

Mechanism Design For Pricing And Bandwidth Allocation In Heterogeneous Wireless Networks To Maximize The Social Welfare

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Abstract—In this paper, we present a mechanism for bandwidth allocation and pricing in heterogeneous wireless networks. A proposed three stage game involves three types of agents 1) Network providers 2) Regulatory agent and 3) The Clients play the game. The goal of our mechanism design is increasing the social welfare means that while the network providers are satisfied from preparing and releasing the bandwidth, the clients deliver their bandwidth with a suitable price. When the Purchasing power in a society increases, we can interfere that the client welfare is increased but the very-low price for the products is not profitable for the producers thus to increase the social welfare we should attend to the welfare of network providers as well as the clients. The proposed mechanism guarantees the maximum payoff for the network providers and offers the clients to weighting and ranking the available networks and then prepares the best request bundle to receive the service. The violation from the proposed mechanism is not profitable for network providers and clients and as it is proved in the paper the allocation result is Pareto efficient.

Keywords- *Heterogeneous wireless networks; Micro Economic; Social welfare; Walrasian Equilibrium.*

I. INTRODUCTION

The existence of different wireless technologies and different service providers from one side and the growth of multi modal smart devices that are able to connect to different available networks simultaneously from the other side generate an environment named heterogeneous wireless network. The concept of 'Always Best Connected' [1], means that the clients with multi modal devices like to receive the service from the network provider that increases their payoff and a client is as enough smart as to select the best network. The payoff of clients may differ according to their application requirement, device state and the manner of the client. So according to the three important profiles 1) Application profile, 2) Device profile and 3) The client profile, the available networks are ranked and weighted by the clients.

One of the most important decisions for the network provider is the pricing mechanism. The network providers have to monitor the state of the network, continuously. The number of clients, total delivered bandwidth and etc. are the acquired

data from the current state of the network and then it should adjust the price and the amount of prepared bandwidth. Through pricing mechanism, a network provider likes to increase its payoff by increasing the price while it knows that clients like to receive (buy) the bandwidth with minimum charge. In trading both buyer and seller should reach to the benefit, in other word the trading must be profitable for both buyer and seller. There are two main categories for pricing [2]:

Category 1: The price is aimed from the intersection point of the demand and supply curves;

Category 2: The competition between agents determines the price;

In first category all products are homogeneous, and the price is unique for the products from all firms in a global market; such markets are named Perfect Competition markets. But in second category, there is a few numbers of firms and each firm can offer its own price for its product because products are not completely similar and the cost of product is different for different firms which is named Oligopoly market. For more details, we refer you to Friedman book on the topic of Oligopoly markets and game theory [3].

In oligopoly markets to the pricing mechanisms are involved with two major behaviors: first, the competition between clients and providers and second, the competition of different providers to increase the portion of the market.

Therefore the heterogeneous wireless system is the same as the oligopoly market, the network providers compete with each other to offer their service to the clients through price adjustment while the network provider has to compete with the client to adjust the price and the amount of delivered bandwidth.

A. Related Works

The pricing problem in wired networks was considered in [4], where price and transmission rate were optimized for elastic traffic but network heterogeneity and service competition were not considered. Niyato and Hossain in [5], suggested a leader follower competitive game model within providers to find the optimum prices according to the concept of oligopoly markets. Sallent et al., in [6], proposed an auction based mechanism that users periodically bid the amount of service,

price and the QoS requirement for the provider. Then, the service provider decides on the resource allocation that maximizes its revenue. To approve this multi-unit sealed-bid auction, they used a manager agent that facilitates negotiation between a mobile user and a service provider. Chan et al., proposed a resource allocation framework for heterogeneous wireless networks based on the concept of demand/supply in microeconomics in [7]. The demand function was obtained by solving a utility maximization problem for a user, whereas the supply function was obtained by solving an optimization formulation to maximize the revenue of a service provider. The price, at which the demand equals the supply, was defined as the equilibrium. This work did not consider the competition among service providers because it assumed the perfect competitive market.

In economic theory, perfect competition (sometimes called pure competition) describes markets such that firms are not large enough to have the market power to set the price of a homogeneous product. In a perfectly competitive market system, Consumers' market behavior is followed by their personal self-interest, and each agent (buyer or seller), cannot effect on prevailing prices. Equilibrium in the market system is achieved when the demands of buyers match the supplies of sellers at prevailing prices in every market simultaneously [8].

Duan et al. in [9] studied the mechanism of pricing in global Wi-Fi, whereas many Wi-Fi providers, provide high performance mobile communication experiences, (e.g., AT&T in US, BT Openzone in UK, and PCCW in HongKong) are deploying a large number of Wi-Fi APs in their local markets. They suggest a two-stage Stackelberg game between provider i and a group of N_i local users to find the price and the amount of released service. In first Stage, the provider i announces the price p_i to maximize its revenue. In second Stage, users decide whether and how much to use the service to maximize their payoffs. Finally the equilibrium price is calculated locally and the welfare of clients increases when such usage based pricing is employed. Again Duan et al. in [10] proposed an extended model. In previous model local providers were ignored while in new suggested pricing model, local Wi-Fi providers exist and they tend to cooperate with global Wi-Fi providers and

give up part of their income to global Wi-Fi provider. They study how a global Wi-Fi provider (e.g., Skype) cooperates with many local providers using their Wi-Fi infrastructures to provide a global Wi-Fi service.

II. SYSTEM MODEL

Figure 1 shows a case of considered environment for the heterogeneous wireless access network with three different service providers. Providers cover different areas with their BTSs. A region is generated by dominating the intersection of coverage areas (e.g., the region #2 is the area that is covered by only Net1-B1 and Net2-B2). Thus we have seven different regions in our sample (Table 1).

Table 1- Dominated regions for environment proposed in Figure 1

Region number	Existing Network	BTS
R1	N1	Net1-B1
R2	N1,N2	Net1-B1, Net2-B1
R3	N1,N2	Net1-B1, Net2-B2
R4	N1,N2,N3	Net1-B1, Net2-B1, Net3-B1
R5	N1,N2,N3	Net1-B1, Net2-B1, Net3-B2
R6	N1,N3	Net1-B1, Net3-B3
R7	N1,N2,N3	Net1-B1, Net2-B2, Net3-B3

Let $\mathcal{K} = \{1, \dots, K\}$ as the index set of K regions. And q_i determines the required amount of bandwidth for client $i \in \mathcal{I}$ where $\mathcal{I} = \{1, \dots, m\}$ is the index set of m clients. Let $p_{j,k}$ denotes the price per unit of bandwidth offered by network provider $j \in \mathcal{J}$ in region k where $\mathcal{J} = \{1, \dots, n\}$ is the index set of n different network providers. Also, $x_{i,j}$ denotes the assigned bandwidth to client $i \in \mathcal{I}$ from network provider $j \in \mathcal{J}$. In the following we present a game model and action plan for all agents to find the $x_{i,j}^*$ and $p_{j,k}^*$ that maximizes the social welfare.

