A clustering based simulated annealing approach for solving an un-capacitated single allocation *p*-hub location problem

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***Abstract*— Hub location problems play main role in large transportation systems and logistic environments. Its applications in real worlds cause that many researchers continue their attentions on this problem. In this paper clustering is applied as a heuristic method to improve the performance of simulated annealing algorithm for solving an un-capacitated single allocation p-hub location problem (USApHLP). For better illustration of the proposed clustering based Simulated Annealing (CbSA) approach, some numerical examples have been solved and the results have been reported in this paper. The comparisons show that CbSA algorithm outperforms the classic simulated annealing approach for the USApHLP.**

Keywords- Hub location problem; Neighborhood search ;Clustering; Simulalted annealing

I. INTRODUCTION

P-hub location problems are one of the most applicable problems which deal with finding best locations for establishing hub centers and economical allocation in a network of nodes. In a p-hub location allocation problem there are a set of nodes (N) which there are flows of communications between each pair of nodes. In this system, the main costs are associated with establishing hub centers among candidate nodes and costs related to allocating all other nodes to P-selected centers. By increasing nodes in a network, mentioned costs will be increased. Since connections costs between hubs will be decreased under a discount factor (α), so in this network an origin node cannot send its own flow to destination node directly and all pairs of nodes should connect to each other only through established hub centers. Discount factor (α) is usually known as transfer cost and its value is often $0 < a < 1$. An uneconomical allocation may lead to increase the total costs seriously without using the hubs. So allocations costs play the main role in total costs in a hub location problem. Clustering is one of the most effective techniques in grouping. The clustering method can be used to determine the allocations based on geographic dispersion of nodes and the hub points. In this paper we will illustrate the clustering based simulated annealing algorithm to improve the solution approach performance for un-capacitated single allocation *p*-hub location problem. This paper has been organized as following: The related literature is reviewed in the next section, and the proposed method is explained in Section 3. The numerical analysis is illustrated in section 4 to show the proposed approach performance. Finally the conclusion will be explained in the last section.

II.Literature Review

There are many researches in the literature which tries to solve the Hub location problems. Generally these methods are classified into exact, heuristic and meta-heuristic approaches. Despite of using exact solution approaches to small-sized hub location problems which can find the global optimum solution, for large sized problem finding the optimal solution in a reasonable computational time is almost intractable. However there are several papers which have used these methods for solving the hub location problem. Krishnamoorthy et al (2009) presented a 2-phase algorithm with branch and bound approach for solving a single assignment *p*-hub center problem [1]. Krishnamoorthy et al (2009) also used branch and bound algorithm to solve the single and multiple allocation un-capacitated *p*-hub center problems [2].

There are many investigations in the literature which have applied the heuristic and meta-heuristics to solve the hub location problems. For example Chin et al (2007) presented a hybrid heuristic algorithm for an un-capacitated single assignment hub location and allocation problem [3]. They also used a hybrid of Simulated Annealing (SA) and Tabu Search (TS) meta-heuristics for solving this problem. Also Silva et al (2009) used a multi stage heuristic method to finding a suitable initial solution point followed by a TS approach [4]. The abilities of meta-heuristics in finding the optimal or at least near to optimal solutions for hub location problem, makes these methods very suitable to solve the large scaled instances. Yilmaz et al (2005) implemented a genetic algorithm on an un-capacitated hub location problem [5]. They illustrated that their algorithm outperforms other existing algorithms and also their

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approach performance is verified in solving the large scaled problems in both computational time and solution quality aspects. Rodriguez et al (2007) solved a capacitated hub location problem with simulated annealing algorithm [6]. Cheng-Chang et al (2012) applied an extended genetic algorithm for the capacitated p-hub median problem [7]. They implemented the proposed method in Chinese air cargo network. By considering the related studies in p-hub location problems, it can be found that proposing new heuristic and meta- heuristics to solve this problem is necessary and valuable. Also table 1 report several approaches implemented in hub location problem during recent years. According to this table clustering based simulated annealing method for solving p-hub location problem has not been applied before in hub problem scope.

