A Bender's Decomposition Algorithm for Multi Objective Stochastic Fuzzy Hub Location Problem

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Abstract- In this paper a multi-objective stochastic fuzzy hub location problem is considered. Characteristics such as emergency conditions and safety are considered in the model. These characteristics are scenario based and introduced as triangular fuzzy numbers. Then, the crisp equivalent mixed integer problem is defined and Bender's decomposition algorithm (BD) is used to solve the proposed model. To evaluate the performance of the proposed model, the results of BD are compared to those of Cplex solver. The results show that BD compared to Cplex better serve the different sizes and different fuzziness.

Key Words-Triangular Fuzzy Number, Multi-Objective Hub location Problem, Chance Constraint, Bender's Decomposition Algorithm.

1 INTRODUCTION

Location problems are divided into two general categories consisting of deterministic and uncertain problems. In previous researches, hub networks are designed by a decentralized management approach [1]. In a study related to air transport networks, airports have been considered as an M/D/C server queue system. Airports existing in the queue are defined as a chance constraint [2]. Also some research considers hub location problem with random demands or random cost [3], [4], [5], [6], [7]. Fuzzy location problems are subsets of uncertain location problems. Maximum covering location problems are modeled with fuzzy travel time [8]. Some author's studied, designed hub network taking into account dynamics set of virtual hubs [9]. In another study, P-hub center problems with minimizing travel time as the objective function with fuzzy time are defined [10]. Some others studied the p-hub center problems to minimize the maximum travel time [11]. Another aspect could be the hub location problems divided into two general categories namely, single and multi-objective problems. In research deal with multi-objective capacitated hub location problems, the second objective minimizes the service time of each node, that it is replaced with capacity constraints [12]. In this paper, we propose a model considering impact of stochastic criteria with fuzzy values such as weather conditions, traffic of the hub network. The proposed model consists of two objective functions. First it minimizes fixed cost and stochastic transportation cost. The second minimizes stochastic fuzzy characteristics of potential hub nodes and links.

1.1 Notation

1.1.1 Parameters

f_i: Set up cost for locating a hub at node, $i \in H$.

$c_{oi}^{k\,s}$: Stochastic transportation cost for commodity k of origin o to hub i based on scenario s.

$c_{ij}^{k\,s}$: Stochastic transportation cost for commodity k of hub link i to j based on scenario s.

$c_{jd}^{k\,s}$: Stochastic transportation cost for commodity k of hub i to destination d based on scenario s.

W_{od}^{ks}: Stochastic flow for commodity $k \in K$ of origin $o \in N$ to destination $d \in N$ based on scenario $s \in s_\pi \left(O_o = \sum_{d \in N} W_{od}^{k\,s}\right)$.

$F_{oijd}^{k\,s}$: Stochastic unit transportation cost for commodity k of origin o to destination d based on scenario s $\left(F_{oijd}^{k\,s} = \gamma c_{oi}^{ks} + \alpha c_{ij}^{ks} + \tau c_{jd}^{ks}\right)$.

p_s: Probability of occurrence of scenario s$\left(\sum_{s \in s_\pi} p_s = 1\right)$.

$\tilde{\pi}_i^s$: Fuzzy risk for hub node i based on scenario s.

$\tilde{\pi}_{oi}^s$: Fuzzy risk for commodity k of origin o to hub i based on scenario s.

$\tilde{\pi}_{ij}^{k\,s}$: Fuzzy risk for commodity k of hub i to hub j based on scenario s.

$\tilde{\pi}_{jd}^{k\,s}$: Fuzzy risk for commodity k of hub j to destination d based on scenario s.

α: Discount factor to hub link $(0 \leq \alpha \leq 1)$.

γ: Discount factor to origin-hub link $(\alpha \leq \gamma)$

τ: Discount factor to hub-destination link $(\alpha \leq \tau)$

ρ_1: Intensity factor of objective function$(1 \leq \rho_1)$.

ρ_2: Intensity factor of chance constraint$(1 \leq \rho_2)$.

β: Service level in chance constraint

δ: Threshold of path risk.

1.1.2 Decision variables

Z_i: Equals 1, if hub facility locates in node i.

$X_{oijd}^{k\,s}$: Fraction of the commodity k flow of origin o to destination d based on scenario s$\left(0 \leq X_{oijd}^{k\,s} \leq 1\right)$.

$X_{oi}^{k\,s}$: Equals 1, if the commodity k flows from origin o to hub i based on scenario s.

$X_{ij}^{k\,s}$: Equals 1, if the commodity k flows from hub i to hub j based on scenario s.

$X_{jd}^{k\,s}$: Equals 1, if the commodity k flows from hub i to destination d based on scenario s.

2 Multi Objective Stochastic Fuzzy Hub Location Problem

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Two assumptions namely, triangle inequality and at most having two hub facilities are considered to be satisfied in the model [13]. The proposed model is based on a two stage stochastic programming.

$$Min\, Z = \sum_{i \in H} f_i Z_i + \sum_{s \in s_\pi} p_s \left(\sum_{o \in N} \sum_{i \in H} \sum_{j \in H} \sum_{d \in N} \sum_{k \in K} W_{od}^{k\,s} F_{oijd}^{k\,s} X_{oijd}^{k\,s} \right). \quad (1)$$

$$Min\, \Gamma = \sum_{i \in H} \sum_{s \in s_\pi} p_s \tilde{\pi}_i^s Z_i + \sum_{s \in s_\pi} p_s \left(\sum_{o \in N} \sum_{i \in H} \sum_{j \in H} \sum_{d \in N} \sum_{k \in K} (\tilde{\pi}_{oi}^{k\,s} + \rho_1 \tilde{\pi}_{ij}^{k\,s} + \tilde{\pi}_{jd}^{k\,s}) X_{oijd}^{k\,s} \right). \quad (2)$$

