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Mohammad Hadi Doroudyan & Amirhossein Amiri

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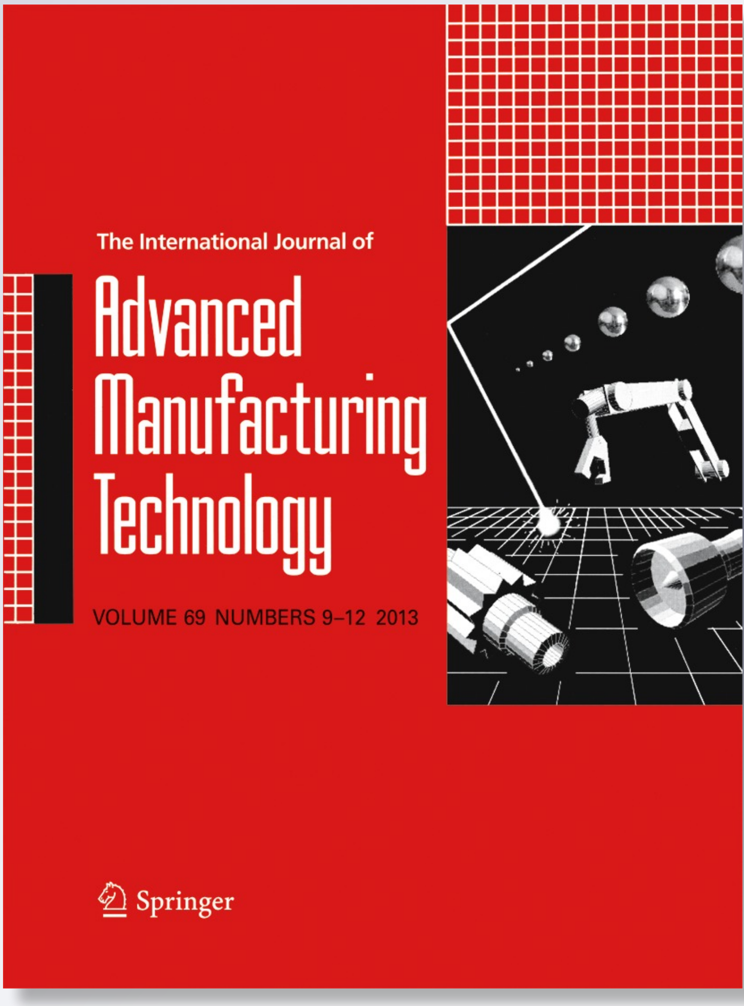

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ORIGINAL ARTICLE

Monitoring multivariate–attribute processes based on transformation techniques

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Abstract Control charts are widely used in monitoring the quality of a product or a process. In most of the cases, quality of a product or a process can be characterized by two or more correlated quality characteristics. Many control charts have been proposed for monitoring multivariate or multi-attribute quality characteristics, separately, but sometimes the correlated variables and attribute quality characteristics represents the quality of a process. In this paper, the use of four transformation methods is proposed to monitor the multivariate–attribute processes. In the first one, the distribution of correlated variables and attribute quality characteristics are transformed to approximate multivariate normal distribution, and then the transformed data are monitored by multivariate control charts including T^2 and MEWMA. Based on the second transformation method, the correlated variables and attribute quality characteristics are transformed, such that the correlation between the quality characteristics approaches to zero, then univariate control charts are used in monitoring the transformed data. In the third and fourth proposed methods, a combination of two transformation methods is used to make the quality characteristics independent and to transform them to normal distribution. The difference between the third and fourth method is the order of using the transformation techniques. The performance of the proposed methods is evaluated by using simulation studies in terms of average run length criterion. Finally, the proposed approach is applied to a real dataset.

M. H. Doroudyan · A. Amiri
Department of Industrial Engineering, Shahed University, Tehran, Iran

M. H. Doroudyan
e-mail: doroudyan@shahed.ac.ir

A. Amiri (✉)
Industrial Engineering Department, Shahed University, Tehran, Iran
e-mail: amirhossein.amiri@gmail.com

A. Amiri
e-mail: amiri@shahed.ac.ir

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1 Introduction

In some statistical process control applications, the quality of a product or a process is characterized by one or more quality characteristics and monitored by control charts under different types of quality characteristics and assumptions. In the case of having more than one quality characteristics, multivariate control charts have been developed to monitor processes considering correlated structure of quality characteristics. Hotelling [1] showed that monitoring correlated quality characteristics separately increases the errors. For details about the errors in control charts, refer to Montgomery [2]. The T^2 control chart [3], multivariate exponentially weighted moving average (MEWMA) proposed by Lowry et al. [4] and multivariate cumulative sum (MCUSUM) by Healy [5] are the most interesting multivariate control charts. Beside control charts in monitoring multivariate processes, other approaches such as artificial neural networks have been investigated by researchers such as Guh [6], Yu and Xi [7], and Atashgar and Noorossana [8]. The last review on the most common methods in monitoring multivariate processes has been done by Bersimis et al. [9]. In some applications, multi-attribute characteristics well describe the quality of a process or a product. Some methods are proposed for monitoring this type of quality characteristics. For example, Niaki and Abbasi [10] proposed normal-to-anything (NORTA) inverse transformation in monitoring multi-attribute quality characteristics. Cozzucoli [11] classified multinomial processes into $k+1$ defect categories and monitored them by multivariate p control chart. Taleb [12] monitored attribute inguistic characteristics in two approaches, consisting of probability and fuzzy theory. Ryan and Woodall [13] proposed control chart for Poisson count data with varying sample sizes. Chiu and Kuo [14] proposed a new control chart for monitoring fraction nonconforming in a bivariate binominal process. Hybrid neural network and simulated annealing is applied in monitoring autocorrelated multi-attribute processes by Niaki and Nasaji [15]. Li and Tsung [16] proposed a new control scheme in monitoring multi-attribute processes based on multiple binomial and Poisson CUSUM charts, together with multiple hypothesis testing technique. For more information about multi-attribute process monitoring, refer to the review paper recently written by Topalidou and Psarakis [17].

In this research, the quality of a product or a process is represented by the combination of variables and attributes quality characteristics which is not much explored in the literature of statistical process control. Kang and Brenneman [18] proposed a method to determine defect rate confidence bound for combined variable and attribute quality characteristics based on independent assumption. But in some cases, independent assumption is violated, and neglecting the correlation leads to misleading results. For example, in plastic manufacturing companies, the number of nonconforming and weight of a product are correlated variable–attribute quality characteristics. As another example, in semiconductor manufacturing process, the impurity of particle counts, the average of oxide thickness, and the range of the thickness are correlated Poisson, Normal, and Gamma distributed quality characteristics. Furthermore, some biomedical, psychological, and health sciences data include both correlated discrete and continuous outcomes [19].

Doroudyan and Amiri [20] proposed NORTA inverse transformation method in monitoring multivariate–attribute quality characteristics. NORTA inverse transforms multivariate–attribute quality characteristics to multivariate normal data, and then multivariate control charts are used to monitor the transformed data. In this paper, the use of two other transformation methods is proposed to monitor multivariate–attribute quality characteristics. In the first one, the skewness reduction approach is applied to transform the distribution of correlated variables and attributes data to approximate multivariate normal distribution, and then multivariate control charts including T^2 and MEWMA are used to monitor transformed data. Based on the second transformation method, multivariate–attribute quality characteristics are transformed, such that the correlation between transformed data approaches to zero. Then, p univariate control charts including bootstrap and exponentially weighted moving average (EWMA) control charts are applied to monitor the transformed data. Note that p is the number of quality characteristics. This transformation technique makes the quality characteristics uncorrelated [21]. Hence, each quality characteristic can be easily monitored by a univariate control chart separately. This simplifies the diagnosis of the quality characteristic responsible for out-of-control state. Finally, two combinations of these transformation methods are proposed to eliminate the skewness of data as well as the correlation between quality characteristics, simultaneously.

