An extended GRA method for MCDM with interval-valued trapezoidal fuzzy numbers and unknown weights

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Abstract -**In this paper, we investigate the multiple attribute decision making (MADM) problems. Criteria values are taken as linguistic variables because of the complexity, fuzziness and uncertainties of the values. Then they can be expressed in generalized interval-valued trapezoidal fuzzy numbers. The purpose of this paper is to develop an extended grey relational analysis (GRA) method for solving MCDM problems with generalized interval-valued trapezoidal fuzzy numbers and unknown information on criteria weights. Then information entropy method is used to determine attributes weights. Finally, a numerical example is illustrated to verify the developed method and to demonstrate its practicality and feasibility.**

Keywords - **Grey relational analysis (GRA), Generalized interval-valued trapezoidal fuzzy number, Multiple criteria decision making (MCDM), Weight information**

I. INTRODUCTION

The reason of using trapezoidal fuzzy number is which can express vagueness information caused by linguistic assessments through transforming them into numerical values. However, in some actual decision making process, it is difficult to determine the specific values for lower and upper bounds of trapezoidal fuzzy numbers. If the values' range is difficult to determine, then it is appropriate to define lower and upper bounds values as an interval. Thus, it can not only avoid the loss of information, but also express the decision making information more precisely. Therefore, linguistic variables were converted to interval-valued trapezoidal fuzzy numbers. The concept of interval-valued fuzzy set is initially proposed by [1-2]. Obviously, the generalized interval-valued trapezoidal fuzzy number is more general form of fuzzy numbers, almost all fuzzy numbers can be viewed as its special case, for example, trapezoidal fuzzy number, generalized trapezoidal fuzzy number, interval-valued triangular fuzzy, etc. So it has great significance to research on the decision making problems in which attribute values are the generalized interval-valued trapezoidal fuzzy numbers. Some researchers declare that there are few researches based on generalized interval-valued trapezoidal fuzzy number [3].

A. The basic concept of the interval-valued trapezoidal fuzzy numbers

A.1. The concept of generalized trapezoidal fuzzy numbers

Definition1[4]. A generalized trapezoidal fuzzy numbers can be defined as a vector $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ and the membership function a(x):$R \rightarrow [0.1]$ is defined as follow:

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-a_1}{a_1-a_2} W_{\tilde{A}}, & x \in [a_1, a_2) \\ W_{\tilde{A}}, & x \in [a_2, a_3] \\ \frac{x-a_4}{a3-a4} W_{\tilde{A}}, & x \in (a_3, a_4] \\ 0, & otherwise \end{cases}$$

Where $a_1 \le a_2 \le a_3 \le a_4$ and $w_{\tilde{A}} \in [0,1]$.

A.1.1 The center of gravity (COG) point of generalized trapezoidal fuzzy numbers.

[5]. proposed the concept of COG point of generalized trapezoidal fuzzy numbers, and suppose that the COG point of generalized trapezoidal fuzzy numbers $\tilde{a}= (a_1, a_2, a_3, a_4; w_{\tilde{a}})$ is $(x_{\tilde{a}}, y_{\tilde{a}})$,then:

$$\begin{cases} y_{\tilde{a}} = \begin{cases} \frac{w_{\tilde{a}} \times \left(\frac{a_3-a_2}{a_4-a_1}\right)}{6} & if\ a_1 \neq a_4 \\ \frac{w_{\tilde{a}}}{2} & if\ a_1 = a_4 \end{cases} \\ x_{\tilde{a}} = \frac{y_{\tilde{a}} \times (a_2+a_3)+(a_1+a_4)\times(w_{\tilde{a}}-y_{\tilde{a}})}{2\times w_{\tilde{a}}} \end{cases} \quad (8)$$

A.2.Generalized interval-valued trapezoidal fuzzy numbers

[6]. represented generalized interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; W_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; W_{\tilde{\tilde{A}}^U}\right)\right]$ as shown in Fig. 2, where $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1, 0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1, 0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \le 1$ and $\tilde{\tilde{A}}^L \subseteq \tilde{\tilde{A}}^U$.