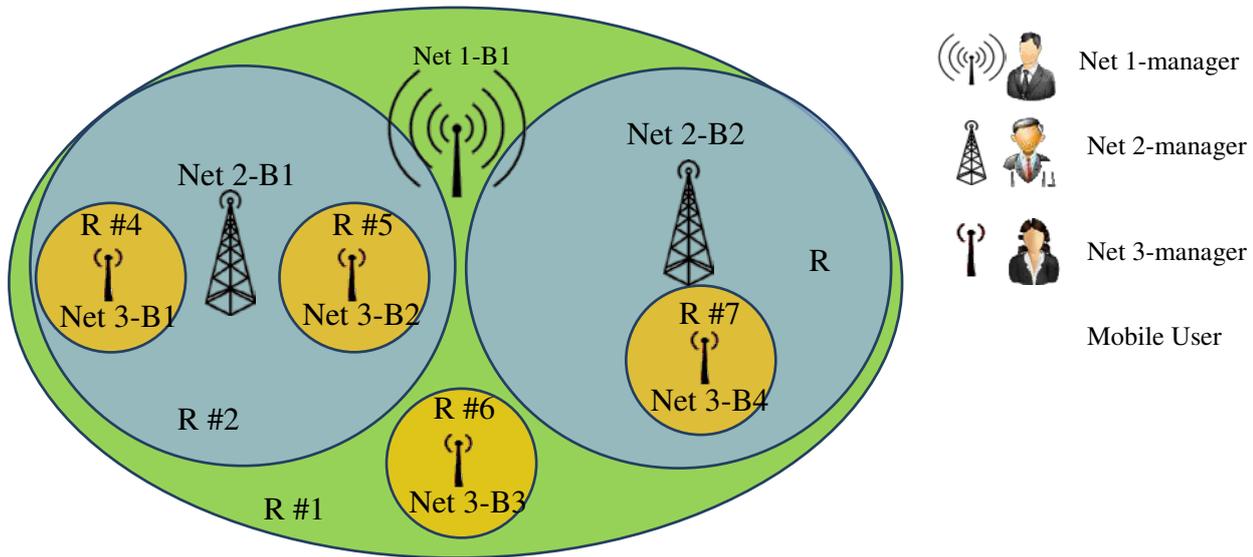


Figure 1- The schema of different regions in heterogeneous wireless networks

There are two types of clients in a region: 1) Direct clients and 2) Guest Clients.

1) Direct Clients: These clients are connected directly to their selected network and the account management and billing is done by the same network provider.

2) Guest Clients: Clients can not directly connect to their selected network so they connect to a middle network as a guest client and the account management and billing is done with the selected network.

A. Solution Overview

Our goal is designing a mechanism to increase the social welfare. So, we should have some tools to guide the equilibrium of the market toward our goal. If we consider the decisions in the heterogeneous wireless network as repeated events, our approach is using a three-stage game in iteration, with regard to the dependency of the iterations. The dependency of iterations means that the payoffs of previous iterations does not affect on the current iteration and in game theory models that means the discount factor of the game is zero ($\delta = 0$). In first and second stages we have a Bargaining-like game the network providers offer their price of a unit of bandwidth to the regulatory agent. The players in the first stage are the network providers. In second stage the regulatory agent considers the rate of penalty or reward for the proposed prices or the regulatory agent may reject the offered price (As the rule of the Bargaining game). Finally in the third stage where we have a leader-follower-like game, the clients decide on the amount of bandwidth to receive from different network providers.

As depicted in Figure 2, in the first stage, the network providers offer the price of a unit of bandwidth according to

the used bandwidth in previous iteration. We assume that the regulatory agent is aware of the Marginal Cost curves and it considers the penalty or reward to encourage the network providers to offer the true values for the prices. In third stage, the clients do prioritization and weighting the available networks and then decide on the amount of service to receive from each of available network providers.

In the following sub parts we present the model and the strategy of providers, regulatory agent and clients and then combine these strategies in a closed form model named DMBAP (Designed Mechanism for Bandwidth Allocation and Pricing) for heterogeneous wireless network.

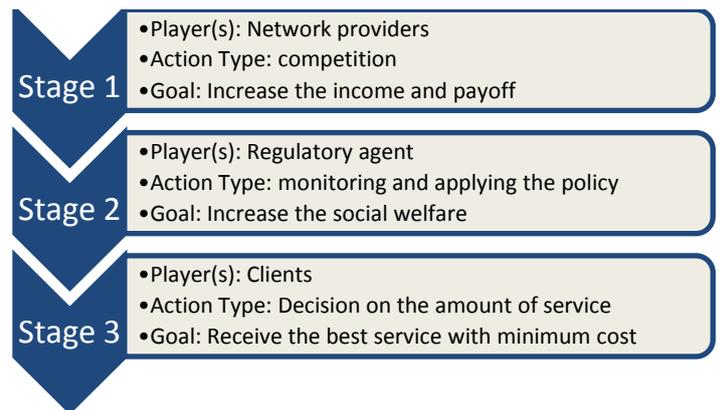


Figure 2- Three stages in one iteration of repeated decisions

The extensive form of our proposed game is showed in Figure 3.

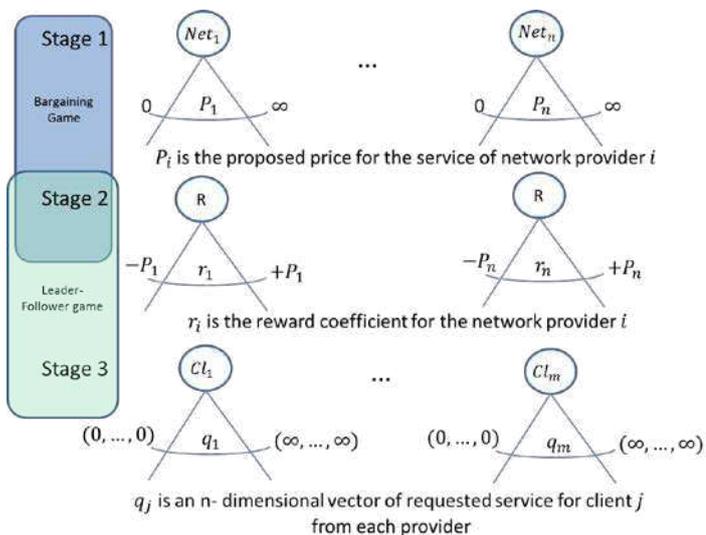


Figure 3- extensive game model for the proposed three stages game

B. Roaming Model

As mentioned before, there are two types of clients for a network provider: Direct Clients and Guest Clients. So each network has three prices for its service:

- P_D : is the price of a unit of bandwidth for clients that are directly connected to one of the BTSs of the network;
- P_M : is the price of a unit of bandwidth for accounting and billing information. The clients whom can not connect directly to one of the BTSs of the network so they should pay only the membership price;
- P_G : is the price of a unit of bandwidth for clients that are connected to a BTS of the network but this network is not their selected one.