TABLE I. SOLUTION METHODS FOR HUB LOCATION PROBLEM BETWEEN 2005 AND 2012

Approach	Method	Hub location problem	Reference number
Meta heuristic	Genetic algorithm	✓	[7][5]
	Simulated annealing	✓	[6]
Heuristic	Hybrid heuristic method	✓	[3]
	Multi stage heuristic	✓	[4]
Exact	Branch and bound	✓	[1][2]
New Approach	CbSA	-	-

II. PROPOSED METHOD

In this paper a new Clustering based Simulated Annealing (CbSA) meta-heuristic is proposed to solve the p-hub location problem. The proposed method consists of two steps. A k-means clustering approach has been applied to find a suitable initial solution point in the first step. As the second step a new SA algorithm is developed to find the optimal or at least near to optimal solution. The CbSA algorithm generates more suitable neighborhood solutions in comparison to the traditional SA. Next sections describe the proposed method in details.

A. Initial solution (S)

In this paper since there are P hubs, we consider the $K=P$ in the K-means algorithm. In each a node with the minimum distance from cluster-center should be selected as a hub node in the corresponding group. Then all other nodes in the cluster should be allocated to selected hub. We name this initial solution as "*S*" and will be entered to the second step described in next section

B. The first neighbor solution (S1)

In this step the hub nodes extracted from the previous step will be fixed for the new solution (S1) however the allocations will be using a heuristic approach. This approach is based on a proximity measure which should be calculated as Eq. (1).

$$PM_{ik} = \frac{w_{ik}}{dis_{ik}} \tag{1}$$

where w_{ik} is the flow to be sent from node i to node k and dis_{ik} is the distance value between the nodes i and the center of cluster k. The pseudo code of the proposed heuristic for generating the new allocation structure can be described as follows:

1- Select one of non-hub nodes (i) randomly.
2- Calculate its proximity measure using Eq. (1) for all i-hub pairs. If there is more than one option as the maximum of proximity, select one of them randomly. This proximity measure between nodes i and cluster j is calculated from Eq. (1). In this equation, k is the hub node in cluster j.
3- Allocate the selected non-hub node (i) with the maximum proximity to the corresponding hub node.
4- If all the nodes are allocated finish the algorithm, else go to 2.

C. The second neighbor solution (s2)

Irrespective of previous hub node in each cluster, in order to make this new solution, a non-hub node should be selected randomly as a hub and other nodes in that cluster are allocated to the selected new hub.

At last two new solutions of S1 and S2 are compared to each other and the solution with the minimum of objective function is selected as the final new solution. However if the objective functions are the same; a solution is selected randomly. This selected solution will be replaced with S for the next CbSA iteration.

D. Solution representation

The representation of a solution can be illustrated according to Figure1. It shows that 1, 3 and 4 are hub nodes, node 2 and 5 have been allocated to hub 4 and 3 respectively.

1	4	3	4	3

Figure 1. Solution representation example in the proposed CbSA algorithm

III. NUMERICAL ILLUSTRATION

Consider there is a set of 10 nodes and their coordinates have been reported in table 1. This example is selected from the dataset named (D103) existed in the literature [8]. For better illustration of the proposed neighborhood solution generating a sample solution (S) and its new solutions (S1&S2) has been depicted in Figure 2.

TABLE II. D103 NODES POSITION IN THE TWO-DIMENSION SPACES.

Nodes	Coordination X	Coordination Y
1	20355.966023	16167.127237
2	39988.592020	19773.197847
3	23572.548971	31529.184022
4	37304.369590	32079.249435
5	24512.513179	38835.826089
6	40867.253869	38565.884488
7	27520.840478	46921.515986
8	36067.877874	44894.490748
9	19345.710923	51972.040940
10	33635.749233	49663.479404

TABLE III. THE WEIGHTS BETWEEN THE POINTS FOR D103 EXAMPLE

	1	2	3	4	5	6	7	8	9	10
1	75.4	36.9	54.5	19.2	20	17.6	56.7	17	18.8	16.3
2	25.7	38.3	25	26.5	18.2	23.1	38.1	20.4	12.4	16.6
3	66.8	39.3	51.1	22.4	24.2	20.5	70.5	20.2	23	19.4
4	17.3	33.1	18.7	24.9	16.3	22.9	40.1	23.1	11.4	18.6
5	17.4	25.1	21.1	19.0	23.4	18.3	73.4	20	23.8	19.4
6	11.6	18.4	12.1	16.1	11.1	16.9	37.1	21.8	9.5	18
7	82.3	77.5	90.4	67.7	95.0	72.6	312	98.8	110	110
8	35.1	41.7	36.3	48.4	35.7	57.5	173	89.9	41.2	91.8
9	12.2	11.9	15.3	10.2	18.9	11.1	67.8	15	23.6	20.3
10	21.7	24.9	22.8	27.7	23.0	32.8	109	50.8	30.2	63.3

