Project critical path assessment in production environments by a new interval-valued fuzzy decision method

Y. Dorfeshan and S.M. Mousavi
Department of Industrial Engineering, Faculty of Engineering, Shahed University,
Tehran, Iran
yahyad_147@yahoo.com, sm.mousavi@shahed.ac.ir

A. Siadat
LCFC, Arts et Métier Paris Tech, Centre de Metz,
Metz, France
ali.siadat@ensam.eu

Abstract

Critical path assessment plays an important role in production project management in order to schedule and control of the projects. In fuzzy sets theory, it is often difficult for an expert to accurately quantify his or her opinion as a number in interval [0,1]. Therefore, it is more appropriate to indicate this degree of certainty by an interval that is why interval-valued fuzzy sets (IVFSs) are used to better address the uncertainty of real world production projects. Also, a new multi-criteria decision making (MCDM) method based on fuzzy preference relation under IVFSs is developed. Moreover, entropy method is extended by means o_{NEAA} distance between each point from nearer and farther point among ideal points under IVFSs. Finally, an application about production projects is solved to better illustrate the calculation and capability of the proposed method.

Keywords
Production projects, critical path assessment, interval-valued fuzzy sets, entropy, decision making method

1. Introduction

Decision-making is often associated with the procedure of selecting the best alternative from the set of feasible alternatives. In many cases when selecting the best alternative, it is necessary to take into account the impact of conflict multiple criteria. Multiple criteria decision making (MCDM) is an important part of today's decision making problem. It has been vastly applied to various areas such as economics, management, production, and engineering (e.g., Salimi et al., 2013; Ebrahimnejad et al., 2014; Roshanaei et al., 2013; Mousavi et al., 2014).

Among many cases, crisp data are incomplete to model real life conditions. In fact, uncertainty plays an important role in decision-making problems. In order to tackle the uncertainty of real world problems, the fuzzy sets theory was proposed by Zadeh (1965). For instance, Chen (2000) proposed extension of TOPSIS for group decision making under fuzzy environments. Tsaur et al. (2002) transformed fuzzy MCDM problem into a crisp one by means of centroid defuzzification and then solve the crisp MCDM problem using the TOPSIS method. Chu and Lin (2003) introduced a fuzzy TOPSIS method for robot selection problems. Zammori et al. (2009) expressed critical path assessment problem as a MCDM problem under a fuzzy environment and solve it by TOPSIS method.

Critical path method (CPM) identifies critical activities on the critical path so that resources may be centralized on these activities in order to reduce the project completion time. Critical path selection problem is an important issue in project management and especially, production and manufacturing project management. Several articles have been

done in recent years. Khalaf (2013) introduced a fuzzy project scheduling based on a ranking function that applied this method in Al-SAMA construction project. Madhuri and Chandan (2016) applied a fuzzy critical path method to manufacturing tugboat, in which linear programing model has been used for determining critical path. Mehlawat and Gupta (2016) presented an MCDM method based on fuzzy preference relation to specify critical path in case study of design and manufacture small electronic components, particularly for the aviation, defense and space industries.

In fuzzy sets theory, it is often difficult for an expert to accurately quantify his or her opinion as a number in interval [0,1]. Then, it is more appropriate to indicate this grade of certainty by an interval (e.g., Mousavi et al., 2013; Vahdani et al., 2014a, 2014b; Mohagheghi et al., 2015; Moradi et al., 2017). Grattan (1976) noted that the showing of a linguistic expression in the form of fuzzy sets is not enough. Interval-valued fuzzy sets (IVFSs) were proposed for the first time by Gorzlczany (1987) and Turksen (1996). In fact, interval-valued fuzzy (IVF) numbers provide more degree of freedom to tackle uncertainty of decision making problem in real world production project management. Also, fuzzy preference relation is one of the ranking category of fuzzy numbers that uncertainties of fuzzy numbers are kept during comparison of fuzzy numbers' process. In order to use advantages of fuzzy relative preference relation and IVF number, in this paper a new MCDM method for assessing and determining critical path under IVFSs is presented. Moreover, to determine the weights of criteria the entropy method based on distances between each point from nearer and farther point among ideal points is developed and added to the presented new MCDM method.

The remainder of the paper is structured as follows. Section 2 discusses and reviews the preliminary and basic knowledge of IVF number. Section 3 introduces the proposed method. Section 4 presents an application to better demonstrate the capability of proposed method and finally, section 5 concludes the paper.