The value of P_M and P_G for the network j is calculated from its P_D as follow:

$$P_G^j = \frac{2}{3}P_D^j, \quad P_M^j = \frac{1}{2}P_D^j \quad (1)$$

Also the Load of the network j is the combination of Load from direct clients, member clients and guest clients as follow:

$$L_j = L_D^j + \frac{2}{3}L_G^j + \frac{1}{3}L_M^j \quad (2)$$

where L_D is the load of the direct clients, L_G is the load of the guest clients and L_M is the load of member clients that could not connect to the network directly.

We suppose that according to the investment of different network providers they had bought the spectrums so they have different abilities to cover clients. Some of them own many spectrums so their clients can connect to them easily. Also some of network providers may have fewer spectrums but they prepare better user facilities that attract clients. Thus, we suppose there is an $n \times n$ Roaming Matrix (RM). The element RM_{ij} determines the rate of clients that use the network j as

the middle network to use the service of network i and RM_{ii} is the rate of clients that are directly connected to the network provider i BTSs and the summation of elements in a row of RM is unique:

$$\forall i \in \{1, \dots, n\} \sum_{j=1}^n RM_{ij} = 1. \quad (3)$$

Suppose a client uses the roaming service and its selected network is i while it connects to a BTS of network j . So this client is a guest in network j and it should pay P_G^j to the network j for its connection and pay P_M^i to the network i for its membership management for a unit of bandwidth.

C. Stage 1: The competition of Network providers

In the first stage of our game we have a competition of different network providers; each network tries to increase its payoff. The network providers should offer the price of a unit of bandwidth that is provided for direct clients (P_D^j). A network offers the price of a unit of bandwidth according to the used amount of bandwidth in previous iteration or previous step. This stage of the game is the same as the Cournot game that all firms should decide simultaneously. The proposed strategy for network providers is using the MC curve to determine the price of a unit of bandwidth:

$$P_j(t) = P_D^j(t) = MC_j(L_j(t-1)). \quad (4)$$

Where $P_j(t)$ and $P_D^j(t)$ are the price of service for direct clients and $L_j(t-1)$ determines the load of the network j in previous step. It is not profitable for a network to violate this mechanism because the regulatory agent would consider the penalty for its.

In micro economic, each firm and supplier has a Marginal Cost (MC) curve which is a U-shaped and specifies the cost of a unit of product against the total amount of the product. In the stationary state of a market, the price of a unit of product is equal to the cost of production of it and thus the amount of product and the price of a unit of product determine a single point on the MC curve [8]. Figure 4 draws a MC curve and it shows how a producer adjusts the price of a unit of product. Suppose that we are a firm and already we are at the position $A(Q_A, P_A)$ in Figure 4, and then with some changes in the market we have to increase our product to Q_B ; hence, the cost of product increases and it is not profitable for us to sell our product with the previous price P_A . So, we have to increase the price of our product to P_B according our MC curve.

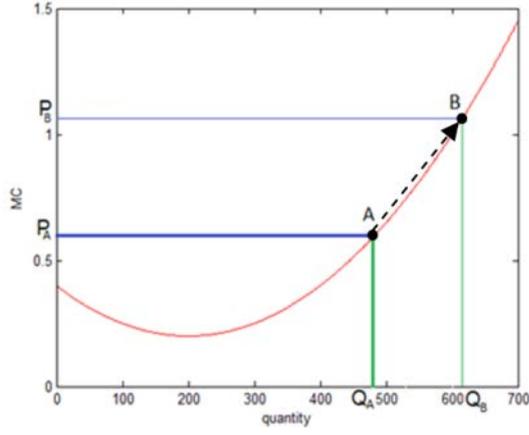


Figure 4- Adjustment of the price of the product according to the changes of the quantity of the release product

The goal of the network provider is to maximize its total profit. Since the profit of a firm is the difference between total revenue and total cost. Total cost (TC) of a firm is the integral of Marginal cost curve plus a static cost ($MC = \partial TC / \partial Q$) and total revenue is the production of the amount of product and the price (Figure 5).

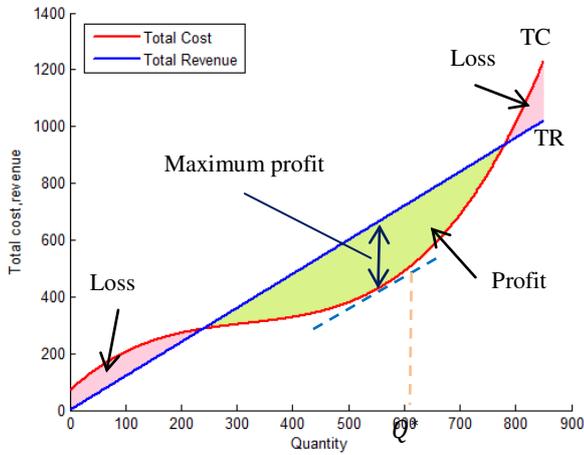


Figure 5-Total Revenue and Total Cost Curves and the maximum profit

The profit of a provider maximizes when the vertical distance between total revenue and total cost is greatest and at that quantity the profit meets its maximum value for a given price. At the best quantity (Q^*) the Marginal Revenue (MR) (the slope of the total revenue curve) and MC (the slope of the total cost curve) are equal, since the marginal revenue is price of bandwidth ($MR(Q^*) = MC(Q^*) = P$). Figure 5 shows that the Total Revenue (TR) and Total Cost (TC) Curves and the quantity where the provider profit is maximized. Thus, if we apply a strategy that guarantees for network provider j that the price of service (P_j) for the amount of service (L_j) be equal or

more than $MC_j(L_j)$, then it is incentive for the network providers to follow such strategy.

Our proposed strategy for network providers is that they announce the price according to the information of previous step of the game. Each network calculates the traffic load according to the relation (2) and using its MC curve it offers the price to the regulatory agent.

D. Regulatory Agent Decision

The goal of the regulatory is increasing the social welfare; means that while the satisfaction of network providers is obtained, the clients receive the service with a suitable price. The quotient of the paid price by the clients to the amount of delivered bandwidth which can be considered as a measure of clients' welfare which is named PBR (Price to Bandwidth Ratio). To evaluate the clients welfare of two different situations, the one with the less PBR is better than the other one because the clients should spend lower amount of money to receive the same service.

