in which an algorithm based on Lagrangian relaxation was proposed. Commercial general-purpose optimisation software can solve small instances of problems; however, computational time with such software becomes prohibitive for reasonably sized instances. For this reason, we will adopt a method for solving the problem based on the Lagrangian relaxation technique. The reader is referred to Refs. [46,47,48]for a detailed discussion on the Lagrangian relaxation methodology.

We consider the Lagrangian relaxation of the problem, which was obtained by dualising the constraints in the sets (25), using multipliers γ^k for all $k \in \mathcal{K}$.

Problem L.

$$\text{Maximize } \mathcal{F} = \sum_{k \in k} \frac{Inc^k}{(1+Ir)^{k-1}} - \sum_{k \in k} \gamma^k \left(\sum_{i \in \mathcal{M} \cup \mathcal{W}p} Co_i . \left(x_i^k - x_i^{k-1} \right) + \sum_{i \in \mathcal{M} \cup \mathcal{W}p} \sum_{o \in \mathcal{O}} CA_{i,o} . \left(y_{i,o}^k - y_{i,o}^{k-1} \right) - DL^k - INV^k \right)$$

Subject to: (2)–(22), (26)–(37)

Problem L can be further decomposed into two subproblems, LR1 and LR2.

Problem LR1.

$$Z_{LR1} = \text{Max} \sum_{k \in k} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M}} \sum_{p \in p_f} \sum_{j \in \mathcal{W}} \frac{1}{(1+Ir)^{k-1}} \mathcal{PR}_p . f_{p,i,j}^{k,t} - \sum_{k \in k} \sum_{i \in \mathcal{M}} \frac{1}{(1+Ir)^{k-1}} \left(Co_i . \left(x_i^{k+1} - x_i^k \right) \right.$$

$$+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{k+1}-y_{i,o}^{k}\right)+CU_i.x_i^k+\sum_{o\epsilon\mathcal{O}} Cop_{i,o}.y_{i,o}^{k}\Bigg)$$

$$-\sum_{k\epsilon k}\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{i\in\mathcal{M}}\frac{1}{(1+Ir)^{k-1}}\left(CP_{p,i}.q_{p,i}^{k,t}+\sum_{j\in\mathcal{W}} CT_{p,i,j}.f_{p,i,j}^{k,t}\right)$$

$$-\sum_{k\epsilon k}\sum_{t\epsilon\mathcal{T}}\sum_{s\epsilon\mathcal{S}}\sum_{p\in p_r}\sum_{i\in\mathcal{M}}\frac{1}{(1+Ir)^{k-1}}\left(CD_{p,s,i}.f_{p,s,i}^{k,t}+\mathcal{P}S_{p,s}.f_{p,s,i}^{k,t}\right)$$

$$-\sum_{k\epsilon k}\sum_{i\in\mathcal{M}}\gamma^k\left(Co_i\cdot\left(x_i^k-x_i^{k-1}\right)+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{k}-y_{i,o}^{k-1}\right)\right)$$

$$-\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{i\in\mathcal{M}}\Bigg(Co_i\cdot\left(x_i^{l+1}-x_i^{l}\right)$$

$$+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{l+1}-y_{i,o}^{l}\right)+CU_i.x_i^l+\sum_{o\epsilon\mathcal{O}} Cop_{i,o}.y_{i,o}^{l}\Bigg)$$

$$-\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{i\in\mathcal{M}}$$

$$\times\left(CP_{p,i}.q_{p,i}^{l,t}+\sum_{j\in\mathcal{W}} CT_{p,i,j}.f_{p,i,j}^{l,t}\right)$$

$$-\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{t\epsilon\mathcal{T}}\sum_{s\epsilon\mathcal{S}}\sum_{p\in p_r}\sum_{i\in\mathcal{M}}$$

$$\times\left(CD_{p,s,i}.f_{p,s,i}^{l,t}+\mathcal{P}S_{p,s}.f_{p,s,i}^{l,t}\right)+\sum_{k\epsilon k}\gamma^k.INV^k$$