The structure of the paper is as follows: the problem is defined in the next section. The proposed techniques are illustrated in Sect. 3. In Sect. 4, the performance of the proposed methods is evaluated by using simulation studies in terms of average run length (ARL) criterion. The application of the proposed methods is shown through a real case, which is inducement of this research in Sect. 5. Conclusions and some suggestions for future research come in the final section.

2 Problem definition

In most of the cases, quality of a product or a process is represented by a vector of correlated variable–attribute quality characteristics $\mathbf{x} = (x_{11} \ldots x_{1m}\, x_{21} \ldots x_{2n})^T$, where x_{1k}'s ($k = 1,2,\ldots m$) and x_{2l}'s ($l = 1,2,\ldots n$) are variables and attribute quality characteristics, respectively. Since there is correlation structure between variable and attribute quality characteristics, monitoring them separately leads to misleading results. In addition, this research is involved in Phase II monitoring of multivariate–attribute quality characteristics. Hence, the mean vector ($\boldsymbol{\mu}_{\mathbf{x}}$) and variance–covariance matrix ($\boldsymbol{\Sigma}_{\mathbf{x}}$) of quality characteristics ($\mathbf{x}$) are known and estimated based on the historical dataset in Phase I. Naturally, variable quality characteristics (x_{1k}'s) follow continuous distributions such as normal and gamma distribution and attribute quality characteristics (x_{2l}'s) follow discrete distributions such as Poisson and binomial distribution. The purpose of this paper is to monitor the mean vector ($\boldsymbol{\mu}_{\mathbf{x}}$) of the multivariate–attribute quality characteristics. Note that shift in the means of some distributions such as gamma and Poisson distributions leads to shift in the covariance matrix as well. However, without loose of generality, it is assumed that the correlation matrix is fixed through the process.

3 Proposed methods

Skewness of distributions in multivariate–attribute quality characteristics, complexity of their joint distributions, and correlation between variables and attribute quality characteristics leads to undesired performance of the traditional control charts. To account for these issues, using two transformation methods is proposed. In the first one, skewness reduction approach is applied to transform the distribution of quality characteristics to approximate normal distribution, and then multivariate control charts are used in monitoring the transformed data. In the second transformation method, multivariate–attribute data are transformed, such that the correlation between quality characteristics approaches roughly to zero, then p univariate control charts are used in monitoring the transformed data (Let define $p=m+n$). Finally, two combinations of these transformation methods is proposed to eliminate skewness of data and correlation between quality characteristics, simultaneously. The proposed methods are described in the next subsections, respectively.

3.1 Root transformation method

Several transformation methods are proposed to reduce the skewness of data and to transform the original data to approximate normal data. For example, arcsine function transformation by Anscombe [22], root square transformation by Ryan [23], Q-transformation by Quesenberry [24], and recently root transformation by Niaki and Abbasi [25]. It is shown that the root transformation performs better than the other methods in monitoring multi-attribute quality characteristics (Niaki and Abbasi [25]).

The root transformation method transforms skewed data to approximate normal distribution based on the skewness reduction approach. Equation (1) shows the transformation formula.

$$\mathbf{y} = (y_1 \cdots y_p)^T = \left(x_1{}^{r_1} \cdots x_p{}^{r_p}\right)^T. \tag{1}$$

In this method, an appropriate vector of powers ($\mathbf{r} = (r_1 \ \ldots \ r_p)^T$) is determined by a root finding method, such that the marginal distribution of the transformed quality characteristics becomes zero skewed. In order to find a proper vector of powers, a bisection method is applied on the historical data for each quality characteristic. Therefore, r_i for ith quality characteristic is equal to the root of the function ($f\,(r_i)$).

Equation (2) shows $f\,(r_i)$, which calculate the value of skewness for the rth root of ith quality characteristic.

$$f(r_i) = \frac{E\left(x_{ij}^r - \sum_{j=1}^{q} \frac{x_{ij}^r}{q}\right)^3}{\left(\frac{1}{q-1}\sum_{j=1}^{q}\left(x_{ij}^r - \sum_{j=1}^{q}\frac{x_{ij}^r}{q}\right)^2\right)^{\frac{3}{2}}}, \tag{2}$$

where, x_{ij} is the jth sample (j = 1, 2, … q) of ith quality characteristic.

The bisection method is based on the changing sign of the function on two sides of the root, and the initial limit should contain the root. In the other words, the sign of the function should be opposite at the two sides of the initial interval. Then, the algorithm continues with bisecting the interval and selecting the one which consists of opposite sign in its two sides as a new interval, until the length of interval becomes less than a predefined value[1].

Although in most of distributions such as Poisson and gamma the possible data are positive, this method has a weakness in finding a proper root for negative data. To overcome this problem, we propose to perform another simple transformation such as $x_{ij} = x_{ij} - \min_j(x_{ij})$ for ith characteristic which has this problem, before root transformation method. As a result, the ith quality characteristic becomes positive, and then the proper root for new data can be found. In addition, when one of the quality characteristics has normal distribution, the root (r_i) is always set equal to one for it. Note that the zero skewed is a required condition for normality, but it is not a sufficient condition. However, it is assumed that the transformed data follows approximate normal distribution, and multivariate control chart is used to monitor the transformed data. Moreover, the mean vector ($\boldsymbol{\mu}_\mathbf{y}$) and covariance matrix ($\boldsymbol{\Sigma}_\mathbf{y}$) of transformed data ($\mathbf{y}$) have changed and should be estimated based on transformed historical data using moment method.

After finding a vector of appropriate powers ($\mathbf{r}$) and estimating the mean vector and variance–covariance matrix of transformed historical data, the transformed quality characteristics are calculated for each sample, and then multivariate control charts such as T^2 and MEWMA are used to monitor the transformed data.

The statistic of T^2 control chart is $T^2 = (\mathbf{y} - \boldsymbol{\mu}_\mathbf{y})^T \boldsymbol{\Sigma}_\mathbf{y}^{-1} (\mathbf{y} - \boldsymbol{\mu}_\mathbf{y})$. The upper control limit (UCL) is equal to $\chi^2_{1-\alpha,\,p}$, where α is the probability of Type I error of control chart, and p is the number of quality characteristics. The control chart signals when the statistic exceeded the UCL. Further properties of T^2 control chart are presented in Mason and Young [3]. In addition, the statistic of MEWMA control chart is $H_i = \mathbf{w}_i^T \boldsymbol{\Sigma}_\mathbf{w}^{-1} \mathbf{w}_i$, in which $\mathbf{w}_i = \lambda\ (\mathbf{y} - \boldsymbol{\mu}_\mathbf{y}) + (1 - \lambda)\ \mathbf{w}_{i-1}$. Likewise, the steady-state of variance–covariance matrix of control statistic is equal to $\boldsymbol{\Sigma}_w = (\lambda/(2-\lambda))\ \boldsymbol{\Sigma}_\mathbf{y}$. The parameter $\mathbf{w}_0$ usually is a p-dimensional zero vector, and λ is a smoothing parameter ($0<\lambda\leq 1$).