We suppose that the regulatory agent receives the commercial model of the network provider then it grants the license to the provider to prepare the service for clients. Thus the regulatory knows the amount of investment of a provider and its pricing model. So, if a network provider offers a price higher than the expected price, then the regulatory agent will consider a penalty for it. Suppose \tilde{P}_j is the predicted price for network provider j by the regulatory and P_j is the offered price by the network provider j ; the amount of penalty for network provider j (ϑ_j) is calculated as follow for a unit of soled bandwidth:

$$\vartheta_j = \begin{cases} 0 & ; \text{ if } P_j \leq \tilde{P}_j \\ 2 * (\tilde{P}_j - P_j) & ; \text{ otherwise.} \end{cases} \quad (5)$$

This form of penalty enforces the network providers to be honest in their price offering.

E. Clients Strategy

Intelligent clients do ranking and weighting the available networks and then send their requests amount of bandwidth to receive the service. A client tries to prepare a request bundle that maximizes its payoff.

We used the TOPSIS¹ approach for weighting and ranking the available network providers. All clients access the networks' status vectors. The network status vector, involves Delay (D), Signal to noise ratio (SNR), Signal Strength (SS), Accessibility (A), Stability (S), Cost (C), Load (L). The TOPSIS approach is a MCDA method for dynamic weighting of available strategies for a problem and in our case it is a method to weighting the available networks according to networks' state vectors. There exists many publication that use TOPSIS to weighting the strategies or solutions [11, 12]. Suppose Q_i is the required bandwidth for client i . It is brief that, this requirement will be provided at any cost by the client

¹Technique for Order Preference by Similarity to Ideal Solution

but the clients tries to minimize its cost while maximizes its payoff.

Suppose $\mathcal{J} = \{1, 2, \dots, n\}$ is the index set of n available network providers and $\mathcal{I} = \{1, 2, \dots, m\}$ is the index set of m active clients. Also assume w_{ij} shows the weight of network provider $j \in \mathcal{J}$ as the sight of client $i \in \mathcal{I}$ and x_j^i is the bandwidth allocated to client i from network provider j .

To initiate the optimization problem we should know the client's budget but we said that the client provide its requirement at any cost that means the budget of client is unlimited. In reality the budget is not unlimited and the client specifies a limited budget for its requirement according to the norm of all providers' prices. Thus the client should calculate the norm of price for a unit of bandwidth. The client is faced with a set of prices received from different network providers. As the site of a client, the price of a network which has a higher weight is more reliable, hence the norm price of a unit of bandwidth is calculated for each client as follow:

$$\bar{P}^i = \frac{\sum_{j=1}^n w_{ij} P_j}{\sum_{j=1}^n w_{ij}} \quad \forall i \in \mathcal{J}. \quad (6)$$

And the budget of client i (H^i) is calculated as follow:

$$H^i = \bar{P}^i Q_i \quad \forall i \in \mathcal{J}. \quad (7)$$

The utility function of client i is diffied as follow:

$$u^i(\mathbf{x}^i) = \left(\sum_{j \in \mathcal{J}} \sqrt[3]{w_{ij} x_j^i} \right)^3 \quad \forall i \in \mathcal{J} \quad (8)$$

$$\sum_{j \in \mathcal{J}} w_{ij} = 1 \quad \forall i \in \mathcal{J}.$$

The used utility function is a popular function which is widely used in micro economic theories and it is familiar with the **CES utility function**. The CES utility functions are suitable mathematical models for natural behavior of clients. Essentially, the CES utility function is the normal weighted average of payoffs and one can easily verify that this utility function represents preferences that are strictly monotonic and strictly concave.

Each user in region k solves the bellow optimization problem to find the best request bundle:

$$\begin{aligned} & \forall i \in \mathcal{J}_k: \text{Maximize } u^i(\mathbf{x}^i) \\ & \text{s.t.} \\ & 1) \mathbf{P}_k \cdot \mathbf{x}^i \leq H^i \\ & 2) \mathbf{x}^i \in \mathbb{R}_+^n. \end{aligned} \quad (9)$$

Where $\mathbf{P}_k = (P_{1,k}, P_{2,k}, \dots, P_{n,k})$ is the price vector of all network providers in region k and \mathcal{J}_k denotes the index set of clients located in region k . The first constraint in(9) simply expresses the client's budget constraint. The solution $\mathbf{x}^i(\mathbf{P}_k, H^i)$ to (9) is the client's request bandwidth bundle, which depends on the prices and the client's budget.

Theorem 1: The client request bundle is unique for each \mathbf{P} and budget H^i

The function $\mathbf{x}^i(\mathbf{P}, H^i)$ (the problem (9)) has a unique answer for each price vector \mathbf{P} .

Proof:

This is a non-linear programming problem with two inequality constraint. As we have noticed, an answer \mathbf{x}^{i*} exists and is unique. After framing the Lagrangian equalities, we acquire:

$$\mathcal{L}(\mathbf{x}^i, \lambda) = u^i(\mathbf{x}^i) - \lambda[\mathbf{P} \cdot \mathbf{x}^i - H^i]. \quad (10)$$

Accepting that the solution \mathbf{x}^{i*} is strictly positive (second constraint), we can use the Kuhn-Tucker method to solve it[13]. If $\mathbf{x}^{i*} \gg \mathbf{0}$ solves(9), then by Kuhn-Tucker Theorem, there exists a $\lambda^* \geq 0$ such that $(\mathbf{x}^{i*}, \lambda^*)$ fulfill the bellow Kuhn-Tucker conditions:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_j^i} = \frac{\partial u^i(\mathbf{x}^{i*})}{\partial \mathbf{x}_j^i} - \lambda^* \mathbf{P}_j = 0, \quad j = 1, \dots, n \quad (11)$$

$$\mathbf{P} \cdot \mathbf{x}^{i*} - H^i \leq 0 \quad (12)$$

$$\lambda^* [\mathbf{P} \cdot \mathbf{x}^{i*} - H^i] = 0. \quad (13)$$

As the utility function $u^i(\mathbf{x}^i)$ is strongly increasing so it is strictly monotonic, thus the constraint(12) is satisfied with equality, and (13) is implicitly established. Hence, these conditions lessen to:

$$\frac{\partial u^i(\mathbf{x}^{i*})}{\partial \mathbf{x}_j^i} - \lambda^* \mathbf{P}_j = 0, \quad j = 1, \dots, n \quad (14)$$

$$\mathbf{P} \cdot \mathbf{x}^{i*} - H^i = 0.$$

Now, suppose that $\nabla u^i(\mathbf{x}^{i*})$ exists and $(\mathbf{x}^{i*}, \lambda^*) \gg \mathbf{0}$ solves(14). Then

$$\nabla u^i(\mathbf{x}^{i*}) = \lambda^* \mathbf{P} \quad (15-a)$$

$$\mathbf{P} \cdot \mathbf{x}^{i*} = H^i. \quad (15-b)$$

If \mathbf{x}^{i*} is not utility-maximising, then there must be some $\mathbf{x}^{i'} \geq \mathbf{0}$ such that

$$\begin{aligned} u^i(\mathbf{x}^{i*}) &< u^i(\mathbf{x}^{i'}) \\ \mathbf{P} \cdot \mathbf{x}^{i'} &\leq H^i. \end{aligned} \quad (16)$$