S.t.: (5),(8), (11),(12), (34)–(36) BC: (10), (26)–(28), (30)–(33)

Problem LR1 is related to the supply and production echelons in the supply chain network. Because the objective function of the main problem is maximising the total net profit of the supply chain and selling products is done at customer zones, here we assumed that, in Problem LR1, the products are purchased directly from the production units. It is obvious that adopting such assumptions causes no change in the nature of the main model because the main model attempts to maximise the production quantity as well.

Some constraints are specific to Problem LR1, and others, listed as BC, are those in which only the parameters and variables of the supply and production echelons are considered.

Problem LR2.

$$Z_{LR1}=\text{Max}\sum_{k\epsilon k}\sum_{t\epsilon\mathcal{T}}\sum_{j\epsilon\mathcal{W}}\sum_{p\in p_f}\sum_{c\epsilon\mathcal{C}}\frac{1}{(1+Ir)^{k-1}}\left(\mathcal{PR}_p.f_{p,j,c}^{k,t}\right)$$

$$-\sum_{k\epsilon k}\sum_{i\in\mathcal{W}p}\frac{1}{(1+Ir)^{k-1}}\Bigg(Co_i\cdot\left(x_i^{k+1}-x_i^{k}\right)$$

$$+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{k+1}-y_{i,o}^{k}\right)+CU_i.x_i^k+\sum_{o\epsilon\mathcal{O}} Cop_{i,o}.y_{i,o}^{k}\Bigg)$$

$$-\sum_{k\epsilon k}\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{j\in\mathcal{W}}\frac{1}{(1+Ir)^{k-1}}$$

$$\times\left(CS_{p,j}\cdot\left(h_{p,j}^{k,t}+\sum_{i\in\mathcal{M}}\frac{f_{p,i,j}^{k,t}}{2\mathrm{A}_{i,j}}\right)+\sum_{c\in\mathcal{C}} CF_{p,j,c}.f_{p,j,c}^{k,t}\right)$$

$$-\sum_{k\epsilon p}\sum_{i\in\mathcal{W}p}\gamma^k\left(Co_i\cdot\left(x_i^k-x_i^{k-1}\right)+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{k}-y_{i,o}^{k-1}\right)\right)$$

$$+\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{t\epsilon\mathcal{T}}\sum_{j\epsilon\mathcal{W}}\sum_{p\in p_f}\sum_{c\epsilon\mathcal{C}}\left(\mathcal{PR}_p.f_{p,j,c}^{l,t}\right)$$

$$-\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{i\in\mathcal{W}p}\Bigg(Co_i\cdot\left(x_i^{l+1}-x_i^{l}\right)$$

$$+\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{l+1}-y_{i,o}^{l}\right)+CU_i\cdot\left(x_i^l+\sum_{o\epsilon\,\mathcal{O}} Cop_{i,o}.y_{i,o}^{l}\right)$$

$$-\sum_{k\epsilon k}\gamma^k.(1-TR).(1-SH).\sum_{l=1}^{k-1}\sum_{t\epsilon\mathcal{T}}\sum_{p\in p_f}\sum_{j\in\mathcal{W}}$$

$$\times\left(CS_{p,j}\cdot\left(h_{p,j}^{l,t}+\sum_{i\in\mathcal{M}}\frac{f_{p,i,j}^{l,t}}{2\mathrm{A}_{i,j}}\right)+\sum_{c\in\mathcal{C}} CF_{p,j,c}.f_{p,j,c}^{l,t}\right)$$

S.t.: (2)–(4), (9), (29), (35), (37).

BC: (10), (26)–(28), (30)–(33).

Problem LR2 is related to warehouses and customer zones in which the parameters and variables of storage, distribution and selling can be seen. In this problem, similar to Problem LR1, some constraints are specific and others (BC) are those in which only the parameters and variables of the warehouse and customer echelons are considered.