[1] For example, to find a root of $f\,(r) = 0$ in the interval of (a_0, b_0), where $f\,(a_0)\,f\,(b_0) < 0$, use the following algorithm:

$$t = 0,$$
while $|b_t - a_t| > \varepsilon$
$$r_{t+1} = (a_t + b_t)\big/2$$
if $(f(r_{t+1})f(a_t) < 0)$, then
$$a_{t+1} = a_t \text{ and } b_{t+1} = r_{t+1}$$
else
$$b_{t+1} = b_t \text{ and } a_{t+1} = r_{t+1}$$
end if
$$t = t + 1$$
end while
$$r^* = r_t$$

Where ε is a small predetermined value which is defined by the user. In general, a root for $f\,(r) \approx 0$ is searched in the initial interval (0, 1].

Moreover, the UCL is determined by simulation, such that the desired in-control average run length (ARL_0) is achieved. When the statistic exceeds the UCL, it is considered as a signal.

3.2 Symmetric root transformation

As mentioned in the first section, the main issue in using univariate control charts for monitoring multivariate processes is that the correlation between quality characteristics are not considered and it leads to increasing false alarms and misleading results. To overcome this problem, there are some methods to transform data, such that the correlation between quality characteristics approaches to zero. The most common of these methods is principle components analysis [26]. Although in this method, normality assumption is not a required condition, but there are problems in finding the source of variation. Albeit, there is a procedure to estimate the source of variation (Montgomery [2]), but it is complicated. In this subsection, using the symmetric root transformation method in Golnabi and Houshmand [21] is proposed to monitor multivariate–attribute process. Golnabi and Houshmand [21] proposed this method in monitoring multivariate normal quality characteristics. Afterwards, Niaki and Abbasi [27] used this method in monitoring multi-attribute quality characteristics. In this paper, the use of this method is proposed to monitor multivariate–attribute processes.

Assume a vector of quality characteristics ($\mathbf{x} = (x_1 \ldots x_p)^T$) represents the quality of a product or a process. Based on the historical data, the mean vector and variance–covariance matrix of quality characteristics are $\boldsymbol{\mu}_\mathbf{x}$ and $\boldsymbol{\Sigma}_\mathbf{x}$, respectively. Based on the symmetric root transformation method, first, the mean vector ($\boldsymbol{\mu}_\mathbf{x}$) is subtracted from the vector of quality characteristics ($\mathbf{x}$). Then, it is multiplied by a symmetric matrix ($\mathbf{C}$). Therefore, a vector of transformed data ($\mathbf{y} = (y_1 \ldots y_p)^T = \mathbf{C}(\mathbf{x} - \boldsymbol{\mu}_\mathbf{x})$) is obtained. Matrix $\mathbf{C}$ is the inverse of the square root of $\boldsymbol{\Sigma}_\mathbf{x}$. Accordingly, the transformed data are uncorrelated. Equation (3) shows the transformation formula.

$$\mathbf{y} = \left(y_1 \cdots y_p\right)^T = \left(\sum\nolimits_{\mathrm{x}}\right)^{\frac{1}{2}} \left(x_1 - \mu_1 \cdots x_p - \mu_p\right)^T. \quad (3)$$

After this transformation, y_i's are independent random variables which can be monitored by univariate control charts. A proof is given in Appendix A, which shows that new variables are uncorrelated with mean vector of zero.

As soon as a univariate control chart signals, the process would be out-of-control, and related quality characteristic is selected as the source of variation. Note that the overall type I error for multiple univariate control charts with α_i type I errors is determined by $1 - \alpha_{\mathrm{overall}} = (1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_p)$. If α_i's are set equivalent, the value of probability Type I error for each control chart (α_i) is computed by Eq. (4) based on desired $\alpha_{\mathrm{overall}}$.

$$\alpha_i = 1 - \sqrt[p]{1-\alpha_{overall}}. \quad (4)$$

Golnabi and Houshmand [21] and Niaki and Abbasi [27] proposed using p univariate X control charts in monitoring uncorrelated transformed variables. Based on our investigations, if original quality characteristics have nonzero skewness, this skewness remains in the transformed data. Since monitoring skewed distributions by using symmetric X control charts leads to inefficiency in detecting some shifts and misleading results, in this research, the use of bootstrap control chart using empirical distribution [28] is proposed instead of symmetric X control chart. In the bootstrap control chart, T samples are regenerated from the in-control distribution of quality characteristics. Then, the T samples are ranked in ascending order. Determine α_l and α_u as Type I error at the lower and upper tail of the control chart. Let define $QL=[T\alpha_v]$ and $QU=[T(1-\alpha_u)]$, where [] is the operator which rounds off the decimal number. Therefore, if x_i is the ith smallest sample, LCL and UCL are x_{QL} and x_{QU}, respectively. Wu and Wang [28] suggested that QL and QU should be not less than 10. Alternatively, the use of p EWMA control charts is proposed, which are robust to normality assumption. The statistic of univariate EWMA control chart proposed by Roberts [29] is $W_i=\lambda(x_i - \mu)+(1-\lambda)W_{i-1}$, in which λ is the smoothing parameter ($0<\lambda\leq 1$), and W_0 usually is set equal to zero. Upper and lower control limit are $\pm L\sigma\sqrt{\lambda/(2-\lambda)}$, where L determined such that the desired ARL_0 is achieved. In this paper, simulation studies are used to determine the control limits of EWMA control chart.

3.3 Combined symmetric root and root transformation

In this section, the combined symmetric root and root transformation method is proposed to eliminate the correlation between the quality characteristics and the skewness of data, simultaneously. Therefore, symmetric root transformation method can be used after root transformation to have symmetric distributions and then employ X control charts. In addition, inverse order of transformation methods can be used; in other words, first, the symmetric root transformation, and then root transformation is employed. In addition, to show the performance of EWMA control chart in normal distributed data, p-EWMA control charts are used to monitor transformed data as well.

3.3.1 Root then symmetric root transformation

First, similar to the root transformation method, the bisection method is applied to find a root vector. Then, the historical data is transformed to approximate multivariate normal distribution by using root transformation method in Eq. (1). The mean vector ($\boldsymbol{\mu}_\mathbf{y}$) and variance–covariance matrix ($\boldsymbol{\Sigma}_\mathbf{y}$) of multivariate normal data are estimated by using moment method. By using symmetric root transformation method in

Eq. (5), the multivariate normal data (**y**) is transformed to approximately uncorrelated normal data (**z**).

$$\mathbf{z} = \left(z_1 \ \cdots \ z_p\right)^T = \left(\boldsymbol{\Sigma}_{\mathbf{y}}\right)^{-\frac{1}{2}}\left(\mathbf{y}-\boldsymbol{\mu}_{\mathbf{y}}\right). \quad (5)$$

The mean (μ_i) and variance (σ_i) of each transformed variable is estimated by using the moment method. Finally, p-X or p-EWMA control charts are designed to monitor the uncorrelated normal data, separately. Based on the desired overall probability of Type I error ($\alpha_{overall}$), the value of probability Type I error for each control chart (α_i) is computed by Eq. (4). Note that the probability of Type I error of univariate control charts (α_i's) is set equivalent.