Because u^i is continuous and $H^i > 0$, for some $t \in [0, 1)$ close enough to one. Letting $\tilde{\mathbf{x}} = t\mathbf{x}^{i'}$ the preceding inequalities imply that

$$u^i(\mathbf{x}^{i*}) < u^i(\tilde{\mathbf{x}}) \quad (17-a)$$

$$\mathbf{P} \cdot \tilde{\mathbf{x}} < H^i. \quad (17-b)$$

As u^i is quasiconcave, we know that if $u(x') \geq u(x)$ then $\nabla u^i(x)(x' - x) \geq 0$, so we have

$$\text{The fact is: } \nabla u^i(\mathbf{x}^{i*})(\tilde{\mathbf{x}} - \mathbf{x}^{i*}) \geq 0 \quad (18-a)$$

$$\nabla u^i(\mathbf{x}^{i*})(\tilde{\mathbf{x}} - \mathbf{x}^{i*})$$

$$(15-a) \rightarrow = (\lambda^* \mathbf{P}) \cdot (\tilde{\mathbf{x}} - \mathbf{x}^{i*}) \quad (18-b)$$

$$= \lambda^* (\mathbf{P} \cdot \tilde{\mathbf{x}} - \mathbf{P} \cdot \mathbf{x}^{i*}) \quad (18-c)$$

$$(15-b), (17-b) \rightarrow < \lambda^* (H^i - H^i) = 0. \quad (18-d)$$

It is brief that the result (18-d) contradicts with the fact at the beginning of the proof. So, the client's problem (9) has a unique solution, $\mathbf{x}^{i*} = \mathbf{x}^i(\mathbf{P}, H^i)$.

□

III. THE INCENTIVE AND PREVENTIVE FACTORS OF THE PROPOSED STRATEGY

A strategy to be valid is required to be incentive for its players. If a strategy could not make enough incentives for the players then it may not be followed by the players and undoubtedly following the strategy action plans would not be profitable for the players and they would select the actions that are not meaningful for the strategy designer.

In our three stage game, the punishment rule applied by the regulatory agent in second stage makes the network providers to be honest in their offering prices. The network providers know that if they do not comply in offering the fair price, they will be penalized. The strategy designer should prove that the rate of penalty for a network who offers a high price is more than its additional income.

Suppose the price of a unit of bandwidth for some network provider j for the load L_j is P_j but the network provider offers the price \check{P}_j ($\check{P}_j > P_j$). Suppose, ψ shows the additional income for the network:

$$\psi_j = (\check{P}_j - P_j) * L_j. \quad (19)$$

According to the strategy of the regulatory agent in (5) the penalty ϑ_j is considered for this network provider and .

$$\vartheta_j = 2 * (\check{P}_j - P_j) * L_j. \quad (20)$$

It is brief that for any $\check{P}_j > P_j$, the amount of penalty is more than the income ($\vartheta_j > \psi_j$) so it is not profitable for the network j to offer a price, higher than the fair price.

IV. REAL INCOME AND REAL PAYMENTS

According to the concept of the roaming service, one client should spend more money if it could not connect to its selected network. Suppose, there are two different network providers, l with similar service price for a unit of bandwidth $P_k = P_l = P$ and the client i selects the network k but according to some limitations it uses the network l BTS. So the client should pay $P_M^k + P_G^l$ for a unit of received bandwidth. Using the relation (1) the payment of the client is $\frac{1}{2}P_D^k + \frac{2}{3}P_D^l$ for a unit of bandwidth. According to the equality of prices for two networks, the client should pay $\frac{1}{2}P + \frac{2}{3}P = \frac{7}{6}P$ for a unit of bandwidth while if the client was connected directly to the network k it did pay the price P for a unit of bandwidth.

Now suppose the network j with direct load L_D^j , guest load L_G^j and membership load L_M^j . Also suppose $L_G^j = L_M^j$ means that the amount of bandwidth for the guest clients who used the network j as a middle network is equal to the amount of bandwidth for the clients of network j whom can not directly connect to network j . In this case using the relation (2), the

real load of network j is $L_j = L_D^j + \frac{1}{3}L_M^j + \frac{2}{3}L_G^j$ and according to the equality of L_G^j and L_M^j , we can write

$$L_j = L_D^j + L_M^j. \quad (21)$$

while the income of the network π_j is $L_D^j * P_D^j + L_G^j * P_G^j + L_M^j * P_M^j$ and using the relations (1),(2) and equality of L_G^j and L_M^j we can write the income of network j as:

$$\pi_j = L_D^j * P_j + L_M^j * \frac{7}{6}P_j. \quad (22)$$

Using the relations (21),(22) the average income per a unit of bandwidth is

$$\frac{\pi_j}{L_j} = \frac{L_D^j + \frac{7}{6}L_M^j}{L_D^j + L_M^j} * P_j. \quad (23)$$

If all clients of network j were directly connected to the network j and there was no any guest client for network j then its load was $L'_j = L_D^j + L_M^j$ and the income was $\pi'_j = (L_D^j + L_M^j) * P_j$ and the average income of the network for selling a unit of bandwidth was P_j . It is brief that in two proposed situations the average income per a unit of bandwidth in (23) is higher than P_j .

V. SIMULATION

A. *Simulation I: All network providers are honest and follow the strategy*

Suppose an environment with three different regions and $m = 50$ clients and $n = 4$ network providers. Number of clients in each region is:

Table 2- Heterogeneous wireless network environment

Region ID	Number Of Clients	Total Request Bandwidth
R1	14	1180
R2	19	897
R3	17	445.8

And suppose the roaming matrix is the same in all regions and it is as follow:

$$\begin{aligned} & \text{Roaming Matrix: } RM \\ & = \begin{bmatrix} 0.6606 & 0.0437 & 0.2382 & 0.0575 \\ 0.3363 & 0.4903 & 0.0854 & 0.0880 \\ 0.1119 & 0.1720 & 0.5901 & 0.1260 \\ 0.0684 & 0.1254 & 0.1727 & 0.6335 \end{bmatrix}. \end{aligned} \quad (24)$$

As the result of simulation is depicted in Figure 6, the deliver price per a unit of bandwidth for all service providers is above their marginal cost that means their payoff is more than their revenue is higher than their expectation. Another result aimed from the Figure 6 is that the price of a unit of bandwidth paid by the clients (P'_j) is not equal to the income of the network for a unit of bandwidth (P_j) because of the roaming capability.

