A Three Echelons Supply Chain Network Design in a Fuzzy Environment Considering Inequality Constraints

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Abstract—**In this paper an optimization approach is applied for designing a multi-product supply chain network in a fuzzy environment. The considered supply chain contains three echelons of suppliers, manufacturers and retailers. In this problem a fuzzy optimization method has been used and fuzzy slack or surplus variables are used in optimization stage. In this paper the decision variables are fuzzy while in the other approach they are certain. Two hypothetical case studies illustrate the application of the proposed method. Finally a sensitivity analysis has been done for the fuzziness and its effect on the supply chain network design has been evaluated. The results show that increasing the fuzziness will increase the total cost and extra allocations will be added to the supply chain network structure.**

Keywords- **supply chain netwrk design, fuzzy optimaization**

I. INTRODUCTION

Supply chain management (SCM) has received a significant amount of attention in recent years. There are three planning levels in SCM: strategic, tactical and operational.

Strategic planning is relevant long term decision e.g. the determination number of facilities, their capacity and their location (Vidal and Goetschalckx [1]). The tactical decisions and operational planning are about inventory, transportation, layout, supplier selection, order allocation and etc. Goetschalckx et al. [2] surveyed the articles about strategic and tactical models. Bashiri et al. [3] analyzed tactical and strategic planning and they introduced a new approach to this. Salema et al. [4] presented a model for the simultaneous design and planning of reverse logistic supply chains.

The purpose of supply chain network design (SCND) is location of supply chain facilities and their allocation to demand nodes. This purpose is one of the strategic planning in a supply chain and it plays vital and critical role (Pishvaee and razmi [5]).

Melo et al. [6] presented a review paper and surveyed the papers about the facility location and supply chain management.

Today one of the basic problems in SCND is uncertainty, lack of the exact and certain information and dynamic and complexity of the supply chain facilities. To overcome this problem, fuzzy theory is suggested in modeling of the SCND.

There are some methods to solve the fuzzy optimization. For example Jimmenz et al. [7] proposed an interactive method to linear programming with fuzzy coefficient. Gu and Wu [8] solved the fuzzy optimization problems using a two-phase approach.

Peidro et al. [9] planned a supply chain under supply, demand and process uncertainties and optimized it using fuzzy mathematical programming. They developed a tactical supply chain model in a fuzzy environment in a multi-product, multi-period supply chain network.

Xu et al. [10] minimized total cost and maximized customer services using spanning tree-based genetic algorithms under random fuzzy environment.

Razmi et al. [11] proposed an approach to supplier evaluation and order allocation. In the problem objective function and constraints are fuzzy. Amin et al. [12] in the similar paper used fuzzy linear programming to supplier selection and determining purchased quantity of product from each supplier.

Jolai et al. [13] modeled a distribution planning problem and solved it using a fuzzy goal programming and genetic algorithm and Particle swarm optimization.

The above mentioned papers involve the tactical planning and the strategic decisions are ignored. Pishvaee and razmi [5] and Pishvaee et al. [14] presented a bi-objective model and minimized total cost and environmental impact by application of a fuzzy optimization. Paksoy and Pehlivan [15] modeled a multi-echelon supply chain with uncertain facility capacities. The research includes tactical decision i.e. transportation options selection and strategic planning simultaneously.

In the papers, the problems are single product and the models are solved using α- cut method and the different solutions are gathered by different values of α and finally the best one is selected. In this paper we optimize the SCND problem without using α cut method. Instead of that we use a fuzzy surplus or slack variable proposed by Kumar et al. [16] in optimization stage, so it makes the problem to be simpler.

In this study, the supply chain is multi-product and it is assumed that the input parameters are triangular fuzzy numbers and the problem is a supply chain network design with three echelons, then the problem is solved by a fuzzy optimization method. The proposed approach in supply chain leads to have inequality constraints.

The structure of this paper is as following: In the next section model formulation is presented. In section III, steps of the solution approach are described. Two hypothetical numerical examples with sensitivity analysis are provided in Section IV and they illustrate the application of the proposed method. Finally the concluding remarks are expressed in the last section.