Using Lagrangian relaxation, we can generate solutions as well as lower bounds for the optimal solution of the main model. The success of this approach depends heavily on the ability to generate good Lagrangian multipliers [10]. Generally, the computation of a good set of multipliers is difficult [47,49]. In this paper, the subgradient method is applied to drive the bounds for the LR. This method is an adaptation of the gradient method in which gradients are replaced by subgradients. The readers are referred to [50], which validated the use of subgradient optimisation schema. The proposed method in this paper takes an integrated approach at the tactical level and a hierarchical approach at the strategic level. The objective of adopting a hierarchical approach at the strategic level is to use the information of the net profit of the supply chain in each strategic period to make decisions about supply chain expansion in the subsequent time periods. In the proposed approach, initially, the Problem LR1 is solved; then, some required data are fed into Problem LR2 as parameters. These data include the location of the established production facilities and the production quantities of each product at each plant.

A feasible solution procedure to the main problem is not automatically available from the solution to the Lagrangian problem. Many scholars usually design a heuristic procedure to ensure the feasibility of the solution [40–42,10,39]. In the proposed approach, by adding some rational constraints to each subproblem, the feasibility of the solution is guaranteed.

To ensure the feasibility of the solutions, some constraints are added to the Problem LR1, and some constraints are changed. To maximise the net profit, Problem LR1 has a tendency to maximise the production quantity, but it should be noted that it is impossible to produce products while ignoring the capacity of warehouses. Constraints (38) and (39) are added to Problem LR1. Constraint (38) limits the production quantity to the capacity of the warehouses. To ensure the feasibility of the solution, a floating variable $SP^{k,t}$ is defined to calculate the vacant capacity of the warehouses. The objective of defining this variable will be discussed in the next section:

$$\sum_{i\in\mathcal{M}}\sum_{p\in p_f} q_{p,i}^{k,t}.V_p\le\sum_{j\in\mathcal{W}p} NU_j\cdot\left(MK_j.x_j^k+\sum_{o\in\mathcal{O}} CK_o y_{j,o}^k\right)+SP^{k,t}\quad\forall k\epsilon\mathcal{K},\ \forall t\epsilon\mathcal{T}\tag{38}$$

$$\sum_{i\in\mathcal{M}\cup\mathcal{W}p} Co_i\cdot\left(x_i^k-x_i^{k-1}\right)+\sum_{i\in\mathcal{M}\cup\mathcal{W}p}\sum_{o\epsilon\mathcal{O}} CA_{i,o}\cdot\left(y_{i,o}^{k}-y_{i,o}^{k-1}\right)\le DL^k+INV^k\quad\forall k\epsilon\mathcal{K}\tag{39}$$

Constraint (39) limits the establishment of the new facilities to the funds supplied from external sources and the cumulative net profit. There are some warehouse variables in these constraints because of the consideration of warehouse capacities during the decision making on the location of the production plants and the amounts of production quantities. Additionally, constraints (10), (3)–(26) and (3)–(30), whose variables and parameters had been limited to supply and production echelons, go back to the initial status.

Similar to Problem LR1, some modifications are performed in Problem LR2 to ensure the feasibility of the solutions. These modifications are accomplished by adding two constraints to Problem LR2:

$$\sum_{i \in \mathcal{M}} \sum_{j \epsilon \mathcal{W}} f_{p,i,j}^{k,t} = \sum_{i \in \mathcal{M}} \overline{q}_{p,i}^{k,t} \quad \forall p \epsilon p_f,\ \forall k \epsilon \mathcal{K},\ \forall t \epsilon \mathcal{T} \tag{40}$$