In Phase II, first, each sample (**x**) is transformed to multivariate normal (**y**) by using root transformation in Eq. (1) based on the vector of power (**r**). Then, multivariate normal sample (**y**) is transformed to approximately uncorrelated normal data (**z**) by using symmetric root transformation in Eq. (5), based on the mean vector ($\boldsymbol{\mu}_{\mathbf{y}}$) and variance–covariance matrix ($\boldsymbol{\Sigma}_{\mathbf{y}}$) of multivariate normal data. Finally, each transformed variable (z_i) is monitored in separate control chart. Whenever each control chart signals, the process is considered in out-of-control state, and related quality characteristic is reported as the source of variation.

3.3.2 Symmetric root then root transformation

In this method, the inverse order of transformation methods is applied. It means, first, the symmetric root transformation in Eq. (3) is implemented on historical data (**x**) based on the mean vector ($\boldsymbol{\mu}_{\mathbf{x}}$) and variance–covariance matrix ($\boldsymbol{\Sigma}_{\mathbf{x}}$) of quality characteristics. Then, a bisection method similar to the root transformation method is employed on each skewed variable to find a proper transformation (**r**). According to the existing some negative data on the skewed variables, a simple transformation such as $y_i = y_i - min\ (y_i)$ is performed before the bisection method. After finding a power vector (**r**), root transformation method in Eq. (6) is applied to the uncorrelated data (**y**) to transform them into approximate normal (**z**).

$$\mathbf{z} = \left(z_1 \ \cdots \ z_p\right)^T = \left(y_1^{r_1} \ \cdots \ y_p{}^{r_p}\right)^T. \quad (6)$$

The mean (μ_i) and variance (σ_i) of each transformed variable is estimated by using moment method. Finally, multiple univariate control charts such as X or EWMA control charts are used to monitor uncorrelated normal data, separately. Note that the probability of Type I error of univariate control charts (α_i's) is determined by using Eq. (4), based on overall desired probability of Type I error (α_{overall}).

In Phase II, first, each sample (**x**) is transformed by using symmetric root transformation in Eq. (3) to approximately uncorrelated data (**y**) based on the mean vector ($\boldsymbol{\mu}_{\mathbf{x}}$) and variance–covariance matrix ($\boldsymbol{\Sigma}_{\mathbf{x}}$) of quality characteristics. Then, each variable (y_i) is transformed to approximate normal (z_i) by using root transformation in Eq. (6), based on the vector of power (**r**). Note that a simple transformation ($y_i = y_i - a_i$) may be needed for some specific variables before root transformation, in which $a_i = min\ (y_i)$ has been determined in the previous step. Finally, each transformed variable (z_i) is monitored by a separate control chart. Whenever each control chart signals, the process is considered in out-of-control state, and related quality characteristic is reported as the source of variation.

In the next section, the performance of the transformation methods are evaluated for monitoring the mean vector in the multivariate–attribute process by using simulation studies through some numerical examples. Then, all methods are compared together: (A) root transformation (T^2 and MEWMA); (B) symmetric root transformation (p-bootstrap and p-EWMA control charts); (C) root then symmetric root transformation (p-X and p-EWMA control charts); (D) symmetric root then root transformation (p-X and p-EWMA control charts). Figure 1 shows the summarized steps of the proposed methods.

4 A numerical example

In this section, the performance of the proposed approaches is evaluated through a numerical example. The normality of the transformed data is tested by using JB normality test (Jarque and Bera [30]), which evaluates the hypothesis that whether the sample skewness and kurtosis are significantly different from their expected normal values by a chi-square statistic. In order to compare the methods, the control limits for all methods are set, such that they have the same ARL_0. Then, these control charts are compared based on the out-of-control average run length (ARL_1) criterion, when the mean vector of quality characteristics changes under different shift scenarios.

In this paper, random vectors are generated using Gaussian copula [31]. Based on this method, in order to generate a vector of random variable $\mathbf{x} = (x_1 \ \ldots \ x_p)^T$ with arbitrary distribution and covariance $\boldsymbol{\Sigma}_{\mathbf{x}}$, the following equation is used:

$$\mathbf{x} = \left(x_1 \quad \cdots \quad x_p\right)^T = \left(F_{x_1}^{-1}(s_1) \quad \cdots \quad F_{x_p}^{-1}(s_p)\right)^T, \quad (7)$$

where, $\mathbf{s} = (s_1 \ \ldots \ s_p)^T$ is a random vector generated from a Gaussian copula, and $F_{x_i}^{-1}(.)$ is the inverse of cumulative distribution function of x_i. It is required to determine the correlation matrix of s_i's (P), such that $\boldsymbol{\Sigma}_{\mathbf{x}}$ is achieved. **P** is set using trial and error in this paper.