S :	1	4	1	4	7	4	7	7	7	7	A
S1:	1	4	1	4	1	7	7	4	4	4	B
S2:	3	6	3	6	10	6	10	10	10	10	C

Figure 2. Initial solution (*S)* and new generated solutions (*S1*, *S2*) by proposed algorithm for the numerical example

According to the figure 2, nodes 1, 4 and 7 are hub nodes and node numbers 2 and 3 are allocated to hubs 4 and1, respectively. In this figure each hub and their allocated nodes are considered as a cluster. In next section we describe how S1 and S2 can be extracted from S
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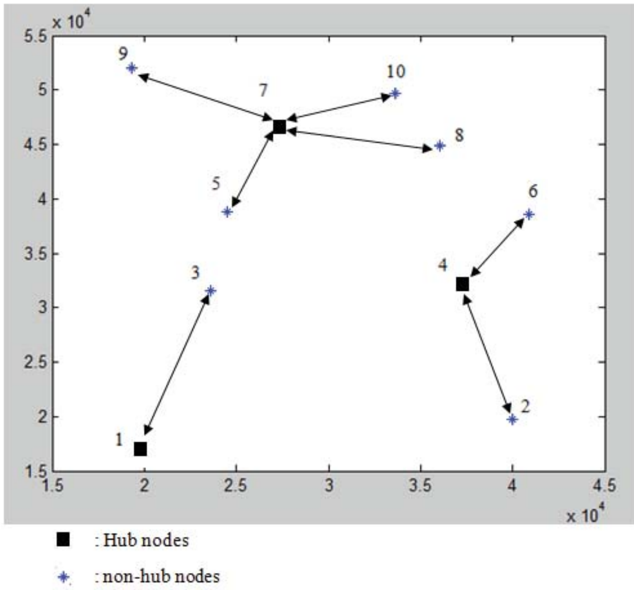

Figure 3. Schematic view of solution S

A. *Illustrating example- generating S1*

For generating S1 from S, we consider the selected hubs in S as S1 hubs and try to change the assignments; these processes can be described as follows:

1. Calculate the proximity measure between all non-hub nodes and hubs using Eq. (1). The results are shown in table 4

TABLE IV. CALCULATED PROXIMITY MEASURES FOR GENERATING THE NEIGHBORHOOD SOLUTION IN THE NUMERICAL EXAMPLE

	2	3	5	6	8	9	10
1	0.0013	0.0043	0.0008	0.0004	0.0011	0.0003	0.0006
4	0.0021	0.0016	0.0013	0.0022	0.0038	0.0004	0.0015
7	0.0013	0.0044	0.0085	0.0024	0.0197	0.0071	0.0163

2. Select a non-hub node randomly and allocate it to the hub which has the maximum proximity measure with this non-hub node. If the candidate hub is not unique select one of them randomly.

3. Update clusters by center point re-computation and calculate proximity measure by considering the new center point in each cluster. For example if in step 2 node number 6 has been selected randomly, it should be allocated to hub number 7 because of maximum

proximity measure with hub 7. Then in the next step all proximity measures with third cluster which have the node number 7 as the hub, should be calculated with considering the center point of nodes 6 and 7. So the allocations will be changed but without changing on hubs. Figure 2-B shows the S1 neighborhood solution generated from S.

B. Illustrating example - generating S2

For generating solution S2, we change the hub nodes in each cluster randomly. For example in the third cluster with nodes 5, 7, 8, 9, 10 node 10 is selected as a new hub which other nodes in this cluster should be assigned to this new hub. Figure 2-C shows S2 solution which is extracted from S by changing the hubs. The allocations in the new solution (S2) will be based on the initial clusters structure.

For the next step the best solution between S1 and S2 will be selected as a new solution to continue the CbSA algorithm. figure 4 shows the structure of generating two heuristic neighbor solutions and selecting one of them as final neighborhood.