Also we can find that the paid price of the bandwidth for the network with the highest price (Network N_3 in our example) is less than its MC curve ($P'_3 < MC(L_3) < P_3$) while for all other networks ($\forall j \in \mathcal{J} \setminus \{3\} P'_j > P_j$). The reason of the later observation is the roaming concept. The clients whom select the network N_3 and they could not connect directly to the network N_3 's access points or BTSs had to connect one of the other networks (about 40% of clients). In our simulation the prices of all other networks is less than the network N_3 ; thus the member clients of network N_3 spent less charge to their middle network as the guest client and so their average paid amount is less than the Marginal Cost of network N_3 (MC_3).

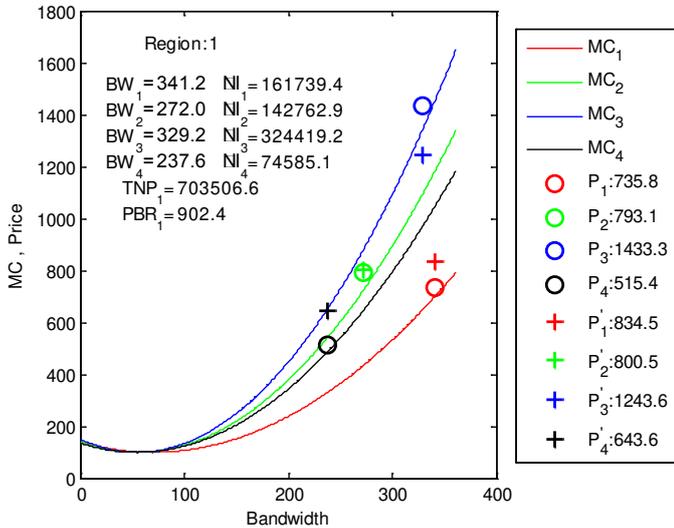


Figure 6- Result of simulation I in region R1

Also it should be added that we used random weights for clients for the features of network-status vectors in our TOPSIS approach. The result of simulation I in region R_2 and R_3 are showed in Figure 7.

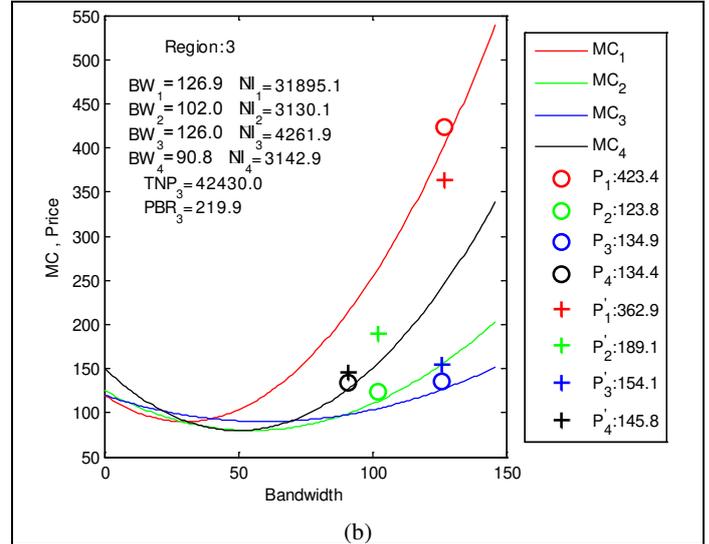
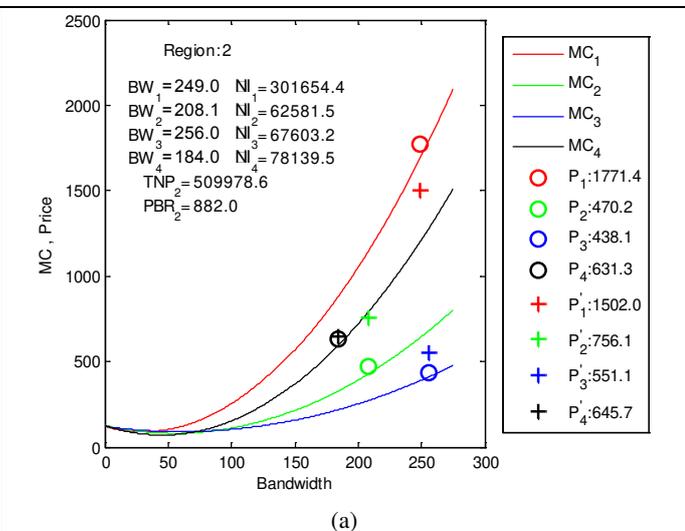


Figure 7- Result of simulation I in regions R_2, R_3

B. Simulation II: All network providers except network 1 are honest

Suppose the environment presented in Table 2 that the network N_1 does not follow the designed strategy and offers a price more than its real marginal cost. According to the outlined strategy, the regulatory agent will consider a penalty for wrongdoing network. Figure 8 shows the result of bandwidth allocation and pricing in region R_1 . The most change rather than Figure 6, is the high price value for network 1. In Figure 6, the price of a unit of bandwidth for network 1 was 735.84 while it is 1029.17 in Figure 8.

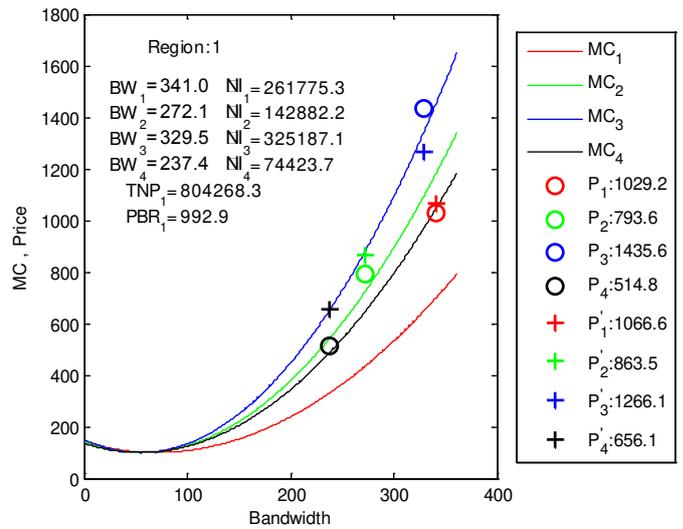


Figure 8- Result of simulation II in region R_1 when network N_1 offers a higher value for the price

The result of simulation II in region R2 and R3 are showed in Figure 9.