$$\sum_{i \in \mathcal{W}p} Co_i \cdot \left(x_i^k - x_i^{k-1}\right) + \sum_{i \in \mathcal{W}p} \sum_{o \epsilon \mathcal{O}} CA_{i,o} \cdot \left(y_{i,o}^k - y_{i,o}^{k-1}\right) \le DL^k + INV^k$$
$$\sum_{i \in \mathcal{M}} Co_i \cdot \left(\overline{x}_i^k - \overline{x}_i^{k-1}\right) + \sum_{i \in \mathcal{M}} \sum_{o \epsilon \mathcal{O}} CA_{i,o} \cdot \left(\overline{y}_{i,o}^k - \overline{y}_{i,o}^{k-1}\right) \tag{41}$$

Constraint (40) limits the distributed quantity of products to the quantity of production in plants. Moreover, constraint (41) limits the establishment of warehouses to the available budget in which the fixed cost of the production facilities that was calculated in Problem LR1 was subtracted. It should be noted that $\overline{q}_{p,i}^{k,t}$, $\overline{x}_i^k$ and $\overline{y}_{i,o}^k$ are the values of the related variables in Problem LR1.

- Solution algorithm

The solution algorithm described in this section was designed based on the subgradient method. The proposed algorithm includes nine steps and attempts to generate feasible solutions to the main problem by using both iterative and hierarchical approaches. In contrast, the proposed algorithm takes a hierarchical approach at the strategic level and an integrated approach at tactical level. The overall solution algorithm can be summarised as follows:

Step 1:

- Set the current iteration (*Iter*) to 1

Set the initial value of the Lagrangian multipliers γ^k to 0, for all $k \epsilon \mathcal{K}$.

Step 2: Set the floating parameter $SP^{k,t}$ to 0, for all $k \epsilon \mathcal{K}$, $t \epsilon \mathcal{T}$ and repeat steps 3–8 while the following conditions are both satisfied:

$$1 - \text{Iter} \le \text{maxiter}$$

$$2 - \text{gap} \ge 0.01$$

where

$$\text{gap} = \frac{F_{Iter+1} - F_{Iter}}{F_{Iter+1}}$$

Parameter F_{Iter} is the objective function of the main problem at iteration *Iter*. Additionally, *maxiter* is the maximum permitted iteration, as determined by the decision maker.

Step 3: Set k to 1 and repeat steps 4–6 while $k \le K$.

Step 4: Solve Problem LR1 only for index k.

Step 5: Derive the required data from the solution of Problem LR1 and feed them to Problem LR2.

Step 6: Solve Problem LR2 only for index k.

Step 7: Derive the final value of the variables from the solutions in steps 4 and 6, according to Table 5.

Table 5
The source of values for decision variables.

*LR*1		*LR*2	
x_i^k	$\forall i \in \mathcal{M}$	x_i^k	$\forall i \in \mathcal{W}$
$y_{i,o}^k$	$\forall i \in \mathcal{M}$	$y_{i,o}^k$	$\forall i \in \mathcal{W}p$
$z_{s,p}^{k,t}$		$f_{p,i,j}^{k,t}$	
$f_{p,s,i}^{k,t}$		$f_{p,j,c}^{k,t}$	
$q_{p,i}^{k,t}$		$h_{p,j}^{k,t}$	

Step 8: By the use of the values that were gathered in step 7, calculate the objective value of the main problem as well as the following functions; then, go back to step 2. In Eq. (43), Δ is the "agility" parameter in the subgradient optimisation.