B.2.3 The distance between two generalized interval-valued trapezoidal fuzzy numbers

Suppose that are two generalized interval-valued trapezoidal fuzzy numbers, then the distance of two generalized interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is calculated as follows[10]:

$$\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; W_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; W_{\tilde{\tilde{A}}^U}\right)\right]$$

$$\tilde{\tilde{B}} = \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; W_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; W_{\tilde{\tilde{B}}^U}\right)\right]$$

$$\begin{aligned} d(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = \frac{1}{8} \times \Big(& \left|W_{\tilde{\tilde{A}}^L} \times a_1^L - W_{\tilde{\tilde{B}}^L} \times b_1^L\right| + \left|W_{\tilde{\tilde{A}}^L} \times a_2^L - W_{\tilde{\tilde{B}}^L} \times b_2^L\right| + \left|W_{\tilde{\tilde{A}}^L} \times a_3^L - W_{\tilde{\tilde{B}}^L} \times b_3^L\right| + \left|W_{\tilde{\tilde{A}}^L} \times a_4^L - W_{\tilde{\tilde{B}}^L} \times b_4^L\right| + \\ & \left|W_{\tilde{\tilde{A}}^U} \times a_1^U - W_{\tilde{\tilde{B}}^U} \times b_1^U\right| + \left|W_{\tilde{\tilde{A}}^U} \times a_2^U - W_{\tilde{\tilde{B}}^U} \times b_2^U\right| + \left|W_{\tilde{\tilde{A}}^U} \times a_3^U - W_{\tilde{\tilde{B}}^U} \times b_3^U\right| + \left|W_{\tilde{\tilde{A}}^U} \times a_4^U - W_{\tilde{\tilde{B}}^U} \times b_4^U\right| \Big) \end{aligned} \quad (9)$$

A.2.4 The center of gravity (COG) point of generalized interval-valued trapezoidal fuzzy numbers.

(1)Utilize the equation (8) to calculate the coordinate of COG

trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U$respectively.

Suppose that there are two values of $(x_{\tilde{\tilde{A}}^L}, y_{\tilde{\tilde{A}}^L}), (x_{\tilde{\tilde{A}}^U}, y_{\tilde{\tilde{A}}^U})$ which belongs to the generalized

(2)The center of gravity of generalized interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ $are(X_{\tilde{\tilde{A}}}, y_{\tilde{\tilde{A}}})$ [7]., then:

$$\begin{cases} x_{\tilde{\tilde{A}}} = \dfrac{(x_{\tilde{\tilde{A}}^L} + x_{\tilde{\tilde{A}}^U})}{2} \\ y_{\tilde{\tilde{A}}} = \dfrac{(y_{\tilde{\tilde{A}}^L} + y_{\tilde{\tilde{A}}^U})}{2} \end{cases} \quad (10)$$

Then $E_{\tilde{\tilde{A}}} = \sqrt{x_{\tilde{\tilde{A}}}^2 + y_{\tilde{\tilde{A}}}^2}$ (11)

B. *Determine the attribute weight*

There are many methods to determine the attribute weight, such as [8-9], information entropy method [10-13] we will adopt information entropy method to determine the attribute weight.

Because the traditional entropy method is generally suitable for attribute value taking the form of crisp number and yet it fail in dealing with the generalized interval-valued trapezoidal fuzzy numbers. In order to use the entropy method, we firstly convert the generalized interval-valued trapezoidal fuzzy numbers. In order to use the entropy method, we firstly convert them into a crisp number by equation (11).