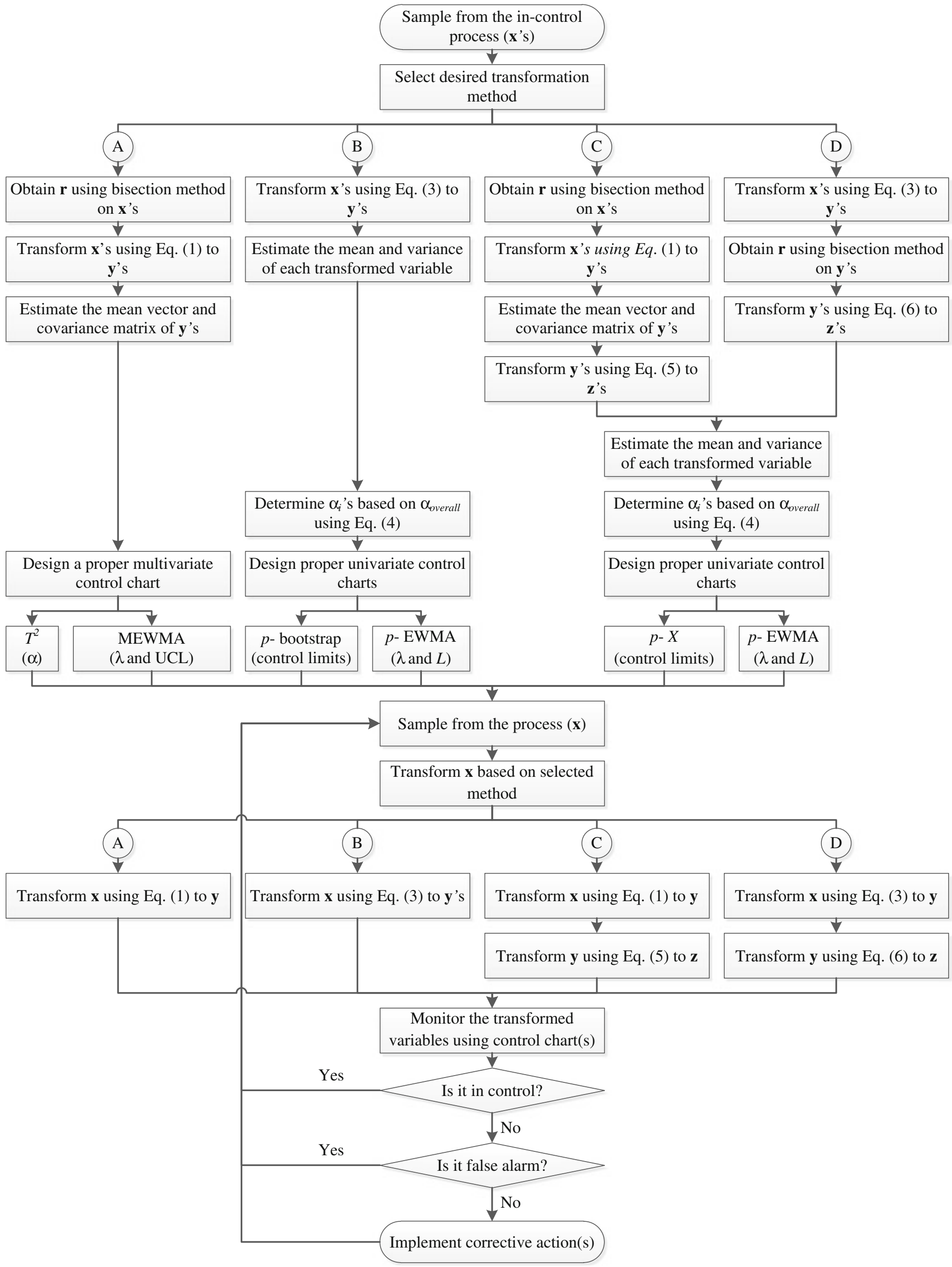

Fig. 1 Flowchart of the proposed methods

4.1 Evaluating performance of the transformation methods

In this example, the quality of a product is represented by a vector of $\mathbf{x} = (x_1\ x_2)^T$. Based on Phase I analysis, x_1 has normal distribution, and x_2 has Poisson distribution. The mean vector and variance–covariance matrix of quality characteristics are as follows:

$$\boldsymbol{\mu}_{\mathbf{x}} = (3 \quad 4)^T \text{ and } \boldsymbol{\Sigma}_{\mathbf{x}} = \begin{pmatrix} 4 & 1.4 \\ 1.4 & 4 \end{pmatrix}.$$

In order to evaluate the performance of the transformation methods, 5,000 random vectors are generated using Gaussian copula [31]. Estimated mean vector and variance–covariance matrix of generated data are:

$$\hat{\boldsymbol{\mu}}_{\mathbf{x}} = (3.02 \quad 4)^T \text{ and } \hat{\boldsymbol{\Sigma}}_{\mathbf{x}} = \begin{pmatrix} 3.98 & 1.41 \\ 1.41 & 3.97 \end{pmatrix}.$$

Skewness of the original data is 0.01 and 0.47 for variable and attributes quality characteristics, respectively. The kurtoses are 3.01 and 3.18, respectively. The proposed transformation techniques are evaluated in below:

(A) Root transformation method

First, the root transformation method is implemented on the historical data. The vector of power obtained as

$\mathbf{r}=(1\ 0.73)^T$. Then, the original data is transformed using Eq. (1). The estimated mean vector and variance–covariance matrix of the transformed data would be:

$$\widehat{\mu}_{\mathbf{y}} = (3.02 \quad 2.67)^T \text{ and } \widehat{\Sigma}_{\mathbf{y}} = \begin{pmatrix} 3.98 & 0.72 \\ 0.72 & 1.06 \end{pmatrix}.$$

The skewness values of transformed data become 0.01 and 0, respectively. Also, their kurtoses have changed to 3.01 and 3.06, respectively. The performance of the root transformation technique in transforming the distribution of quality characteristics to normal distribution is evaluated by using the JB test. The results confirmed the normality of variable quality characteristic and non-normality of attribute quality characteristic, while the normality of the transformed data are accepted with P-values of 0.91 and 0.71 for variable and attribute quality characteristic, respectively. The results show that the root transformation performs well in transforming the correlated variable and attribute quality characteristics to the approximate multivariate normal distribution.

(B) Symmetric root transformation method

According to the mean vector and variance–covariance matrix of quality characteristics, the historical data is transformed by using Eq. (3). The mean vector and variance–covariance matrix of the transformed data are estimated by using moment method as follows:

$$\widehat{\mu}_{\mathbf{y}} = (0.01 \quad 0)^T \text{ and } \widehat{\Sigma}_{\mathbf{y}} = \begin{pmatrix} 1 & 0 \\ 0 & 0.99 \end{pmatrix}.$$

The results show that the symmetric root transformation performs well in eliminating the correlation between the variable and the attribute quality characteristics and transforms them to the approximately uncorrelated data. Note that the skewness of transformed data are equal to 0.01 and 0.48, respectively, similar to the original data, and their kurtoses equal to 3.04 and 3.24. Likewise, the normality of transformed data is evaluated by using the JB test. The results confirmed the normality of the first transformed data with P-value of 0.82 and non-normality of the second transformed data, like the original quality characteristics. The results show that although the symmetric root transformation is able to diminish the value of correlation between the correlated data, but the skewness of original data remains in the transformed data.

(C) Root then symmetric root transformation method

First, the root transformation method is applied to the historical data. Then, the symmetric root transformation method (Eq. 5) is implemented on the multivariate normal data ($\mathbf{y}$) to transform them to approximately uncorrelated normal data ($\mathbf{z}$). The mean vector ($\mu_{\mathbf{z}}$) and variance–covariance matrix ($\Sigma_{\mathbf{z}}$) of transformed data ($\mathbf{z}$) are estimated as follows:

$$\widehat{\mu}_{\mathbf{y}} = (0 \quad 0)^T \text{ and } \widehat{\Sigma}_{z} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

After transformation, the skewness values of transformed data become 0.01 and −0.01, respectively. In addition, the kurtoses of transformed data have changed to 3.03 and 3.14, respectively. The results of the JB test confirmed the normality of transformed data with P-values of 0.80 and 0.11, respectively. The results show that this approach performs well in transforming the distribution of quality characteristics to the approximately uncorrelated normal data.

(D) Symmetric root then root transformation

In this method, first, the quality characteristics are transformed by symmetric root transformation method (Eq. 3) to nearly uncorrelated data ($\mathbf{y}$). Then, the uncorrelated data ($\mathbf{y}$) are transformed to approximately normal distribution by the root transformation method (Eq. 6). Note that after utilizing the symmetric transformation method, some of the data become negative, which is problematic for the root transformation method, hence a simple transformation is implemented on the second variable as $y_2=y_2-min\ (y_2)$. The root $\mathbf{r}=(1\ \ 0.59)^T$ is obtained in the root transformation method. The mean vector ($\mu_{\mathbf{z}}$) and variance–covariance matrix ($\Sigma_{\mathbf{z}}$) of the transformed data are estimated by using moment method as follows:

$$\widehat{\mu}_{\mathbf{z}} = (0.01 \quad 1.74)^T \text{ and } \widehat{\Sigma}_{z} = \begin{pmatrix} 1 & 0 \\ 0 & 0.16 \end{pmatrix}.$$

The skewness values of transformed data become to 0.01 and 0, respectively. Also, their kurtoses become 3.04 and 3. Likewise, the results of the JB test confirm the normality of transformed data with P-values of 0.83 and 0.99. The results show that the proposed method performs well in transforming the correlated mixed quality characteristic to the approximate independent normal variable.

4.2 Monitoring transformed data

In this section, the performance of the proposed methods is evaluated in detecting shifts in the mean vector of the example under different scenarios explained in the previous section. The control charts are designed, such that ARL_0 values for all methods to be equivalent. Then, all methods are compared based on the ARL_1 criterion.