II. MODEL FORMULATION

In this study it is assumed that the multi-product supply chain network has three echelons i.e. suppliers, manufacturers and retailers. The manufacturers buy some of the components from suppliers and assemble the components and manufacture the products, and then the products are transported to retailers.

The following notations are applied in the formulation of the model

A) Indices

C index of customers/retailers c=1,…,C

I index of plants i=1,…,I

K index of components/part k=1,…,K

P index of products p=1,…,P

V index of suppliers v=1,…,V

B) Parameters

SC_{vk} buying and shipping cost of component k from supplier v

F_v^v fixed cost of opening the supplier v

$\widetilde{M}_{pi}$ manufacturing cost of product p by plant i

$\widetilde{F}_{pi}^p$ setup cost of manufacturing of product p by plant i

$\widetilde{T}_{ic}$ transportation cost of products from plant i to retailer c

C_{vk}^s maximum capacity of component k in supplier v

$\widetilde{C}_{pi}^m$ maximum capacity of plant i for manufacturing of product p

$\widetilde{D}_{pc}$ demands for product p from retailer c

C) Variables:

y_{kvi} quantity of component k transported from supplier v to plant i

$\widetilde{x}_{pi}$ quantity of product p manufactured by plant i

$\widetilde{l}_{pic}$ quantity of product p transported from plant i to retailer c

n_{pi} 1, if product p is manufactured by plant i, 0 otherwise

u_v 1, if supplier v supply a component, 0 otherwise

The problem is formulated as flows:

$$\min\ TC = \sum_k \sum_v SC_{vk} \sum_i y_{kvi} \oplus \sum_v F_v^v \times u_v \oplus \sum_p \sum_i \widetilde{M}_{pi} \otimes \widetilde{x}_{pi} \oplus \sum_p \sum_i \widetilde{F}_{pi}^p \otimes n_{pi} \oplus \sum_i \sum_c \widetilde{T}_{ic} (\sum_p \widetilde{l}_{pic}) \quad (1)$$

The objective function (1) minimizes buying and shipping cost from suppliers to plants, fixed cost of opening, manufacturing cost, setup cost of manufacturing and transportation cost of products from plants to retailers.

Constraints:

$$y_{kvi} \le u_v \times C_{vk}^s \qquad \forall i,k,v \qquad (2)$$

$$\widetilde{x}_{pi} \le \sum_v y_{kvi} \qquad \forall k,p,i \qquad (3)$$

$$\widetilde{x}_{pi} \le n_{pi} \times \widetilde{C}_{pi}^m \qquad \forall i,p \qquad (4)$$

$$\sum_c \widetilde{l}_{pic} \le \widetilde{x}_{pi} \qquad \forall i,p \qquad (5)$$

$$\sum_i \widetilde{l}_{pic} \ge \widetilde{D}_{pc} \qquad \forall p,c \qquad (6)$$

$$n_{pi},\ u_v \in \{0,1\} \qquad \forall p,i,v \qquad (7)$$

$$y_{kvi}, \widetilde{x}_{pi}, \widetilde{l}_{pic} \ge 0 \qquad \forall k,v,i,p,c \qquad (8)$$

Equation (2) is capacity constraint on suppliers. If $u_{v=}0$ the supplier is not allocated but if $u_{v=}1$ the supplier has the components under its capacity. Constraint (3) shows that quantities of inputs components to plants should be larger than the used components in product. Constraint (4) is similar to constraint (2) but it limits the manufacturers to production base on their capacity. Constraint (5) is similar to constraint (3) and it states that the outputs products should be larger than the manufactured products, because some of them are damaged and wastes. Constrain (6) is as the retailers satisfaction constraint and it ensures that the demands satisfied. Constraints (7) and (8) ensure the binary and non-negativity restrictions on the decision variables.

The objective function and constraints are linear. The flowing solution approach can be used to optimize.