2. Preliminary

The IVFSs were presented by Gorzalczany (1987); then, Yao and Lin (2002) described the interval-valued trapezoidal fuzzy numbers (Fig. 2). An interval-valued trapezoidal fuzzy number is defined as follows:

$$\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \tag{1}$$

which contains two parts, namely the lower value, $\tilde{A}^L$ and upper value $\tilde{A}^U$ that $\tilde{A}^L \subset \tilde{A}^U$.

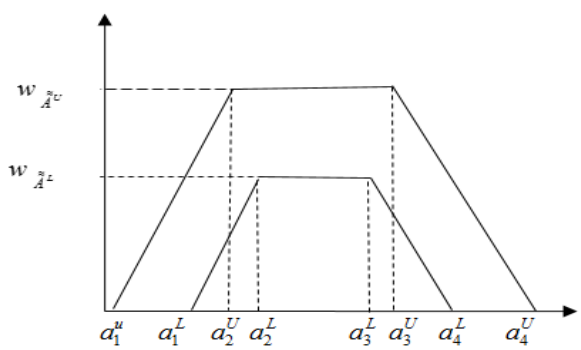

Fig. 1. Interval-valued fuzzy number

If two interval-valued fuzzy numbers can be defined as follows:

$$\tilde{A} = [\tilde{A}^L \subset \tilde{A}^U] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \tag{2}$$

$$\tilde{B} = [\tilde{B}^L \subset \tilde{B}^U] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{B}^U}\right)\right] \tag{3}$$

where, $0 \le a_i^L \le 1$ and $0 \le a_i^U \le 1$ for all i=1,2,…,n, and $0 \le w_{\tilde{A}^L} \le w_{\tilde{A}^U} \le 1$ and $0 \le w_{\tilde{B}^L} \le w_{\tilde{B}^U} \le 1$. Then, the arithmetic operations are defined as below (Chen and Sanguansat, 2011):

Addition:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \\ &\oplus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{B}^U}\right)\right] = \\ &\left[\begin{array}{l}\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\left\{w_{\tilde{A}^L}, w_{\tilde{B}^L}\right\}\right), \\ \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\left\{w_{\tilde{A}^U}, w_{\tilde{B}^U}\right\}\right)\end{array}\right]\end{aligned} \tag{4}$$

Subtraction:

$$\begin{aligned}\tilde{A} \Theta \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \\ &\Theta \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{B}^U}\right)\right] = \\ &\left[\begin{array}{l}\left(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\left\{w_{\tilde{A}^L}, w_{\tilde{B}^L}\right\}\right), \\ \left(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min\left\{w_{\tilde{A}^U}, w_{\tilde{B}^U}\right\}\right)\end{array}\right]\end{aligned} \tag{5}$$

Multiplication:

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \\ &\otimes \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{B}^U}\right)\right] \cong \\ &\left[\begin{array}{l}\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min\left\{w_{\tilde{A}^L}, w_{\tilde{B}^L}\right\}\right), \\ \left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min\left\{w_{\tilde{A}^U}, w_{\tilde{B}^U}\right\}\right)\end{array}\right]\end{aligned} \tag{6}$$

Division:

$$\begin{aligned}\tilde{A} \phi \tilde{B} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}^U}\right)\right] \\ &\phi \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{B}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{B}^U}\right)\right] \cong \\ &\left[\begin{array}{l}\left(a_1^L / b_4^L, a_2^L / b_3^L, a_3^L / b_2^L, a_4^L / b_1^L; \min\left\{w_{\tilde{A}^L}, w_{\tilde{B}^L}\right\}\right), \\ \left(a_1^U / b_4^U, a_2^U / b_3^U, a_3^U / b_2^U, a_4^U / b_1^U; \min\left\{w_{\tilde{A}^U}, w_{\tilde{B}^U}\right\}\right)\end{array}\right]\end{aligned} \tag{7}$$

Also, Euclidean distance between $\tilde{A}$ and $\tilde{B}$ can be defined as follows (Ashtiani et al., 2009; Chen, 2000):

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{8}\left[\begin{array}{l}(a_1^L - b_1^L)^2 + (a_2^L - b_2^L)^2 + (a_3^L - b_3^L)^2 + (a_4^L - b_4^L)^2 + (a_1^U - b_1^U)^2 \\ +(a_2^U - b_2^U)^2 + (a_3^U - b_3^U)^2 + (a_4^U - b_4^U)^2\end{array}\right]} \tag{8}$$