(A) Root transformation

In the previous subsection, an appropriate transformation vector is determined, and then the mean vector and variance–covariance matrix of transformed data are estimated. The UCL of T^2 control chart is set to $\chi^2_{0.995,2}$=10.59 to obtain the ARL_0=200. The value of ARL_0 is obtained equal to 210 by simulation runs. In addition, UCL in MEWMA control chart is determined by using simulation with 10,000 replications equal to 9.82 for desired $ARL_0 \approx 210$. The smoothing parameter in this control chart is set equal as 0.2.

(B) Symmetric root transformation

In order to achieve the desired overall ARL_0 (210) or the same probability of Type I error (0.0048), the value of ARL_0 or probability of Type I error for each control chart should be set equal to 419 or 0.0024, respectively. Thus, in the two-bootstrap control charts, the control limits for the first transformed variable is symmetric and equal to 0.01±3.04. The lower and upper control limits for the second transformed variable are −2.21 and 3.64, respectively. The control limits for 2-EWMA control charts are ± 0.967 and ± 0.98 for the first and second transformed variables, respectively. The smoothing parameter is set equal to λ=0.2.

(C) Root then symmetric root transformation

The control limits for 2-X control charts are set ± 3.05 and ± 2.973 for the first and second transformed variables, respectively, to obtain the overall $ARL_0 \approx 210$. Similarly, the control limits for 2-EWMA control charts are set ± 0.97 for both transformed variables by setting the smoothing parameter equal to 0.2.

(D) Symmetric root then root transformation

In order to reach ARL_0=210, control limits for 2-X control charts are 0.01±3.04 and 1.74±1.178 for the first and second transformed variables, respectively. The control limits for 2-EWMA control charts would be ± 0.967 and ± 0.385 for the first and second transformed variables, respectively, in which the smoothing parameter is equal to 0.2.

The values of ARL_1 for different shift scenarios in the mean vector of quality characteristics are reported in Tables 1 and 2, when the process is monitored by Shewhart-type and EWMA-based control charts, respectively. In each scenario (θ_1 θ_2) shows the magnitude of shift in unit of σ in the mean of distributions from $(\mu_1 \mu_2)^T$ to $(\mu_1+\theta_1 \ \ \mu_2+\theta_2)^T$. In order to outline the results, Tables 3 and 4 show the best method in detecting each shift scenario for Shewhart-type and EWMA-based control charts, respectively.

In all methods, the EWMA-based control charts perform better than the Shewhart-type control chart in the small and medium size of shifts. While in large size of shifts, Shewhart-type control charts performs better.

According to the Tables 1 and 2, the root transformation method in the simultaneous shifts in both quality characteristics performs better than the other methods. This is

Table 1 ARL_1 values when the process is monitored by Shewhart-type control charts

Method	A	B*				C*				D*			
Control chart	T^2	2-bootstrap				2-X				2-X			
Different scenarios	ARL_1	ARL_1	#1	#2	#3	ARL_1	#1	#2	#3	ARL_1	#1	#2	#3
(σ 0)	36.75	34.44	77	22.47	0.53	38.38	82.70	17.04	0.26	35.79	81.20	18.39	0.41
(2σ 0)	5.63	5.42	91.48	7.04	1.48	5.78	93.59	5.73	0.68	5.44	93.75	5.32	0.93
(3σ 0)	1.83	1.81	94.59	2.25	3.16	1.87	97.24	1.78	0.98	1.82	95.65	1.83	2.52
(−σ 0)	34.51	38.21	91.57	8.22	0.21	37	85.62	14.02	0.36	37.49	89.85	9.93	0.22
(0 σ)	29.17	33.11	8.36	91.36	0.28	26.20	6.05	93.76	0.19	27.95	6.87	92.83	0.30
(0 2σ)	6.33	6.85	2.46	97.06	0.48	5.68	1.42	98.31	0.27	5.93	1.73	97.74	0.53
(0 3σ)	2.60	2.74	1.16	98.14	0.70	2.41	0.45	99.18	0.37	2.47	1.05	98.21	0.74
(0 -σ)	74.83	54.17	16.55	83.10	0.35	93.91	24.33	75.16	0.51	83.70	25.39	73.90	0.71
(σ σ)	23.14	28.69	39.18	60.08	0.74	27.05	41.12	57.48	1.40	25.60	34.73	64.64	0.63
(2σ 2σ)	4.06	5.54	45.73	50.14	4.13	5.32	54.98	39.70	5.32	5.17	43.39	52.68	3.93
(3σ 3σ)	1.60	2.03	49.88	35.20	14.92	1.95	59.79	23.61	16.60	1.92	46.88	38.20	14.92
(−σ -σ)	41.52	55.09	91.33	8.66	0.01	56.86	96.45	3.55	0	58.89	95.72	4.26	0.02
(σ σ)	10.84	10.58	36.32	60.86	2.82	14.18	41.36	56.73	1.91	12.54	44.75	52.44	2.81
(−σ σ)	9.33	14.07	47.73	50.50	1.77	11.59	32.69	65.34	1.97	12.98	44.16	53.84	2.00
(2σ -σ)	2.79	3.11	61.15	29.54	9.31	3.52	66.31	28.81	4.88	3.19	65.58	27.34	7.08
(−σ 2σ)	3.29	4.56	19.92	75.40	4.68	3.80	10.69	85.76	3.55	4.36	17.36	77.53	5.11

(*Asterisk*) these methods are able to diagnose the source of variation including: first (#1), second (#2), and both (#3) quality characteristics

Table 2 ARL_1 values when the process is monitored by EWMA-based control charts

Method	A	B*				C*				D*			
Control chart	MEWMA	2-EWMA				2-EWMA				2-EWMA			
Different scenarios	ARL_1	ARL_1	#1	#2	#3	ARL_1	#1	#2	#3	ARL_1	#1	#2	#3
(σ 0)	9.48	9.34	98.56	1.21	0.23	9.43	96.32	3.14	0.54	9.23	96.33	3.09	0.58
(2σ 0)	3.53	3.46	99.78	0.10	0.12	3.50	98.61	0.59	0.8	3.45	98.4	0.6	1
(3σ 0)	2.29	2.24	99.93	0	0.07	2.28	99.15	0.17	0.68	2.24	98.33	0.27	1.4
(−σ 0)	9.04	8.94	96.68	2.87	0.45	9.12	96.01	3.41	0.58	9.12	97.76	1.85	0.39
(0 σ)	10.06	8.97	2.19	97.40	0.41	9.69	1.89	97.81	0.3	10.01	2.68	96.99	0.33
(0 2σ)	4.08	3.61	0.48	99.09	0.43	3.96	0.38	99.34	0.28	4.05	0.69	98.75	0.56
(0 3σ)	2.73	2.35	0.13	99.36	0.51	2.62	0.04	99.79	0.17	2.70	0.22	99.22	0.56
(0 -σ)	8.32	10.07	2.47	97.03	0.50	8.17	1.47	98.14	0.39	8.04	1.68	97.98	0.34
(σ σ)	7.88	8.31	42.36	53.08	4.56	8.92	58.71	36.83	4.46	8.83	47.42	48.04	4.54
(2σ 2σ)	3.22	3.39	41.30	43.33	15.37	3.63	63.12	22.59	14.29	3.67	49.73	34.10	16.17
(3σ 3σ)	2.17	2.25	37.31	33.97	28.72	2.68	43.45	30.53	26.02	2.43	48.47	22.88	28.65
(−σ -σ)	7.11	9.67	63.21	33.98	2.81	8.39	52.48	42.75	4.77	8.60	50.06	46.17	3.77
(σ -σ)	4.30	5.50	49.25	39.54	11.21	4.98	31.31	57.90	10.79	4.84	33.02	55.33	11.65
(−σ σ)	4.60	5.06	44.85	45.90	9.25	5.33	40.14	50.79	9.07	5.34	51.35	39.64	9.01
(2σ -σ)	2.65	3.07	86.13	4.26	9.61	3.04	66.71	14.90	18.39	2.91	65.18	14.26	20.56
(−σ 2σ)	2.95	3.06	12.62	75.06	12.32	3.26	9.83	81.05	9.12	3.36	17.26	69.09	13.65