III. THE SOLUTION APPROACH

The proposed procedure adopted from Kumar et al. [16] which contains flowing steps:

Step 1 Planning and formulation of the model

Step 2 Transformation of all constraints to equality constraints as follows:

$$\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j \le \tilde{b}_i \rightarrow \sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j \oplus \tilde{s}_i = \tilde{b}_i \qquad (9)$$

$$\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j \ge \tilde{b}_i \rightarrow \sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j = \tilde{b}_i \oplus \tilde{s}_i \qquad (10)$$

where $\tilde{s}_i$ is the fuzzy slack or surplus and a non-negative fuzzy number.

Step 3 Exhibition of the fuzzy parameters and fuzzy decision variables by triangular fuzzy numbers in the objective function and in all the constraints such as (11)

$$\sum_{j=1}^{n}(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)\oplus(s_i,t_i,u_i)=(b_i,g_i,h_i) \qquad (11)$$

Step 4 Transformation of the model to crisp programing as follows:

Assume that $(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(m_{ij},n_{ij},o_{ij})$

$$\sum_{j=1}^{n} m_{ij} = b_i \qquad \forall i=1,2,\cdots,m \qquad (12)$$

$$\sum_{j=1}^{n} n_{ij} = g_i \qquad \forall i=1,2,\cdots,m \qquad (13)$$

$$\sum_{j=1}^{n} o_{ij} = h_i \qquad \forall i=1,2,\cdots,m \qquad (14)$$

$$y_j - x_j \ge 0 \quad z_j - y_j \ge 0 \quad \forall j=1,2,\cdots,n \qquad (15)$$

In addition the objective is changed as flows:

$$\sum_{j=1}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\rightarrow\sum_{j=1}^{n}\frac{p_jx_j+2q_jy_j+r_jz_j}{4} \qquad (16)$$

Step 5 Optimization of the crisp programming model to find the optimal value of x_j, y_j, z_j.

Step 6 Setting the obtained solutions in $\tilde{x}_j=(x_j,y_j,z_j)$ for all of the fuzzy decision variables. So the final solution will be as a fuzzy value.

IV. THE FIRST HYPOTHETICAL CASE STUDY

Suppose that there are two suppliers, two plants and two retailers in the supply chain network, the plants manufacture two kinds of products which both of them are assembled from three kinds of components. The following tables show some of the first hypothetical input data.

TABLE I. MANUFACTURING COST IN THE FIRST HYPOTHETIZED EXAMPLE

$\tilde{M}_{pi}$	MF1	MF2
P1	(25,30,35)	(20,25,30)
P2	(35,40,45)	(30,35,40)

TABLE II. TRANSPORTATION COST IN THE FIRST HYPOTHETIZED EXAMPLE

$\tilde{T}_{ic}$	RT1	RT2
MF1	(8,9,10)	(11,12,13)
MF2	(10,11,12)	(9,10,11)

TABLE III. MAXIMUM CAPACITY OF THE MANUFACTURER IN THE FIRST HYPOTHETIZED EXAMPLE

$\tilde{C}^{m}_{pi}$	MF1	MF2
P1	(2700,2800,2900)	(2900,3000,3200)
P2	(3900,4000,4100)	(3900,4000,4100)

TABLE IV. DEMANDS OF RETAILERS IN THE FIRST HYPOTHETIZED EXAMPLE

$\tilde{D}_{pc}$	RT1	RT2
P1	(1900,2000,2100)	(1300,1400,1500)
P2	(2400,2500,2600)	(1900,2000,2100)

According to the proposed procedure the fuzzy model was transformed to crisp models. The obtained results from Lingo8.0 are shown in table V.