$$\gamma_{\text{iter}+1}^k = \gamma_{\text{iter}}^k + \theta_{\text{iter}} \left[\sum_{i \in \mathcal{M} \cup \mathcal{W}p} Co_i \cdot \left(x_i^k - x_i^{k-1}\right) + \sum_{i \in \mathcal{M} \cup \mathcal{W}p} \sum_{o \epsilon \mathcal{O}} CA_{i,o} \cdot \left(y_{i,o}^k - y_{i,o}^{k-1}\right) - DL^k - INV^k \right] \quad \forall k \epsilon \mathcal{K} \tag{42}$$

$$\theta_{\text{iter}} = \Delta \frac{\overline{\text{F}} - \text{F}_{\text{iter}}}{\sum_{k \epsilon k} \left(\sum_{i \in \mathcal{M} \cup \mathcal{W}p} Co_i \cdot (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \mathcal{W}p} \sum_{o \epsilon \mathcal{O}} CA_{i,o} \cdot \left(y_{i,o}^k - y_{i,o}^{k-1}\right) - DL^k - INV^k \right)^2} \tag{43}$$

$$Iter = Iter + 1$$

Step 9: Calculate the value of parameter $SP^{k,t}$ by using the following equation and repeat steps 3–7.

$$SP^{k,t} = \sum_{j \in \mathcal{W}} \left(MK_j x_j^k + \sum_{0 \in \mathcal{O}} CK_o . y_{j,o}^k \right) - \sum_{i \in \mathcal{M}} \sum_{p \in p_f} V_p . q_{p,i}^{k,t} \tag{44}$$

As mentioned before, constraint (38) was added to Problem LR1 to ensure the feasibility of the solutions. This constraint limits the production quantities based on the maximum inventory. Although this constraint guarantees feasibility, it causes a low production quantity and, as a result, a low profit for the supply chain. According to this situation, after obtaining an appropriate arrangement of facilities during steps 1–8, the vacant capacity of warehouses is calculated using parameter $SP^{k,t}$; then, the solution is improved significantly during the internal loop.

5. Computational results

The computational analysis presented in this section is to evaluate the performance of the proposed solution approach. The proposed algorithm is coded in GAMS. The subproblems of the algorithm have been solved by the use of the CPLEX MIP solver. The algorithm was run on a Dual core 2.26 GHz processor with 2 GHz of RAM. Instances of Table 1 are solved with the same inputs of Section 3. As mentioned before, increasing the size of the problem, the total number of variables, the number of binary variables, and the number of constraints result in a significant increase in the CPU time.

The results illustrated in Table 6 confirm that the proposed algorithm can effectively reduce the solution time. We observe that, in small instances and because of the iterative nature of the proposed algorithm, the solution time of this algorithm is more than the solution time of commercial software. In larger instances, we observe that the proposed algorithm has notably better performance in terms of the solution time. Another observation is that the solution time of the commercial software increases exponentially, while the solution time of the proposed algorithm increases linearly. Because the commercial software is unable to find a feasible solution in the problems of class (L), good performance of the proposed algorithm in large-scale problems can be proved.

In comparison to the solution quality, a 4% average gap for all of the solved instances confirms that the proposed algorithm based on the subgradient method has a relatively good solution quality. In some instances, the proposed algorithm could find a better solution than commercial software.

It can be observed from the computational results that the proposed algorithm could make the most important strategic decisions during steps 1–8, but at the tactical level, the decision-making process is not accomplished successfully. This scenario results from solving the subproblems hierarchically as well as resulting from the existence of some rational constraints that were added to the subproblems to ensure the feasibility of the solution. We observed that, after obtaining the most appropriate Lagrangian multipliers, in step 9, the proposed algorithm could make improvements in the objective function significantly by calculating the vacant capacity of warehouses using Eq. (44) and solving the subproblems again. Table 6 illustrates a comparison of the solution time between CPLEX and the proposed procedure. A comparison of the solution quality based on the objective function value is illustrated in Table 7. The following relations are used to calculate the gaps:

$$Gap^{C} = \frac{\text{Best Possible Solution} - \text{CPLEX Solution}}{\text{Best Possible Solution}} \quad (45)$$

$$Gap^{LR} = \frac{\text{Best Possible Solution} - \text{LR Solution}}{\text{Best Possible Solution}} \quad (46)$$

In total, 50 instances were designed and solved. Fig. 6 shows some comparison statistics that concern the solution time and quality. This figure shows that there are seven instances (14%) in which both the solution time and the quality of the proposed procedure are worse than the solution of the commercial software. All of these instances are related to problems P1 and P2. There are three instances (6%) in which the proposed procedure could reach a better solution quality with more solution time. There are also 27 instances (54%) in which the proposed procedure could make improvements in the solution time but with a worse solution quality. In 13 instances (26%), the proposed algorithm could reach a better solution quality in less solution time.

