Rich capacitated vehicle routing problem with multiple commodity

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Abstract: Capacitated vehicle routing problem (CVRP) is an especial case of popular VRP. Considering other aspects in this problem can lead to have a rich VRP. In this paper an extension of such problem is proposed as a Mixed-integer linear programming (MILP) model when we have multiple different commodities. Comparing to the VRP studies that concentrate on the idealized models, the rich VRP involves related constraints encountered in the real life and suggest practical solution. Increasing the importance of reverse logistics activities motivates us to study on VRP with simultaneous pick-up and delivery activities. The objective of this problem is to determine the best set of routes that fulfill delivery and pick-up demands of customers, with considering of time windows restrictions and multi commodity constraints. The performance of the proposed model was analyzed on a variety of instances.

Keywords: Rich vehicle routing problem; Time windows; Delivery and pick-up; Multiple commodity

1. INTRODUCTION

The capacitated vehicle routing problem (CVRP) originally presented in [1] it has taken in a lot of attention in the OR articles. According to many applications of various kinds of VRP authors have worked on extend VRP model that is called as rich VRP with solution approaches for them. (For more researches see [2])

VRP with time windows (VRPTW) includes the routing of a set of vehicles with predetermined limited capacity and predefined time windows. We have a single depot and a set of identified dispersed customers with known demands. Usually customers require to meeting their needs within a certain period of time, if the distribution of products can't be served to the demand point in a specific given time, the customers refuse to accept product, such problems are called hard time windows.

VRP with delivery and pick-up (VRPDP) is an important extension of VRP model that has lots of applications in the real life. For example in grocery distribution systems the final goods should be distributed while the empty bottles or cans or expired goods should be collected in the same time [3].

The structure of this paper is as follows. In section 2 the literature review for extensions of VRP is presented. In Section 3 the proposed mathematical model is presented. In the last section, we illustrate the results of numerical test problems and finally conclusion is drawn.

2. LITERATURE REVIEW

We have these three kinds of VRPDP in the literature:

1. VRP with backhauls (VRPB):
   In this condition all deliveries must be completed before starting pick-up activities.
2. VRP with mixed delivery and pick-up (VRPMDP):
   In this condition only delivery or pickup can be done at each node but not both of them.
3. VRP with simultaneous delivery and pick-up (VRPSDP):
   Delivery and pick-up of the goods at each node can be executed. [3]

In this paper we preferred to discuss about the last one because of the importance and generality of VRPSDP, although [4] use the VRPB approach with the exact techniques and they used an integer programming to solve VRPB. The study of [5] introduces the first problem of VRPSD with 22 customers and 2 vehicles. In mentioned study customers were clustered into groups and each group was solves as a separate travelling sales man problem (TSP).

3. MATHEMATICAL MODEL AND PROBLEM DEFINITION

3.1. Model Assumptions

We have the following assumptions:

- Each route starts and finishes at the central depot
- Every customer is visited only once by one route

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- Deliveries and pick-ups must be done simultaneously and pick-up goods must back to the central depot
- Demand or pick-up of each customer must not exceed capacity of vehicle
- All customer delivery and pick-up for their demands of all commodities are completely served
- Customers have time window requirements for the goods demand

3.2. Symbol description

i, j, h: For customers from *2, …, n* (1 shows the depot)

k is set of vehicles from *k=1, …,m*

r is a set of product *r=1,..., p*

M is an enough large number

o is number of nodes

C_{ij} , Distance between node *i* and *j*

Q_{ij} , Unite cost of distance between node *i* and *j*

cap_{rk} , Capacity of vehicle *k* for product *r*

d_j , Demand of customer *j*

p_j , Pick up amount of customer *j*

f_k , Fixed cost of each vehicle

t_{ij} , Travelling time between node *i* and *j*

a_j, s_j, z_j Early start, service time and latest start time of customer *j*, respectively

y_{ij} , Is calculated by this equation:

$y_{ij} = z_i + t_{ij} - a_j$

x_{ijk} : Binary variables that indicate route between node *i* and *j* by vehicle *k*

b_i : Customer *i* is visited at this moment

il_k : Initial load of vehicle *k* when tour begins

l_j : Load of vehicle after visiting customer *j*

st_j : Variable for sub-tour elimination

3.3. Mathematical model

The objective function of our model is given bellow:

$$\min \sum_{k=1}^{m}\sum_{j=2}^{n} f_k x_{1jk} + \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} Q_{ij} x_{ijk} \quad (1)$$