Subject to:

$$\sum_{i \in H} \sum_{j \in H} X_{oijd}^{k\,s} = 1 \qquad o,d \in N, k \in K, s \in s_\pi \quad (3)$$

$$\sum_{j \in H} X_{oijd}^{k\,s} \le Z_i \qquad o,d \in N, i \in H, k \in K, s \in s_\pi \quad (4)$$

$$\sum_{i \in H} X_{oijd}^{k\,s} \le Z_j \qquad o,d \in N, j \in H, k \in K, s \in s_\pi \quad (5)$$

$$X_{oi}^{k\,s} \le Z_i \qquad o \in N, i \in H, k \in K, s \in s_\pi \quad (6)$$

$$X_{ij}^{k\,s} \le Z_l \qquad l \equiv i,j\;, i,j \in H, k \in K, s \in s_\pi \quad (7)$$

$$X_{jd}^{k\,s} \le Z_j \qquad d \in N, j \in H, k \in K, s \in s_\pi \quad (8)$$

$$X_{oijd}^{k\,s} \le X_{oi}^{k\,s} \qquad o,d \in N, i,j \in H, k \in K, s \in s_\pi \quad (9)$$

$$X_{oijd}^{k\,s} \le X_{ij}^{k\,s} \qquad o,d \in N, i,j \in H, k \in K, s \in s_\pi \quad (10)$$

$$X_{oijd}^{k\,s} \le X_{jd}^{k\,s} \qquad o,d \in N, i,j \in H, k \in K, s \in s_\pi \quad (11)$$

$$X_{oi}^{k\,s} \le \sum_{j \in H} \sum_{d \in N} X_{oijd}^{k\,s} \qquad o \in N, i \in H, k \in K, s \in s_\pi \quad (12)$$

$$X_{ij}^{k\,s} \le \sum_{o \in N} \sum_{d \in N} X_{oijd}^{k\,s} \qquad i,j \in H, k \in K, s \in s_\pi \quad (13)$$

$$X_{jd}^{k\,s} \le \sum_{o \in N} \sum_{i \in H} X_{oijd}^{k\,s} \qquad d \in N, j \in H, k \in K, s \in s_\pi \quad (14)$$

$$p\left((\tilde{\pi}_{oi}^{k\,s} + \rho_2 \tilde{\pi}_{ij}^{k\,s} + \tilde{\pi}_{jd}^{k\,s}) X_{oijd}^{k\,s} \ge \delta \right) \le \beta \qquad o,d \in N, i,j \in H, k \in K, s \in s_\pi \quad (15)$$

$$X_{oijd}^{k\,s}, X_{jd}^{k\,s}, X_{ij}^{k\,s}, X_{oi}^{k\,s} \ge 0 \qquad i,j \in H, O,d \in N, k \in K, s \in s_\pi \quad (16)$$

$$z \in \mathbb{B}^{|H|}. \quad (17)$$

Equation (1) relates to minimizing total costs as the first objective. Equation (2) is related to minimizing risk of hub nodes and links for the second objective. Constraint (3) shows that there is only one path from each origin o ∈ N to destination d ∈ N. Creating correct paths is guaranteed by constraints (4)-(15). The risk limit for origin o to destination d is seen Constraint (16). Other constraints define non-negativity and binaries of variables. We don't define $X_{oijd}^{k\,s}, X_{oi}^{k\,s}, X_{ij}^{k\,s}, X_{jd}^{k\,s}$ as binary variable, directly. Since in the optimal solution these variables take 0 or 1. For solving the problem, chance constraints should appear in the linear form. Thus, it assumes that the number of scenarios is large enough. Through difuzzification, we define the equivalent crisp model [14]. Finally, we define the equivalent single objective model, by using importance coefficients $\theta_1$andθ_2.

3 SOLUTION METHOD

In fact, our proposed model is a mixed integer programming problem. We use BD to solve it.

3.1 Bender's Decomposition Algorithm

First, the sub-problem defined in $X_{oijd}^{k\,s}, X_{oi}^{k\,s}, X_{ij}^{k\,s}, X_{jd}^{k\,s}$ space variables for any constant vector$\hat{z} \in \mathbb{B}^{|H|}$. Assume that $\bar{u}^i \in E(p_D),\ i = 1,..,16$ show extreme points of dual problem and $\bar{\bar{u}}^i \in R(p_D)\,, i = 1,..,16$ show extreme rays. We define optimality and feasibility cuts to each iteration by$\bar{u}^i\ and\ \bar{\bar{u}}^i$. Also the Relax master problem is capable of defining. Then BD algorithm is implemented.

3.2 Numerical experiment

The model is solved by BD and CPLEX solver. Various network sizes and deferent fuzziness are considered. We observed BD results are superior to CPLEX solver ones.

4 CONCLUSION

We introduced a multi-objective stochastic fuzzy hub location problem. First objective defined minimizing total costs and second objective minimizes risk of hub nodes and links. In follow, stochastic fuzzy criteria's such as emergence conditions, traffic considered were as risk measure. These criteria presented in triangle fuzzy numbers. BD is used to solve the model. Finally the results of numerical examples show that in this context BD works better than CPLEX solver.

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