A comparison of the solution time and quality for all 50 instances is illustrated in Fig. 7. In these charts, the horizontal and vertical axis values were calculated according to the following formulas:

$$\Delta S = \textit{solution time (CPLEX)} - \textit{solution time(Proposed procedure)} \quad (47)$$

$$\Delta F = F^{*}(\textit{Proposed procedure}) - F^{*}(\textit{CPLEX}) \quad (48)$$

In chart (a), it appears that, for all instances of size S1 and S2, the performance of the proposed procedure is dominated by

Table 6
Comparison of solution time (s).

Class	Problem	CPLEX			LR		
		Min	Ave.	Max	Min	Ave.	Max
S	P1	2.8	3.56	4.1	10.4	10.76	11.2
	P2	13.8	16.52	17.7	20.8	22.22	23.2
	P3	33.8	39.76	46.2	33.5	33.8	34.5
	P4	88.5	97.04	108.9	53.6	55.24	56.6
	P5	191.1	208.1	228.4	74.6	76.68	78.3
M	P6	320.8	408.64	469.1	107.7	114.52	124.9
	P7	675.6	732.52	788	137.7	143.38	144.5
	P8	1020.8	1220.48	1305.3	184	186.28	187.7
L	P9	7335	7991.6	8469	623	739.8	823
	P10	> 3 h	> 3 h	> 3 h	1399	1491	1543

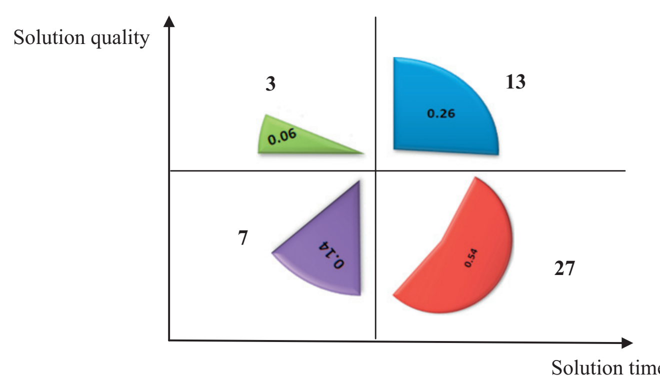

Fig. 6. Comparison statistics of the solution time and quality.

Table 7
Comparison of solution quality.