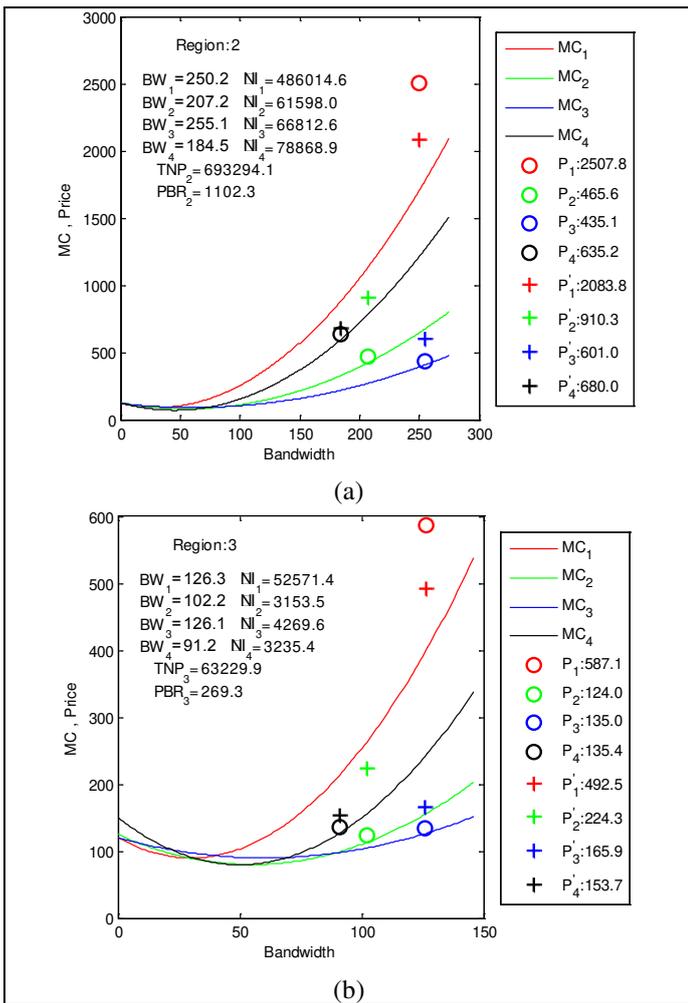


Figure 9- Result of simulation II in region 2, 3 when network N1 offers a higher value for the price

Table 3 shows the payoff of the Network N1 in two different simulations. We can find that while the behavior and request of clients and the state vector of network providers instead of price and load have no change, the network income of network N1 increases when it offers a higher price. The regulatory agents explores that the network N1 is offering an out-of-range price for a unit of bandwidth so it considers a penalty for the network N1. The comparison of the result of two simulation shows that it is not profitable for network providers to offer a higher value for the price of a unit of bandwidth.

Table 3- Comparison the payoff of the network N1 in two simulations in region R1

	Total bandwidth	Price Per unit of BW	Total Revenue	Total Cost	Network Income	Network Penalty	Network Payoff
Sim I	341	735.84	251068.6	89329	161739	0.0	161739.3
Sim II	341	1029.17	350998.7	89223	261775	170729	91046.34

Figure 10 shows the total cost curve, total income and total revenue of network N1 in region 1 in simulation I, II. The point A(341.199,251068.60) shows the state where network N1 follows the proposed strategy and offers a trust price for a unit of bandwidth and it gets the payoff 161739.28 but if the network N1 violates the proposed strategy and offers a higher price value, its income increases and moves to point B(341.047,350995.28) but the regulatory agent considers 102436.06 unit of money as the penalty; so the state of network N1 moves to C(341.047,180269.21) and payoff of network N1 reduces to 91046.34.

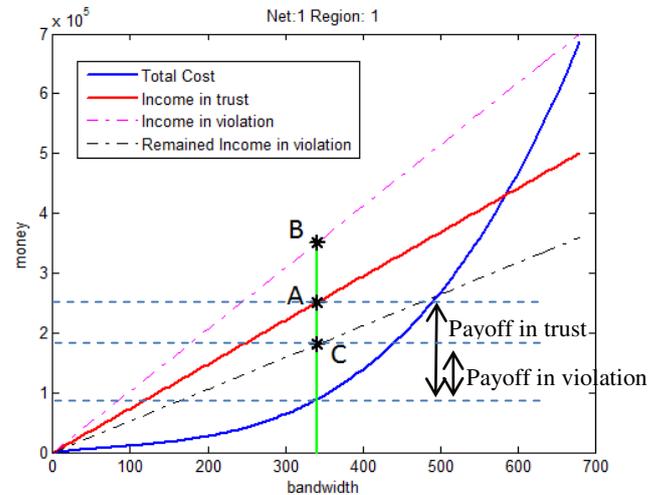


Figure 10- Total Income and Final Payoff of network N1 in two simulations

Table 4 shows the income of all networks in two simulations. As depicted in Table 4, the network payoff of network N1 decreases if it violates the proposed strategy and offering an out-of-range price for a unit of bandwidth is not profitable for any network.

Table 4- summarization of payoffs for all networks in all regions in two simulations

□

	Simulation #	Total Delivered	Total Cost	Total Income	Total Penalty	Network Payoff
Net 1	Sim I	746195	250507	495688	0.0	495688
	SimII	1052684	252323	800361	506013	294347
Net 2	Sim I	326172	117743	208428	0.0	208428
	Sim II	325055	117421	207633	0.00	207633
Net 3	Sim I	600999	204743	396256	0.0	396256
	Sim II	600984	204715	396269	0.00	396269
Net 4	SimI	250773	94946	155827	0.0	155827
	Sim II	251736	95208	156527	0.00	156527

As mentioned before, the value of PBR shows the welfare of clients. As depicted in Table 5, the PBR value in the Simulation I is less than what is in the result of the Simulation II so we can say that the proposed strategy is more incentive for clients.

Table 5- clients' welfare in two simulations

	PBR (Region R1)	PBR (Region R2)	PBR (Region R3)	Total PBR
Simulation I	902.4	882.0	219.9	807.4
Simulation II	992.9	1102.3	269.3	922.1

VI. THE ALLOCATION BASED ON DMBAP APPROACH IS PARETO EFFICIENT

Essentially, Pareto efficiency, or Pareto optimality, is a state of allocation of resources in which it is impossible to increase the payoff of any one individual by changing the allocated resources without decreasing the payoff of at least one individual.