C. An extended GRA method for MCDM with Generalized interval-valued trapezoidal fuzzy number assessments

Step 1. Normalizing As follows:

$$\tilde{\tilde{X}}_{ij} = \left[\left(X_{ij1}^L, X_{ij2}^L, X_{ij3}^L, X_{ij4}^L; W_{ij}^L\right), \left(X_{ij1}^U, X_{ij2}^U, X_{ij3}^U, X_{ij4}^U; W_{ij}^U\right)\right] = \left[\left(\left(\frac{a_{ij1}^L}{m_j}, \frac{a_{ij2}^L}{m_j}, \frac{a_{ij3}^L}{m_j}, \frac{a_{ij4}^L}{m_j}; W_{ij}^L\right), \left(\frac{a_{ij1}^U}{m_j}, \frac{a_{ij2}^U}{m_j}, \frac{a_{ij3}^U}{m_j}, \frac{a_{ij4}^U}{m_j}; W_{ij}^U\right)\right)\right]$$

for benefit attributes, where $m_j = max_j(a_{ij4}^U)$ (9)

$$\tilde{\tilde{X}}_{ij} = \left[\left(X_{ij1}^L, X_{ij2}^L, X_{ij3}^L, X_{ij4}^L; W_{ij}^L\right), \left(X_{ij1}^U, X_{ij2}^U, X_{ij3}^U, X_{ij4}^U; W_{ij}^U\right)\right] = \left[\left(\left(\frac{n_j}{a_{ij1}^L}, \frac{n_j}{a_{ij2}^L}, \frac{n_j}{a_{ij3}^L}, \frac{n_j}{a_{ij4}^L}; W_{ij}^L\right), \left(\frac{n_j}{a_{ij1}^U}, \frac{n_j}{a_{ij2}^U}, \frac{n_j}{a_{ij3}^U}, \frac{n_j}{a_{ij4}^U}; W_{ij}^U\right)\right)\right]$$

for cost attributes, where $n_j = min_j(a_{ij1}^L)$ (10)

Step 2. Determine the reference series. The reference series can be defined as:

$X_0 = (X_{01}, X_{02}, \ldots., X_{03})$
$=[(1,1,1,1;1), (1,1,1,1;1)]$

Step 3. Calculate the distance between the reference value and each comparison value. The distance between the reference value and each comparison value can be calculated using Equation (9)

Step4. Calculate the grey relational coefficient as follows:

$$\varepsilon_{ij} = \frac{\delta_{min}+\zeta\delta_{max}}{\delta_{ij}+\zeta\delta_{max}}, \ i = 1.2,.,m, j = 1,2,..,n. \quad (12)$$

here, suppose that $\zeta = 0.5$

Step 5. Estimate the grey relational grade as follows:

$$\gamma_i = \sum_{j=1}^{n} W_j \varepsilon_{ij} \quad (13)$$

Table 2

The evaluation information of four alternatives given by the DM

	c_1	c_2	c_3	c_4	c_5
a_1	VG	VG	VG	VG	VG
a_2	G	VG	VG	VG	MG
a_3	VG	MG	G	G	G
a_4	G	F	F	G	MG

III*: Numerical example*

We use the data from Ashtiani et al[14] to illustrate the proposed decision-making method with unknown weight values in spite of the mentioned study.

according to equation (8),(10),(11),(12),(13) and (15) we get the following weights:

$W_1 = 0.19, W_2 = 0.21, W_3 = 0.21, W_4 = 0.19,$ $W_5 = 0.20$; Using calculated weights the grey relational grades are computed using equation (13):

$\gamma_1 = 1, \gamma_2 = 0.8709, \gamma_3$=0.7625,$\gamma_4 = 0.5385$;

Therefore, the most suitable alternative is a_1.

IV. Conclusion

As can be seen in Table 2, alternative a_1 in every measure of performance is best compared to other alternative, therefore alternative a_1 would be the best alternative, also results obtained from our proposed method is alternative a_1 That verify the developed approach.

Further more, we can extend the developed models and procedures to deal with the MCDM with interval-valued trapezoidal intuitionistic fuzzy numbers.

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