Class	Problem	CPLEX						Proposed procedure					
		Objective function			Gap^{c}			Objective function			Gap^{LR}		
		Worst	Ave.	Best	Min	Ave.	Max	Worst	Ave.	Best	Min	Ave.	Max
S	P1	3.41×10^7	3.95×10^7	4.60×10^7	0.000092	0.059562	0.089385	3.37×10^7	3.71×10^7	4.08×10^7	0.095574	0.112542	0.127665
	P2	2.25×10^7	2.52×10^7	2.96×10^7	0.027782	0.060698	0.090552	2.35×10^7	2.48×10^7	2.67×10^7	0.02574	0.070183	0.169463
	P3	2.08×10^7	2.33×10^7	2.96×10^7	0.021684	0.060773	0.092036	2.20×10^7	2.34×10^7	2.54×10^7	0.005705	0.047396	0.158477
	P4	2.13×10^7	2.39×10^7	2.73×10^7	0.025007	0.042182	0.05998	2.02×10^7	2.34×10^7	2.64×10^7	0.029094	0.060302	0.099018
	P5	1.67×10^7	1.81×10^7	2.19×10^7	0.013016	0.02249	0.040381	1.50×10^7	1.63×10^7	1.82×10^7	0.00353	0.113632	0.178844
M	P6	1.19×10^7	1.29×10^7	1.49×10^7	0.011497	0.018737	0.024562	9.89×10^6	1.11×10^7	1.23×10^7	0.054154	0.156994	0.185014
	P7	3.48×10^7	3.87×10^7	4.12×10^7	0.028516	0.035685	0.043214	3.61×10^7	3.92×10^7	4.17×10^7	0.006158	0.023633	0.073552
	P8	2.93×10^7	3.26×10^7	3.40×10^7	0.019478	0.022585	0.027465	2.80×10^7	3.10×10^7	3.42×10^7	0.011181	0.068283	0.184847
L	P9	1.68×107	1.88×10^7	2.05×10^7	0.029745	0.0474994	0.091072	1.49×10^7	1.70×10^7	1.89×10^7	0.07704	0.094945	0.122195
	P10	NS	NS	NS	NS	NS	NS	5.47×10^6	5.67×10^6	5.92×10^6	NA	NA	NA

NS, No solution found; NA, cannot be computed according to the NS.

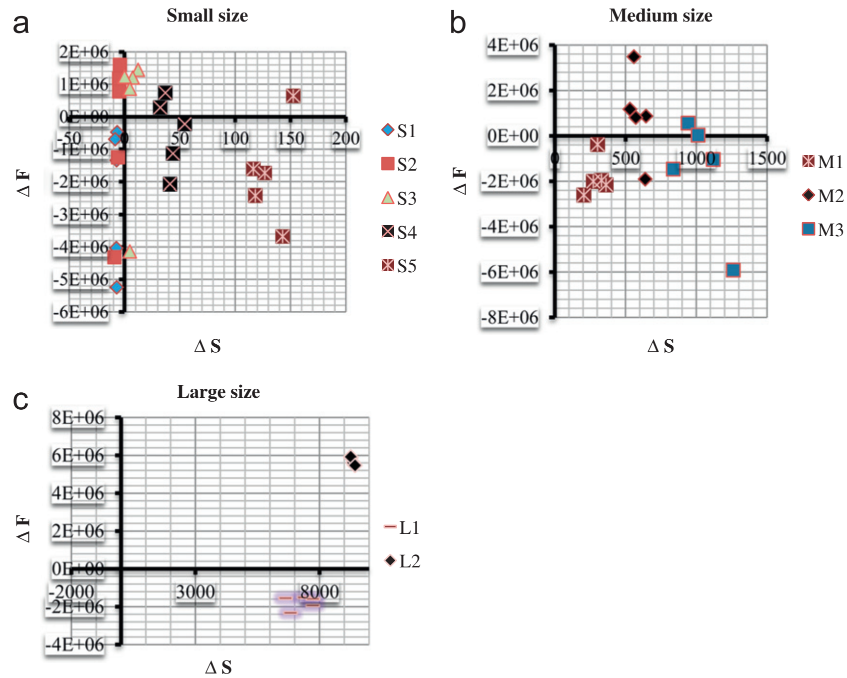

Fig. 7. Comparison of the solution time and the quality for all of the instances.

CPLEX, especially in terms of the solution time. In larger instances in the small class (S3, S4 and S5) and in all of the instances in the medium and large classes, the proposed procedure dominates CPLEX in terms of the solution time.

It is obvious that ΔS has a positive correlation with the size of the instances, implying that, when the size of the instances becomes larger, the value of ΔS increases. In these charts, the objects in small size instances are close to the vertical axis, and, by increasing the size of the instances, the distance of the objects from the vertical axis becomes larger.