For two interval-valued triangular fuzzy numbers, extended fuzzy preference relation $F(\tilde{A},\tilde{B})$ is defined by the membership function:

$$\mu_F(\tilde{A}^L,\tilde{B}^L)=\int_0^1\left(\left((\tilde{A}-\tilde{B})_\alpha^L\right)^L+\left((\tilde{A}-\tilde{B})_\alpha^U\right)^L\right)d\alpha=(a_1^L-b_4^L+a_2^L-b_3^L$$
$$+a_3^L-b_2^L+a_4^L-b_1^L)/2$$
$$\mu_F(\tilde{A}^U,\tilde{B}^U)=\int_0^1\left(\left((\tilde{A}-\tilde{B})_\alpha^L\right)^U+\left((\tilde{A}-\tilde{B})_\alpha^U\right)^U\right)d\alpha=(a_1^U-b_4^U+a_2^U-b_3^U \tag{9}$$
$$+a_3^U-b_2^U+a_4^U-b_1^U)/2$$

Where, the preference intensity function of a triangular fuzzy number $\tilde{A}$ over the $\tilde{B}$ is defined as follows:

$$P(\tilde{A}^L,\tilde{B}^L)=\begin{cases}\mu_F(\tilde{A}^L,\tilde{B}^L), & if \quad \mu_F(\tilde{A}^L,\tilde{B}^L)\geq 0\\ 0, & otherwise\end{cases}$$
$$P(\tilde{A}^U,\tilde{B}^U)=\begin{cases}\mu_F(\tilde{A}^U,\tilde{B}^U), & if \quad \mu_F(\tilde{A}^U,\tilde{B}^U)\geq 0\\ 0, & otherwise\end{cases} \tag{10}$$

3. Proposed decision method

In this section, in order to assess and determine critical path of production project management, a new MCDM method based on fuzzy preference relation is developed under IVF numbers. In fact, this method is based on fuzzy preference relation (Mehlawat and Gupta, 2016) and its extensions under IVF numbers. The major advantages of proposed method for assessing and determining critical paths in production project management is strength and weakness scores based on relative comparisons by using fuzzy preference relation and relative comparison of the performances of production project paths. Also, a new entropy method based on distance between each point from nearer and farther point among ideal points under IVF numbers is extended for specifying weights of criteria.

Step 1: Gather expert's opinion on ratings efficient criteria toward each activity. Also, the qualitative criteria and their weights are explained as linguistic variables and are converted to equivalent IVF numbers which are shown in Table 1.

Table 1: Linguistic variables

LINGUISTIC VARIABLES	IVF NUMBERS
ABSOLUTELY POOR (AP)	((0,0,0,0;1),(0,0,0,0;1))
VERY POOR (VP)	((0.0075,0.0075,0.015,0.0525;0.9),(0,0,0.02,0.07;1))
POOR (P)	((0.0875,0.12,0.16,0.1825;0.9),(0.04,0.1,0.18,0.23;1))
MEDIUM POOR (MP)	((0.2325,0.255,0.325,0.3575;0.9),(0.17,0.22,0.36,0.42;1))
MEDIUM (M)	((0.4025,0.4525,0.5375,0.5675;0.9),(0.32,0.41,0.58,0.65;1))
MEDIUM GOOD (MG)	((0.65,0.6725,0.7575,0.79;0.9),(0.58,0.63,0.8,0.86;1))
GOOD (G)	((0.7825,0.815,0.885,0.9075;0.9),(0.72,0.78,0.92,0.97;1))
VERY GOOD (VG)	((0.9475,0.985,0.9925,0.9925;0.9),(0.93,0.98,1,1;1))
ABSOLUTELY GOOD (AG)	((1,1,1,1;1),(1,1,1,1;1))

Step 2: Construct decision matrix by considering all possible paths of production project network as alternatives.

$$Y = (\tilde{f}_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{pmatrix} \tilde{f}_{11} & \cdots & \tilde{f}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{f}_{m1} & \cdots & \tilde{f}_{mn} \end{pmatrix} \end{array} \tag{11}$$

Where, $A_1, A_2, ..., A_m$ are all possible paths (alternatives) and $C_1, C_2, ..., C_n$ are evaluation criteria. Also, $0 \le j \le n, 0 \le i \le m$.