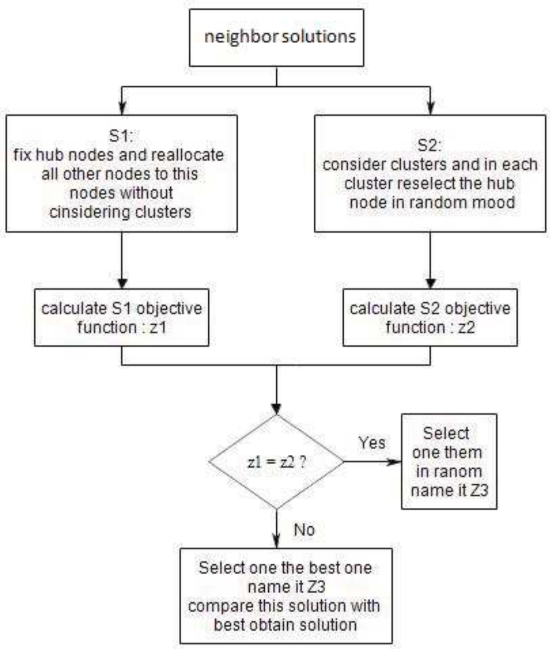

Figure 4. The structure of creating neighbor solutions in presented approach

The flowchart of the proposed CbSA is shown in figure 5. In this flowchart T and T0 are the initial and frozen temperatures, respectively. Nlimit indicates the number of accepted un-suitable solutions according to the acceptance probability. Moreover; n-over shows the numbers of searching neighborhoods in each temperature.

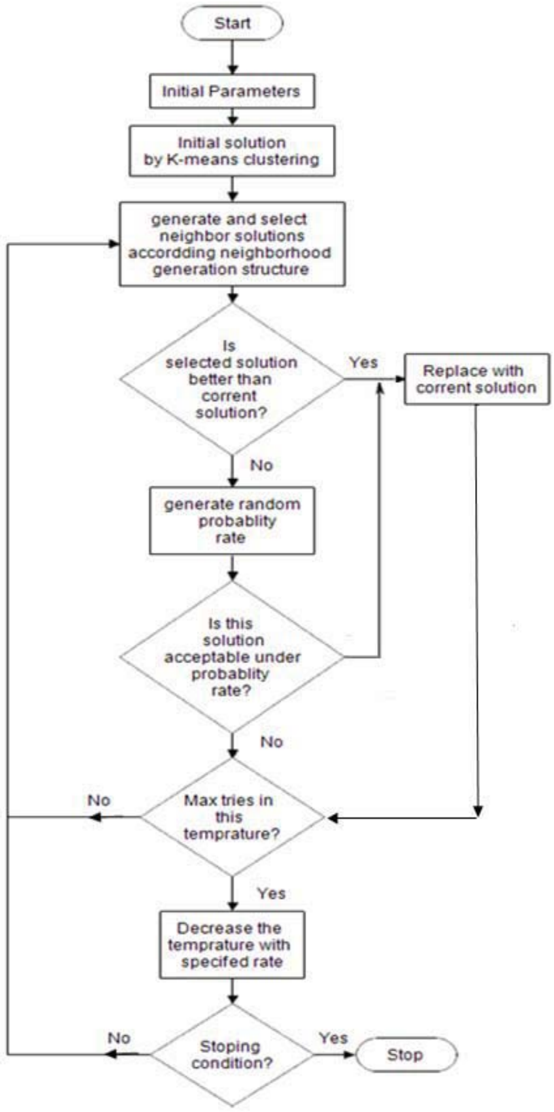

Figure 5. The flow chart of the proposed CbSA algorithm

IV. COMPUTATIONAL RESULT

In this section we present the result of the proposed solution method for *p*-hub location problem by considering of some instances from the literature without capacity information [8].

Table 5 Shows a summary of results of selected instances for usual SA algorithm and proposed CbSA approach which has been coded in MATLAB R2009. In this table the results indicate the average of 10 runs, and the best observed solution during 10 runs. The distances between nodes have been multiplied by 10^{-4} for simplification. The comparison shows the advantages of proposed CbSA. In order to evaluate the performance of CbSA method, we analyzed the relationship between the numbers of nodes, hubs and obtained results from SA and CbSA.