6. Conclusions

In this paper, a mixed integer linear programming model was developed for designing and planning expansions of a four echelon multiple commodity supply chain with a long-term horizon. Different resolutions for strategic and tactical decisions were considered in the proposed model. In addition, this model makes some decisions about supplier selections, production facility locations, warehouse locations, the amount of raw materials to be supplied from each supplier, the amount of each product to be produced at each facility, the amount of each manufactured product to be sent to each customer zone and the expansion plans over a long-term horizon. In the proposed model, the expansion of a supply chain is planned according to the cumulative net profit and funds supplied from external sources.

To evaluate the performance, some test problems were designed and solved by the CPLEX solver. The results showed that the solution time of CPLEX in real-size problems is not reasonable; as a result, we proposed a solution method that is based on a Lagrangian relaxation approach. In the proposed method, the feasibility of the solutions was guaranteed by making some modifications to the subproblems. The results of the computational analysis confirmed the efficiency of the proposed approach.

The proposed model in this paper could be improved by adding discount policies, which have special importance in the current competitive world. We also suggest considering the lot size production situation in which the production variable cost is dependent on the production quantity. Another direction for improvement could be to consider various financial aspects, such as loan management, while planning expansions of the supply chain.

Appendix A. Data generator commands

$R^{k,t}_{s,p} = \text{ceil}(\text{uniform}(10{,}000,20{,}000))$
$MK_i = \text{ceil}(\text{uniform}(500,1000))$
$NK_i = 1.5 * MK_i$
$MU_i = 0.1$
$NU_i = 0.9$
$CK_o = \text{ceil}(\text{uniform}(200,500))$
$D^{k,t}_{c,p} = \text{ceil}(\text{uniform}(2000,4000))$
$B_{p',p} = \text{ceil}(\text{uniform}(1,3))$
$WL_{p,i} = \text{ceil}(\text{uniform}(2,5))$
$V_p = \text{ceil}(\text{uniform}(0,3))$
$MO_{s,p} = 100$
$A_{i,j} = \text{ceil}(\text{uniform}(10,15))$
$\mathcal{PR}_p = \text{ceil}(\text{uniform}(80,100))$
$\mathcal{PS}_{p,s} = \text{ceil}(\text{uniform}(5,10))$
$Co_i = \text{ceil}(\text{uniform}(0,90)) + \text{ceil}(\text{uniform}(100,110) * \text{sqrt}(MK_i))$
$Co_{prw} = \text{ceil}(\text{uniform}(0,50)) + \text{ceil}(\text{uniform}(40,60) * \text{sqrt}(MK_i))$
$CA_{i,o} = \text{ceil}(\text{uniform}(0,30)) + \text{ceil}(\text{uniform}(10,20) * \text{sqrt}(CK_o))$
$CA_{i,o} = \text{ceil}(\text{uniform}(0,10)) + \text{ceil}(\text{uniform}(5,15) * \text{sqrt}(CK_o))$
$CU_i = \text{ceil}(\text{uniform}(0,20)) + \text{ceil}(\text{uniform}(10,20)) * \text{sqrt}(MK_i))$
$CU_{prw} = \text{ceil}(\text{uniform}(0,10)) + \text{ceil}(\text{uniform}(5,10) * \text{sqrt}(MK_i))$
$Cop_{i,o} = \text{ceil}(\text{uniform}(0,5)) + \text{ceil}(\text{uniform}(5,8)) * \text{sqrt}(CK_o))$
$Cop_{prw,o} = \text{ceil}(\text{uniform}(0,5)) + \text{ceil}(\text{uniform}(3,5) * \text{sqrt}(CK_o))$
$CP_{p,i} = \text{ceil}(\text{uniform}(10,20))$
$CS_{p,j} = \text{ceil}(\text{uniform}(2,5))$
$CT_{p,i,j} = \text{ceil}(\text{uniform}(1,3))$
$CD_{p,s,i} = \text{ceil}(\text{uniform}(1,3))$
$CF_{p,j,c} = \text{ceil}(\text{uniform}(1,3))$

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