The constraints have been drawn here:

$$\sum_{i=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1; \quad j = 2,...,n \quad (2)$$

$$\sum_{j=2}^{n} x_{1jk} \le 1; \quad k = 1,...,m \quad (3)$$

$$\sum_{i=1}^{n} x_{ihk} = \sum_{j=1}^{n} x_{hjk}; \quad h = 2,...,n\ ;\ k = 1,...,m \quad (4)$$

$$il_{rk} = \sum_{i=1}^{n}\sum_{j=2}^{n} d_{rj} x_{ijk}; \quad r = 1,..., p;\ k = 1,...,m \quad (5)$$

$$l_{rj} - il_{rk} + d_{rj} - p_{rj} + M(1 - x_{1jk}) \ge 0\ ;\ r = 1,..., p;\ j = 2,...,n\ ;\ k = 1,...,m \quad (6)$$

$$l_{rj} - l_{ri} + d_{rj} - p_{rj} + M(1 - \sum_{k=1}^{m} x_{ijk}) \ge 0; \quad r = 1,..., p;\ i = 2,...,n\ ;\ j = 2,...,n \quad (7)$$

$$st_j - st_i - 1 + o(1 - \sum_{k=1}^{m} x_{ijk}) \ge 0;\ i = 2,...,n\ ;\ j = 2,...,n \quad (8)$$

$$b_i + s_i + t_{ij} - y_{ij}(1 - x_{ijk}) \le b_j\ ;\ i = 2,...,n;\ j = 2,...,n;\ k = 1,...,m \quad (9)$$

$$b_j - a_j - M(1 - x_{1jk}) \le 0\ ;\ j = 2,...,n;\ k = 1,...,m \quad (10)$$

$$il_{rk} \le cap_{rk}\ ;\ r = 1,..., p;\ k = 1,...,m \quad (11)$$

$$l_{rj} \le cap_{rk}\ ;\ r = 1,..., p;\ j = 2,...,n;\ k = 1,...,m \quad (12)$$

$$x_{ijk} \in \{0,1\}\ ;\ i = 1,...,n\ ;\ j = 1,...,n\ ;\ k = 1,...,m \quad (13)$$

$$b_i \ge 0;\ st_i \ge 0;\ a_i \le b_i \le z_i\ ;\ i = 2,...,n \quad (14)$$

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

For better illustration of the proposed model, some numerical instances were solved by GAMS software. The analysis of results shows that the proposed model has similar results to single commodity rich VRP in two cases. First when there is only one vehicle with sufficient capacity and second, when we must use *n-1* vehicles that is one vehicle for each customer. For example in test problems of 3 and 4 the objective function value of our proposed model is equal to the single commodity model Table1 illustrates the results of some numerical instances and confirm the result of sensitivity analysis. Moreover we concluded that existing of an un-capacitated vehicle will not guaranty to have similar results of both models because of time windows constraints.

For future researches adding the simultaneous split and delivery for the rich VRP is recommended. Moreover proposing a solution method for large scaled instances of rich VRP according to the proposed model can be another future study.

TABLE 1
THE RESULTS OF PROPOSED MODEL FOR SOME NUMERICAL INSTANCES

Test Problem	Number of used vehicle	Number of nodes	Number of commodities	Objective value of the proposed model	Objective value of the classic model	Capacity of vehicle in classic model	Capacity of vehicle in proposed model
1	2	4	1	130	130	10	10
2	1	4	2	130	180	10	r1=r2=10
3	1	6	3	120	120	Large number*	r1=r2=r3=10
4	5	6	3	310	310	MVODOP **	r1=r2=r3=10
5	4	6	3	180	250	MVODOP **	Varied
6	2	8	2	180	150	Large number*	Vareid
7	4	8	3	250	310	MVODOP **	r1=r2=r3=10

*At least sum of delivery and pick up of all nodes

**Maximum value of demand or pick up of nodes

5. REFERENCES

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