Step 3: Compute normalized decision matrix as follows:

$$\begin{aligned} \tilde{N}_{ij} &= \left[\left(\frac{a_{ij1}^L}{a^+}, \frac{a_{ij2}^L}{a^+}, \frac{a_{ij3}^L}{a^+}, \frac{a_{ij4}^L}{a^+}\right), \left(\frac{a_{ij1}^U}{a^+}, \frac{a_{ij2}^U}{a^+}, \frac{a_{ij3}^U}{a^+}, \frac{a_{ij4}^U}{a^+}\right)\right] \quad for \quad j \in benefit \\ \tilde{N}_{ij} &= \left[\left(\frac{a^-}{a_{ij4}^L}, \frac{a^-}{a_{ij3}^L}, \frac{a^-}{a_{ij2}^L}, \frac{a^-}{a_{ij1}^L}\right), \left(\frac{a^-}{a_{ij4}^U}, \frac{a^-}{a_{ij3}^U}, \frac{a^-}{a_{ij2}^U}, \frac{a^-}{a_{ij1}^U}\right)\right] \quad for \quad j \in \cos t \end{aligned} \tag{12}$$

Where, $a^+ = \max_i \left\{a_{ij4}^U\right\}, a^- = \max_i \left\{a_{ij1}^U\right\}$.

Step 4: In this step, an IVF entropy method is presented based on ratio of distances between each point from nearer and farther point among ideal points under IVFSs which is adopted from Zamri and Abdullah (2013) to determine weights of efficient criteria.

Step 4-1. Compute the value of ϕ_j presented as:

$$\phi_j = \frac{O_j}{\sum_{i=1}^{m} O_j} \tag{13}$$

Where

$$o_j = \frac{\sqrt{\sum_{j=1}^{n}\begin{bmatrix} (n_j^{near,L} - n_{1j})^2 + (n_j^{near,L} - n_{2j})^2 + (n_j^{near,L} - n_{3j})^2 \\ +(n_j^{near,L} - n_{4j})^2 + (n_j^{near,U} - n_{1j})^2 + (n_j^{near,U} - n_{2j})^2 \\ +(n_j^{near,U} - n_{3j})^2 + (n_j^{near,U} - n_{4j})^2 \end{bmatrix}}}{\sqrt{\sum_{j=1}^{n}\begin{bmatrix} (n_j^{far,L} - n_{1j})^2 + (n_j^{far,L} - n_{2j})^2 + (n_j^{far,L} - n_{3j})^2 + \\ (n_j^{far,L} - n_{4j})^2 + (n_j^{far,U} - n_{1j})^2 + (n_j^{far,U} - n_{2j})^2 + \\ (n_j^{far,U} - n_{3j})^2 + (n_j^{far,U} - n_{4j})^2 \end{bmatrix}}} \tag{14}$$

Step 4-2. Calculate the entropy value as follows:

$$EN_j = \left[-P\sum_{i=1}^{m} \phi_{ij} \ln \phi_{ij}\right] \tag{15}$$

P is a constant set as $(\ln(m))^{-1}$.

Step 4-3. Compute the degree of divergence as follows:

$$1-EN_j=\left[1-\left[-P\sum_{j=1}^{n}\phi_j\ \ln\phi_j\right]\right] \tag{16}$$

Step 4-4. Calculate the weights of efficient criteria by the following:

$$w'_j=\frac{\left[1-\left[-P\sum_{j=1}^{n}\phi_j\ \ln\phi_j\right]\right]}{\sum_{j=1}^{n}\left[1-\left[-P\sum_{j=1}^{n}\phi_j\ \ln\phi_j\right]\right]} \tag{17}$$

Step 5: Multiply the weight which has been obtained from IVF entropy method and equivalent interval-valued of linguistic weights that expert allots to each criterion. The final weight (FW) is computed by using the following:

$$F\tilde{W}_j=w'_j\otimes\tilde{w}_j=\begin{bmatrix}(w'_j\times w^L_{1j},w'_j\times w^L_{2j},w'_j\times w^L_{3j},w'_j\times w^L_{4j};W^L_{\tilde{A}}),(w'_j\times w^U_{1j},w'_j\times w^U_{2j},\\ w'_j\times w^U_{3j},w'_j\times w^U_{4j};W^U_{\tilde{A}})\end{bmatrix} \tag{18}$$

Step 6: Calculate the strength matrix S_{ij} by means of Eq. (19).