Figure 6 gives a visual comparison between results obtained from classic SA and CbSA. As it is clear in figure 6 which CbSA has obtained better results than classic SA and it means the efficiency of using clustering method in hub location problems.

We considered 5 categories for applied instances related to nodes. Except the first category which has 10 nodes and three hubs, in other categories there are two types of problems related to number hubs. Figure 7 presents the difference in obtained results between SA and CbSA in selected instances. Also illustrates the relationship between problem size and results which it could be said by increasing the size of problems the difference in results will be increased in both of 3_hubs and 5_hubs instances.

TABLE V. COMPARISON OF RESULTS FROM CLASSIC SA AND PROPOSED CBSA FOR SELECTED INSTANCES

Instance name	SA			Proposed CbSA		
	Average	Best solution	Time (s)	Average	Best solution	Time (s)
D103	13706.8	13618	24	13679	13618	43
D203	17891.1	17274	26	15830	15631	54
D205	16855.5	15797	37	13021.9	12867	63
D253	19272.2	17908	42	18215.9	16644	78
D255	17929.1	17738	47	14032.1	13157	82
D403	21318	20978	67	19335	18299	102
D405	20688.1	20139	68	15797	14761	108
D503	21538	21138	79	19277.4	16921	132
D505	21367	20001	81	15745	14925	142

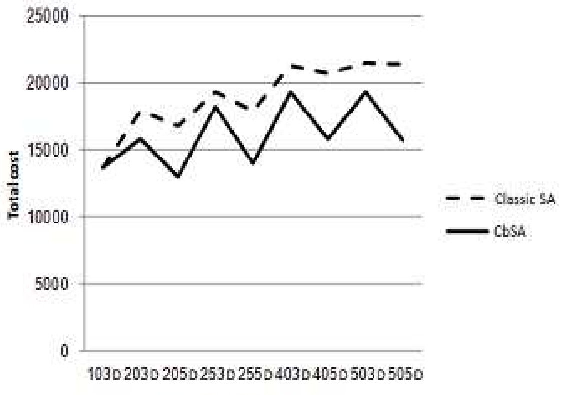

Figure 6. Comparison between obtain results of SA and CbSA

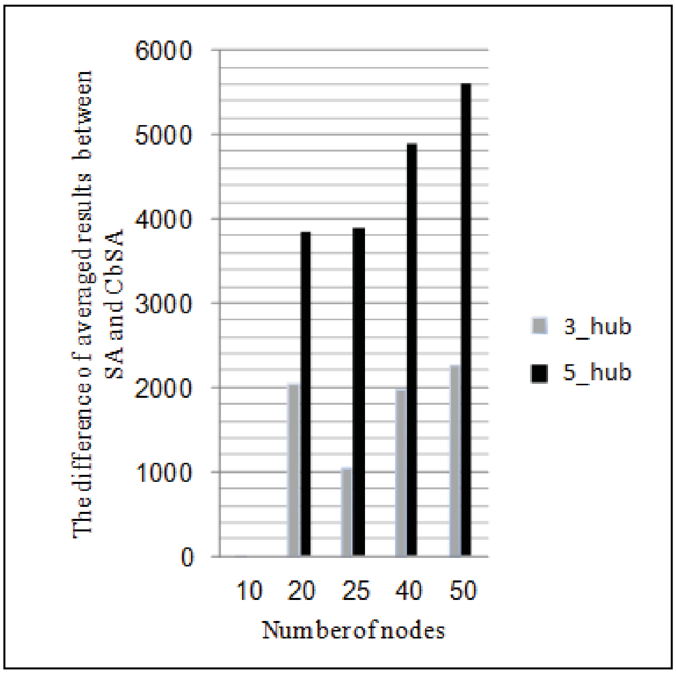

Figure 7. The relationship between problem size and the difference in results between SA and CbSA

V. CONCLUTIONS

In this paper a new approach named CbSA has been presented for solving an un-capacitated single allocation p-hub location problem which is based on the hybrid of a heuristic clustering method and simulated annealing algorithm. In order to evaluate the performance of proposed method some numerical examples taken from the literature have been analyzed and compared with the traditional SA algorithm. The efficiency of proposed solution method has been verified in both computational time and solution quality aspects as well.

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