TABLE V. THE OBTAINED RESULTS FOR DECISION VARIABLES IN THE FIRST HYPOTHETIZED EXAMPLE

$y_{111} = y_{211} = y_{311}$	600
$y_{112}= y_{212}=y_{312}$	4100
$\tilde{x}_{11}$	(300,400,400)
$\tilde{x}_{12}$	(2900,3000,3200)
$\tilde{x}_{21}$	(400,500,600)
$\tilde{x}_{22}$	(3900,4000,4100)
$\tilde{l}_{111}$	(300,400,400)
$\tilde{l}_{211}$	(400,500,600)
$\tilde{l}_{121}$	(1600,1600,1700)
$\tilde{l}_{221}$	(2000,2000,2000)
$\tilde{l}_{122}$	(1300,1400,1500)
$\tilde{l}_{222}$	(1900,2000,2100)
u_1	1
$m_{11}= m_{12}= m_{21}= m_{22}$	1

The quantities of other components transported from suppliers to plants are zero.

The optimal solution is that the first supplier transports the entire components to both of the plants and both of them produces two types of products, and then the first plant transports the manufactured products to the first retailer, and the second plant sends the goods to both of the retailers.

Finally a sensitivity analysis for the fuzziness and its effect on the supply chain network design has been studied. The fuzziness of the data is increased i.e. table I is transformed to table VI. Different solution is obtained, the first plant transports finished products to the second retailer. So it is clear that the total cost is more.

TABLE VI. THE FUZZY MANUFACTURING COSTS WITH HIGHER FUZZINESS IN THE FIRST HYPOTHETIZED EXAMPLE

$\widetilde{M}_{pi}$	MF1	MF2
P1	(20,30,40)	(15,25,35)
P2	(30,40,50)	(25,35,45)

The two problem results are summarized in figure I and table VII. It is clear that by increasing the fuzziness in the parameters an extra allocation has been added in the first structure of the supply chain in the first hypothetical example.

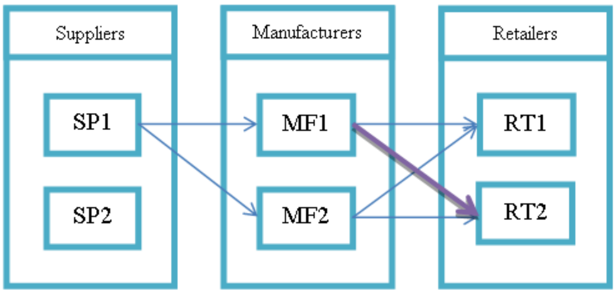

Fig. 1. The optimal supply chain networks for initial fuzzy parameters and with higher fuzziness in the hypothesized example (the dark line has been added to previous structure)

TABLE VII. OBTAINED RESULTS FOR THE FIRST INITIAL SUPPLY CHAIN STRUCTURE AND THE PARAMETERS WITH INCREASED FUZINESS

	Crisp	Fuzzy I	Fuzzy II
Objective function	418150	378017	407400
Total number of variables	30	146	182
Total number of constraints	36	100	136
Computational time (second)	0	1	1
Total number of facility	5	5	5
Total number of links	5	5	6

V. THE SECOND HYPOTHETICAL CASE STUDY

As the second example, the first hypothetical case study is extended, the number of suppliers and manufacturers are increased to five. Then the fuzziness of the data is increased, and the problem is optimized based on mentioned method. Table VIII shows the results. It is clear that the proposed approach can be applied for the larger problems. Moreover the results show that increasing fuzziness will lead to establishing of more facilities and more allocation links, so the total costs will be increased when the fuzziness is increased.

TABLE VIII. OBTAINED RESULTS FOR THE SECONED CASE STUDY UNDER INCREASED FUZZINESS

	Fuzzy I	Fuzzy II
Objective function	174505	430620
Total number of suppliers	1	3
Total number of manufactures	2	2
Total number of links	5	8

VI. CONCLUSION

One of the most important concepts in supply chain network design is the uncertainty, to solve this problem fuzzy theory and fuzzy optimization can be used. In this paper a simple method was introduced to optimize a multi -product supply chain network design in a fuzzy environment. The different of this method from other approach is the fuzziness of decision variables. The results of this study show that every supply chain structure can be analyzed in a fuzzy environment using the inequality approach. Moreover the results show that increasing the uncertainty in the fuzzy parameters can increase the total cost and extra structures and allocations should be added to the supply chain. As a future research designing of a closed loop supply chain network is suggested. Also the analysis of using other fuzzy optimization approaches can be another future study in this area.