$$\begin{aligned}S^L_{ij}&=\sum_{i\neq K}P(\tilde{N}^L_{ij},\tilde{N}^L_{Kj})\\ S^U_{ij}&=\sum_{i\neq K}P(\tilde{N}^U_{ij},\tilde{N}^U_{Kj})\\ S_{ij}&=(S^L_{ij}+S^U_{ij})/2\end{aligned} \tag{19}$$

Step 7: Compute the weakness matrix I_{ij} by using Eq. (20).

$$\begin{aligned}I^L_{ij}&=\sum_{i\neq K}P(\tilde{N}^U_{kj},\tilde{N}^U_{ij})\\ I^U_{ij}&=\sum_{i\neq K}P(\tilde{N}^U_{kj},\tilde{N}^U_{ij})\\ I_{ij}&=(I^L_{ij}+I^U_{ij})/2\end{aligned} \tag{20}$$

Step 8: Calculate the weighted strength matrix $\tilde{\tilde{S}}_i$ and weighted weakness matrix $\tilde{\tilde{I}}_i$ as follows:

$$\tilde{\tilde{S}}_i=\sum_{j=1}^{n}\left[\frac{(S^L_{ij}+S^U_{ij})}{2}\otimes F\tilde{\tilde{W}}_j\right]=\sum_{j=1}^{n}\left[\frac{\sum_{i\neq K}P(\tilde{N}^L_{ij},\tilde{N}^L_{kj})+\sum_{i\neq K}P(\tilde{N}^U_{ij},\tilde{N}^U_{kj})}{2}\otimes F\tilde{\tilde{W}}_j\right] \tag{21}$$

$$\tilde{\tilde{I}}_i = \sum_{j=1}^{n}\left[\frac{(I_{ij}^L + I_{ij}^U)}{2} \otimes F\tilde{\tilde{W}}_j\right] = \sum_{j=1}^{n}\left[\frac{\sum_{i\neq K} P(\tilde{N}_{kj}^L, \tilde{N}_{ij}^L) + \sum_{i\neq K} P(\tilde{N}_{kj}^U, \tilde{N}_{ij}^U)}{2} \otimes F\tilde{\tilde{W}}_j\right] \tag{22}$$

Step 9: Compute the strength indexes S_i^L, S_i^U from the fuzzy weighted strength and weakness indices by using the following:

$$\begin{aligned} S_i^L &= \sum_{i\neq K} P(\tilde{S}_i^L, \tilde{S}_k^L) + \sum_{i\neq K} P(\tilde{I}_k^L, \tilde{I}_i^L) \\ S_i^U &= \sum_{i\neq K} P(\tilde{S}_i^U, \tilde{S}_k^U) + \sum_{i\neq K} P(\tilde{I}_k^U, \tilde{I}_i^U) \end{aligned} \tag{23}$$

Step 10: Calculate the weakness indexes I_i^L, I_i^U from the fuzzy weighted strength and weakness indices by using the following:

$$\begin{aligned} I_i^L &= \sum_{i\neq K} P(\tilde{S}_k^L, \tilde{S}_i^L) + \sum_{i\neq K} P(\tilde{I}_i^L, \tilde{I}_k^L) \\ I_i^U &= \sum_{i\neq K} P(\tilde{S}_k^U, \tilde{S}_i^U) + \sum_{i\neq K} P(\tilde{I}_i^U, \tilde{I}_k^U) \end{aligned} \tag{24}$$

Step 11: Aggregate lower and upper value of strength and weakness indexes as follows:

$$\begin{aligned} I_i &= (I_i^L + I_i^U)/2 \\ S_i &= (S_i^L + S_i^U)/2 \end{aligned} \tag{25}$$

Step 12: Aggregate strength and weakness indexes for obtaining total performance (p_i) as follows:

$$p_i = \frac{S_i}{S_i + I_i} \tag{26}$$

Step 13: Rank alternatives with larger total performance index to get higher level.

4. Application

In this section, an example of production project management to better address the calculation and capability of proposed method is designed and solved. Fig. 2 shows the network of production project. Also, Table 2 illustrates the expert's opinion on ratings versus criteria. Moreover, this table demonstrates expert's opinion about weight of criteria. In this example, time, cost, risk, quality and safety criteria are considered as efficient criteria for assessing critical paths.