(*Asterisk*) these methods are able to diagnose the source of variation including: first (#1), second (#2), and both (#3) quality characteristics

confirmed in both Shewhart-type and EWMA-based control charts, except in (−σ −σ) shift of Shewhart-type control chart.

In comparison, among all methods, it seems there are some kinds of trade-off in detecting shifts. It means one method performs better in some shifts but cannot detect the other shifts well. On the other hand, the number of proposed methods and investigated shift scenarios are relatively large. Therefore, a multicriteria decision making technique is used to compare the methods and summarize the results. In this study, technique for order preference by similarity to ideal solution (TOPSIS) (Munier [32]), as one of the most usual methods for decision making, is used. In this method, the value of ARL_1 in each shift scenario is considered as a criterion. Thus, in each shift scenario, the least and the maximum ARL_1 are the positive and negative ideal, respectively. The weight of all criteria is set equal to 1.

Based on the TOPSIS method, the order of the methods in terms of ARL performance is A, B, C, and D for the Shewhart-type control charts and A, D, C, and B for EWMA-based control charts. As a result, the root transformation method (A) in both Shewhart-type and EWMA-based control charts is selected as the best method. However, this method has a weakness in diagnosing the source of variation. To compare the methods in terms of diagnosing ability, the percentage of correct diagnosing is calculated in each method. Therefore, the rank of methods in terms of the performance in diagnosing the changed quality characteristics are B, C, and D, with percentage of 47.67, 47.57, and 47.48°%, respectively, for Shewhart-type control chart and D, C, and B for EWMA-based control chart, with percentage of 55.73, 55.13, and 55.10°%, respectively.

5 Real dataset

In this section, the root transformation method (A) is applied to a real dataset from a plastic automotive parts manufacturing

Table 3 The best method in the terms of ARL_1 criterion in Shewhart-type control charts

(σ 0)	(2σ 0)	(3σ 0)	(−σ 0)	(0 σ)	(0 2σ)	(0 3σ)	(0 -σ)
B	B	B	A	C	C	C	B
(σ σ)	(2σ 2σ)	(3σ 3σ)	(−σ -σ)	(σ -σ)	(−σ σ)	(2σ -σ)	(−σ -2σ)
A	A	A	A	B	A	A	A

Table 4 The best method in terms of the ARL_1 criterion in EWMA-based control charts

(σ 0)	(2σ 0)	(3σ 0)	(−σ 0)	(0 σ)	(0 2σ)	(0 3σ)	(0 -σ)
D	D	D-B	B	B	B	B	D
(σ σ)	(2σ 2σ)	(3σ 3σ)	(−σ -σ)	(σ -σ)	(−σ σ)	(2σ -σ)	(−σ -2σ)
A	A	A	A	A	A	A	A

process that is a typical high-quality process. The aim of the process is to cast the raw materials (such as polyvinyl chloride, polypropylene, polyamide 6, acrylonitrile butadiene styrene, polyol, and isocyanate) into plastic automotive parts with predefined qualifications. The manufacturing product includes armrest, steering wheel, center console, sun visor, and pillar cover. The quality of finished parts, before assembling, in terms of appearance and functional performance is inspected by sampling. The dimensional, the surface, and the mechanical properties are the important quality characteristics of each part; Kamal et al. [33] and Yang and Gao [34] showed that weight of the product is also a critical quality characteristic, which has correlation with other (surface and mechanical) quality characteristics.

In this dataset (given in Table 5 of Appendix B), the weight of products (in unit of 0.01 g) and the defect counts are considered as the quality characteristics (denoted as x_1 and x_2). The dataset includes 200 samples of size 1. The historical data is used to estimate the parameters of the process. The JB test confirms the normality of weight quality characteristics (x_1) with the P-value of 0.40. Also, based on the goodness-of-fit test for Poisson distribution, the number of defects follows Poisson distribution with the P-value of 0.47. The estimation of mean vector and covariance matrix of the historical data are:

$$\widehat{\mu}_{\mathrm{x}} = \begin{pmatrix} 42664 & 1.90 \end{pmatrix}^T \text{ and } \widehat{\Sigma}_{\mathrm{x}} = \begin{pmatrix} 9.20 & -1.46 \\ -1.46 & 1.66 \end{pmatrix}.$$

The skewness of data is −0.1 and 0.54 for variable and attributes quality characteristics, respectively. Their kurtoses are also equal to 2.60 and 3.11, respectively. The root transformation method (A), which has the best performance among the other proposed methods, is used to monitor the process.

First, the root vector is obtained as $\boldsymbol{r}=(1\ 0.76)^T$, and then observations are transformed by using Eq. (1). The mean vector and covariance matrix of transformed data are estimated using moment method as follows:

$$\widehat{\mu}_{\mathrm{y}} = \begin{pmatrix} 42664 & 1.54 \end{pmatrix}^T \text{ and } \widehat{\Sigma}_{\mathrm{y}} = \begin{pmatrix} 9.20 & -1.02 \\ -1.02 & 0.79 \end{pmatrix}.$$

The results of JB test confirm the normality of transformed data. Therefore, UCL of T^2 control chart is set equal to 10.59 for the ARL_0=200. In addition, UCL and smoothing parameter in the MEWMA control chart are set equal to 9.82 and 0.2, respectively.

In Phase II, each sample is transformed by using Eq. (1). Then, designed T^2 and MEWMA control charts are used to monitor transformed samples. The results show that all samples are in the control region. Therefore, the process is in-control state. Figures 2 and 3 show the trend of statistic in T^2 and MEWMA control charts, respectively.