REFERENCES

[1] C.J. Vidal and M. Goetschalckx, "Strategic production-distribution models: A critical review with emphasis on global supply chain models", European Journal of Operational Research, vol. 98, num. 1, pp. 1-18, 1997.

[2] M. Goetschalckx, Carlos J.Vidal and Koray Dogan, "Modeling and design of global logistics systems: A review of integrated strategic and tactical models and design algorithms", European Journal of Operational Research, vol. 143, pp. 1–18, 2002.

[3] M. Bashiri, H. Badri, J. Talebi, "A new approach to tactical and strategic planning in production–distribution networks', Applied Mathematical Modelling, vol. 36, pp. 1703–1717, 2012

[4] M. I. G. Salema, A. P. Barbosa-Povoa, A. Q. Novais, "Simultaneous design and planning of supply chains with reverse flows: A generic modelling framework", European Journal of Operational Research, vol. 203, pp. 336–349, 2010.

[5] M. S. Pishvaee and J. Razmi, "Environmental supply chain network design using multi-objective fuzzy mathematical programming", Applied Mathematical Modelling, in press, 2011.

[6] M. T. Melo, S. Nickel and F. Saldanha-da-Gama ,"Facility location and supply chain management – A

review", European Journal of Operational Research, vol. 196, pp. 401–412, 2009.

[7] M. Jimenez, M. Arenas, A. Bilbao and M. Victoria Rodrıguez, "Linear programming with fuzzy parameters: An interactive method resolution', European Journal of Operational Research, vol 177, pp. 1599–1609, 2007.

[8] S-M. Guu and Y-K. Wu, Two-phase approach for solving the fuzzy linear programming problems, Fuzzy Sets and Systems, vol. 107, pp. 191-195, 1999.

[9] D. Peidro, J. Mula, R. Poler and J-L. Verdegay, "Fuzzy optimization for supply chain planning under supply, demand and process uncertainties", Fuzzy Sets and Systems, vol. 160, pp. 2640–2657, 2009.

[10] J. Xu, Q. Liu and R. Wang, "A class of multi-objective supply chain networks optimal model under random fuzzy environment and its application to the industry of Chinese liquor", Information Sciences, vol. 178, pp. 2022–2043, 2008.

[11] J. Razmi, M. J. Songhori and M. H. Khakbaz, "An integrated fuzzy group decision making/fuzzy linear programming (FGDMLP) framework for supplier evaluation and order allocation", The International Journal of Advanced Manufacturing Technology, vol. 43, pp. 590-607, 2009.

[12] S. H. Amin, J. Razmi, G. Zhang, "Supplier selection and order allocation based on fuzzy SWOT analysis and fuzzy linear programming", Expert Systems with Applications, vol. 38, pp. 334–342, 2011.

[13] F. Jolai, J. Razmi and N. K. M. Rostami, "A fuzzy goal programming and meta heuristic algorithms for solving integrated production:distribution planning problem", Central European Journal of Operations Research, Vol. 19, Num. 4, 2011.

[14] M. S. Pishvaee, S.A. Torabi and J. Razmi, "Credibility-based fuzzy mathematical programming model for green logistics design under uncertainty", Computers & Industrial Engineering, vol. 62, pp. 624–632, 2012.

[15] T. Paksoy and N.Y. Pehlivan, "A fuzzy linear programming model for the optimization of multi-stage supply chain networks with triangular and trapezoidal membership functions", Journal of the Franklin Institute, vol. 349, pp. 93–109, 2012.

[16] A. Kumar, J. Kaur and P. Singh, "Fuzzy Optimal Solution of Fully Fuzzy Linear Programming Problems with Inequality Constraints", International Journal of Mathematical and Computer Sciences, 2010.