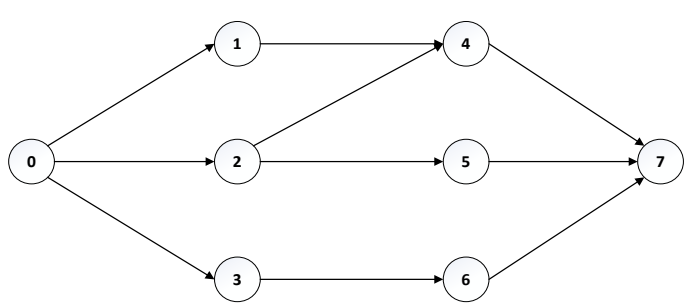

Fig. 2. Production project network

Table 2. IVF-ratings of activities on the time (days), cost (100 $), risk, quality and safety criteria

ACT.	TIME	COST	RISK	QUALITY	SAFETY
0-1	((2.5,3.5,4.5,5.5;0.9),(2,3,5,6;1))	((7,9,10,12;0.9),(6,8,11,13;1))	M	MP	P
0-2	((4.5,5.5,6.5,8;0.9),(4,5,7,9;1))	((6,8,9,11;0.9),(5,7,10,12;1))	MP	MG	P
0-3	((3.5,5,6,8;0.9),(3,4,7,9;1))	((2,3.5,4.5,7;0.9),(1,3,5,8;1))	MG	M	M
1-4	((3.5,5,6,7.5;0.9),(3,4,7,8;1))	((4.5,6,7,9;0.9),(4,5,8,10;1))	M	MG	MG
2-4	((4.5,5.5,6.5,7.5;0.9),(4,5,7,8;1))	((5,6.5,7.5,10;0.9),(4,6,9,11;1))	M	P	MP
2-5	((2.5,3.5,4,4.5;0.9),(2,3,4,5;1))	((6,7.5,8.5,10;0.9),(5,7,9,11;1))	P	M	M
3-6	((1.5,2.5,3.5,4.5;0.9),(1,2,4,5;1))	((3,4.5,5.5,7.5;0.9),(2,4,6,8;1))	G	M	MP
4-7	((3,5,6,7.5;0.9),(2,4,7,8;1))	((6,8,9,11;0.9),(5,7,10,12;1))	G	MG	G
5-7	((5.5,6.5,7.5,8.5;0.9),(5,6,8,9;1))	((4,7,8,11;0.9),(3,6,9,12;1))	P	M	MG
6-7	((2.5,3,3.5,4.5;0.9),(2,3,4,5;1))	((2,3.5,4.5,7;0.9),(1,3,5,8;1))	MP	MP	M
W	VG	M	G	VG	MG

Step 1: Convert expert's opinion to the equivalent IVF number by means of Table 1.
Step 2: Identify the all possible paths of production project network and consider these paths as alternatives. Then, construct decision matrix based on all possible path and efficient criteria such as time, cost, quality, risk and safety by using Eq. (11).
Step 3: Calculate decision matrix based on Eq. (12).
Step 4: Compute IVF entropy method for determining weights of criteria by using Eqs. (13) to (17) which is illustrated in Table 3.

Table 3. Final weights of criteria

CRITERIA	WEIGHT
TIME	0.226
COST	0.315
RISK	0.118
QUALITY	0.13
SAFETY	0.211

Step 5: Multiply weights of criteria which have been obtained from entropy method in the equivalent IVF number (linguistic variable) which is allot to each criterion by expert. This step is done by means of Eq. (18).
Step 6: Calculate the strength matrix S_{ij} by means of Eq. (19) which is shown in Tables 4-1 and 4-2.
Step 7: compute the weakness matrix I_{ij} by using Eq. (20) which is illustrated in Tables 4-1 and 4-2.

Table 4-1. Strength and weakness matrixes

STRENGTH MATRIX									
S_{i1}		S_{i2}		S_{i3}		S_{i4}		S_{i5}	
S_{11}	0.215	S_{12}	0.69	S_{13}	1.28	S_{14}	0	S_{15}	0
S_{21}	0.705	S_{22}	0.64	S_{23}	0.94	S_{24}	0.16	S_{25}	0.39
S_{31}	0.525	S_{32}	0.6	S_{33}	0	S_{34}	0.03	S_{35}	0.28
S_{41}	0	S_{42}	0	S_{43}	1.36	S_{44}	1.04	S_{45}	0.51