6 Conclusions and recommendations for future research

In most of product and service environments, two or more quality characteristics represent the quality of a product or a process. Despite the existing dense literature in multivariate or multi-attribute processes monitoring, literature on monitoring correlated variable and attribute quality characteristics is sparse. In this paper, the use of transformation methods is proposed to monitor multivariate–attribute processes. Four methods are utilized to design schemes in monitoring multivariate–attribute processes, which are (A) root transformation, (B) symmetric root transformation, (C) root then symmetric root transformation, and (D) symmetric root then root transformation. After transforming the data, Shewhart-type and EWMA-based control charts are suggested to monitor the transformed data. The performance of the proposed methods are evaluated and compared through a numerical example, and the results are discussed. It is concluded that the method A has the best performance in both Shewhart-type and EWMA-based control charts, but this method is unable to detect the source of variation. Therefore, using the method B is suggested with p-bootstrap control charts for detecting large shifts in the process means, and method D with p-EWMA control charts for the process with small shifts. Finally, the proposed approach (method A as the best method) is applied to a real data. As the future research,

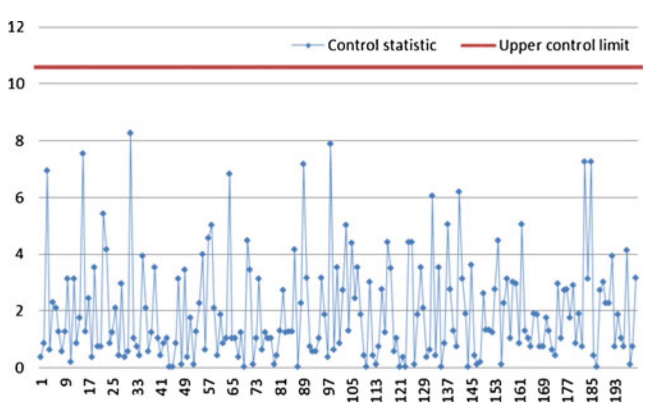

Fig. 2 T^2 control chart for real dataset in section 5

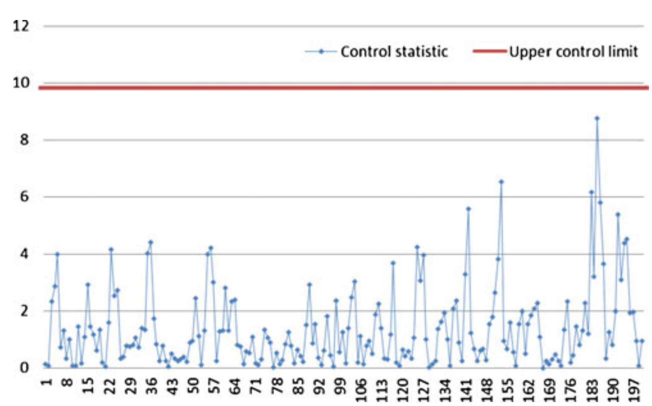

Fig. 3 MEWMA control chart for real dataset in section 5

using the other transformation methods is suggested to monitor correlated variable and attribute data. Issues on the monitoring covariance matrix in multivariate–attribute processes can also be investigated in the future research.

Appendix A: Proof that the transformed data are uncorrelated after symmetric root transformation

$$E\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}(\mathbf{x}-\boldsymbol{\mu}_{\mathbf{x}})\right) = \boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}E(\mathbf{x}-\boldsymbol{\mu}_{\mathbf{x}}) = \boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}(E(\mathbf{x})-\boldsymbol{\mu}_{\mathbf{x}}) = \boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}(\boldsymbol{\mu}_{\mathbf{x}}-\boldsymbol{\mu}_{\mathbf{x}}) = \boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}(0) = 0$$

$$Cov\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}(\mathbf{x}-\boldsymbol{\mu}_{\mathbf{x}})\right) = Cov\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\mathbf{x}-\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\boldsymbol{\mu}_{\mathbf{x}}\right) = Cov\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\mathbf{x}\right) = \left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\right)(\boldsymbol{\Sigma}_{\mathbf{x}})\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\right)' =$$
$$= \left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\right)\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{\frac{1}{2}}\boldsymbol{\Sigma}_{\mathbf{x}}^{\frac{1}{2}}\right)\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\right) = \left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\boldsymbol{\Sigma}_{\mathbf{x}}^{\frac{1}{2}}\right)\left(\boldsymbol{\Sigma}_{\mathbf{x}}^{\frac{1}{2}}\boldsymbol{\Sigma}_{\mathbf{x}}^{-\frac{1}{2}}\right) = \mathbf{I}*\mathbf{I} = \mathbf{I}$$

Appendix B: The dataset for real case in Sect. 5

Table 5 The real dataset

n	x_1	x_2	n	x_1	x_2	n	x_1	x_2	n	x_1	x_2	n	x_1	x_2	n	x_1	x_2
1	42665	1	35	42658	3	69	42664	2	103	42662	0	137	42670	0	171	42662	1
2	42662	3	36	42661	1	70	42659	1	104	42662	1	138	42667	3	172	42666	2
3	42670	3	37	42666	1	71	42669	2	105	42662	5	139	42662	1	173	42662	2
4	42666	2	38	42665	3	72	42663	2	106	42661	4	140	42663	3	174	42659	2
5	42663	4	39	42668	0	73	42667	1	107	42668	0	141	42671	0	175	42661	2
6	42661	1	40	42667	1	74	42665	0	108	42660	2	142	42667	0	176	42669	1
7	42661	3	41	42662	2	75	42666	2	109	42664	1	143	42660	3	177	42667	3
8	42666	1	42	42664	3	76	42665	3	110	42664	2	144	42664	2	178	42668	1
9	42661	3	43	42667	1	77	42661	2	111	42666	0	145	42659	4	179	42660	4
10	42667	0	44	42664	2	78	42667	1	112	42664	1	146	42662	2	180	42662	3
11	42665	2	45	42664	2	79	42663	2	113	42663	2	147	42663	2	181	42660	3
12	42667	0	46	42662	3	80	42664	1	114	42663	3	148	42665	2	182	42663	1
13	42662	3	47	42665	0	81	42662	1	115	42667	3	149	42664	4	183	42656	4
14	42668	1	48	42663	2	82	42669	1	116	42665	3	150	42667	2	184	42665	0
15	42660	0	49	42669	2	83	42665	3	117	42661	5	151	42667	2	185	42656	4
16	42661	3	50	42665	1	84	42661	3	118	42664	0	152	42665	3	186	42664	1
17	42661	4	51	42668	1	85	42661	3	119	42666	1	153	42667	3	187	42664	2
18	42665	1	52	42663	2	86	42669	0	120	42667	1	154	42659	1	188	42669	1
19	42668	0	53	42661	3	87	42664	2	121	42664	2	155	42663	2	189	42666	0
20	42663	3	54	42668	2	88	42668	2	122	42665	1	156	42668	2	190	42668	2
21	42663	3	55	42666	4	89	42660	6	123	42664	2	157	42665	0	191	42668	2
22	42671	1	56	42666	2	90	42660	1	124	42661	5	158	42661	2	192	42670	1
23	42669	0	57	42658	4	91	42663	3	125	42661	5	159	42666	0	193	42663	3
24	42664	3	58	42662	0	92	42666	1	126	42663	2	160	42659	2	194	42666	3
25	42665	3	59	42661	1	93	42666	1	127	42666	3	161	42664	3	195	42667	1
26	42661	1	60	42664	1	94	42667	1	128	42668	0	162	42670	0	196	42663	1
27	42664	1	61	42660	2	95	42665	4	129	42661	1	163	42662	1	197	42663	0
28	42659	2	62	42664	3	96	42660	2	130	42665	1	164	42667	1	198	42663	2
29	42665	1	63	42661	2	97	42665	1	131	42666	2	165	42663	1	199	42663	3
30	42666	1	64	42666	5	98	42657	1	132	42658	1	166	42660	3	200	42665	4
31	42664	6	65	42667	1	99	42666	2	133	42662	2	167	42666	3			
32	42661	2	66	42661	2	100	42668	0	134	42668	0	168	42663	3			
33	42663	3	67	42665	1	101	42664	3	135	42664	2	169	42663	1			
34	42662	2	68	42665	3	102	42669	1	136	42664	3	170	42668	1			

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