Table 4-2. Continued

WEAKNESS MATRIX									
I_{i1}		I_{i2}		I_{i3}		I_{i4}		I_{i5}	
I_{11}	0.37	I_{12}	0	I_{13}	0.03	I_{14}	0.51	I_{15}	0.99

I_{21}	0	I_{22}	0.02	I_{23}	0.37	I_{24}	0.29	I_{25}	0.04
I_{31}	0.06	I_{32}	0.06	I_{33}	3.18	I_{34}	0.43	I_{35}	0.15
I_{41}	1.05	I_{42}	1.85	I_{43}	0	I_{44}	0	I_{45}	0

Step 8: Calculate the weighted strength matrix $\tilde{\tilde{S}}_i$ and weighted weakness matrix $\tilde{\tilde{I}}_i$ by using Eqs. (21) and (22).

Step 9: Compute the strength indexes S_i^L, S_i^U from the fuzzy weighted strength and weakness indices by means of Eq. (23).

Step 10: Calculate the weakness indexes I_i^L, I_i^U from the fuzzy weighted strength and weakness indices by using Eq. (24).

Step 11: Aggregate lower and upper values of strength and weakness indexes by means of Eq. (25) which is demonstrated in Table 5.

Step 12: Aggregate strength and weakness indexes for obtaining total performance (p_i) by using Eq. (26) which is illustrated in Table 5.

Step 13: Alternatives with larger total performance index get higher level in the ranking order. The ranking of critical path is shown in Table 5.

Table 5. Final ranking of each path

LOWER AND UPPER OF STRENGTH AND WEAKNESS INDEXES AND TOTAL PERFORMANCE														FINAL RANKING
S_1^L	0.057	S_1^U	0.057	I_1^L	0.43	I_1^U	0.43	S_1	0.057	I_1	0.43	P_1	0.12	3
S_2^L	0.81	S_2^U	0.81	I_2^L	0	I_2^U	0	S_2	0.81	I_2	0	P_2	1	1
S_3^L	0	S_3^U	0	I_3^L	0.91	I_3^U	0.91	S_3	0	I_3	0.91	P_3	0	4
S_4^L	0.31	S_4^U	0.31	I_4^L	1.46	I_4^U	1.46	S_4	0.31	I_4	1.46	P_4	0.17	2

5. Conclusion

In this paper, a new multi-criteria decision making (MCDM) method based on fuzzy preference relation concept has been developed. Main advantages of the proposed method for assessing critical paths in production project management was strength and weakness scores based on relative comparisons using fuzzy preference relation, and relative comparison of the performances of production project paths. Also, interval-valued fuzzy sets (IVFSs) provide more degree of freedom to cope uncertainty of real world production projects that is why the MCDM method under IVFSs has been extended. Moreover, a new IVF entropy method by means of distance between distance between each point from nearer and farther point among positive ideal point and negative ideal point concepts under IVFSs has been presented. An application about critical path selection by considering efficient criteria such as time, cost, risk, quality and safety by using the proposed method has been solved to better illustrate its capability. The proposed method was useful to the project managers in terms of providing the total performance score of each path to measure performance of the paths on various criteria in a relative procedure. The results obtained in this study assisted the project managers to specify the critical path and also provide information regarding those activities which were critical enough to be given remarkable importance in their execution so that the project goals can be better attained.

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Biography

Yahya Dorfeshan is currently an MSc student at the Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran. His main research interests include quantitative methods in project management, fuzzy sets theory, multi-criteria decision-making under uncertainty, and applied operations research. He has published several papers in international journals and conference proceedings.

Seyed Meysam Mousavi is an Assistant Professor at the Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran. He received a PhD degree from the School of Industrial Engineering at University of Tehran, Iran, and is currently a member of Iran's National Elite Foundation. He is now the head of Industrial Engineering Department at Shahed University and a member of the Iranian Operational Research Association. His main research interests include cross-docking systems planning, quantitative methods in project management, engineering optimization under uncertainty, logistics planning and scheduling, multiple criteria decision making under uncertainty, and applied soft computing. He has published many papers and book chapters in reputable journals and international conference proceedings.

Ali Siadat is a Professor at Laboratoire de Conception, Fabrication Commande, Arts et Metier Paris Tech, Centre de Metz in Metz, France. His current research interests include: computer aided manufacturing, modeling and optimization of manufacturing processes, decision making in manufacturing, inspection planning, and operations research applications. He has published numerous papers and book chapters in the aforementioned fields.