Lemma 1: The MC curve is monotonic and strict ascending in contract region of a network provider.

Proof: Consider the Marginal cost curve in Figure 11, the are two highlighted points l_0 and l_s . l_0 is the amount of load where the slope of marginal cost is zero ($\frac{d}{dl}MC(l_0)=0$) and the l_s is a value of load where the total revenue is equal to total cost:

$$\int_0^{l_s} MC(L) dL = l_s * MC(l_s). \quad (25)$$

So in any load further than l_s which is called contract area, the total revenue is more than the total cost. It is brief that for any $l > l_s$, the slope of the marginal cost is positive and the MC curve is strict ascending in contract area.

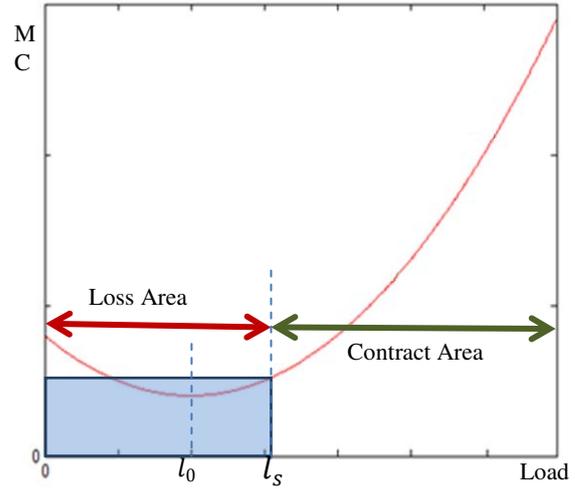


Figure 11-Contract area where the activity is profitable for the network provider

Lemma 2: The total revenue curve is monotonic and strict ascending in contract region

Proof: Suppose two different load l_1, l_2 in contract region of a network provider $l_s < l_1 < l_2$ and suppose the price of a unit of bandwidth is equal to the marginal cost according to the proposed strategy. Note that another mechanism for pricing is not profitable for the network providers why the regulatory agent would consider a penalty for the wrong-doing network. Thus, $P_1 = MC(l_1), P_2 = MC(l_2)$. Total revenues for the assumed loads are:

$$\begin{aligned} \pi(l_1) &= TR(l_1) - TC(l_1) \\ &= MC(l_1) * l_1 - \int_0^{l_1} MC(l)dl - SC \\ \pi(l_2) &= TR(l_2) - TC(l_2) \\ &= MC(l_2) * l_2 - \int_0^{l_2} MC(l)dl - SC \end{aligned} \quad (26)$$

Where SC is the static cost. Hence, $\pi(l_2) - \pi(l_1)$ calculated as follow:

$$\begin{aligned} \pi(l_2) - \pi(l_1) &= MC(l_2) * l_2 - MC(l_1) * l_1 \\ &\quad - \int_{l_1}^{l_2} MC(l)dl \end{aligned} \quad (27)$$

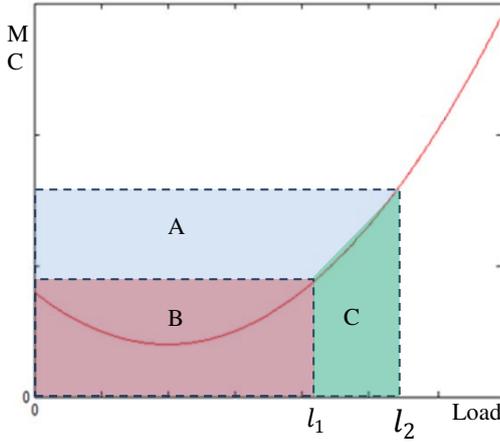


Figure 12-Two different load in contract area of a network provider and graphical display the payoffs

To evaluate the relation (27) pay attention to the Figure 12, where the schema of marginal cost curve of a sample network provider is drawn. The relation (27) has three parts; the first part is $MC(l_2) * l_2$ that is equal to the total area A , B and C. the second part is $MC(l_1) * l_1$ which is equal to the area B and the third part $\int_{l_1}^{l_2} MC(l) dl$ is equal to the area C. So, $\pi(l_2) - \pi(l_1)$ is equal to the area A and for any $l_1, l_2 \in (l_s, \infty)$ and $l_1 < l_2$ the result of $\pi(l_2) - \pi(l_1)$ is positive in other word:

$$\forall l_1, l_2 \in (l_s, \infty) , l_1 < l_2 : \pi(l_2) > \pi(l_1) \quad (28)$$

This means that the payoff functions for network providers are strict ascending in their contract region. \square

Theorem 2: The result of bandwidth allocation is Pareto efficient

The increase of an individual payoff needs to payoff reduction of one or some of clients or network providers.

Proof:

There are two different ways to redistribute the current allocation to increase the payoff of one client or one network provider:

- 1- Change the allocated bandwidth with no change in prices,
- 2- Change the allocated bandwidth with change in prices.

The first issue is impossible because according to the Theorem 1, the request bundle $\mathbf{x}^i(\mathbf{P}, H^i)$ of the client i is unique for any given price vector \mathbf{P} . So, any other allocation will reduce the clients' payoff and it is not profitable for clients unless we change the prices. If we use the second issue and change the prices to \mathbf{P}' to meet the allocation $\mathbf{X}' \neq \mathbf{X}$, where the payoff of at least one individual is further than its previous state and the payoff of other clients and networks is not decreased.

We know that the overall allocations are equal in both situations and only the distribution of the badwidth may change, thus we can write:

$$\sum_{i=1}^m \sum_{j=1}^n x_j^i = \sum_{i=1}^m \sum_{j=1}^n x_j'^i. \quad (29)$$

Let the load of network j is $L_j = \sum_{i=1}^m x_j^i$ and when $\mathbf{X}' \neq \mathbf{X}$ and prices are not $\mathbf{P}' \neq \mathbf{P}$ then the load of networks could not be equal and $\mathbf{L}' \neq \mathbf{L}$ where \mathbf{L} is a n -dimensional vector that shows the load of all networks. When we have $\mathbf{L}' \neq \mathbf{L}$ and the summation of loads are equal ($\sum_{j=1}^n L_j = \sum_{j=1}^n L_j'$) then it is brief that there exists at least two network h, k that $L_h > L_h'$ and $L_k < L_k'$. Suppose π_j, π_j' denote the payoff of network j in when allocations are \mathbf{X}, \mathbf{X}' .

According to the Lemma 2 when $L_k < L_k'$ then π_k' is less than π_k ($\pi_k' < \pi_k$) that means the network k loses the payoff in new allocation. Thus, we can not find any allocation that increases the payoff of at least one of the individuals (network or client) and the payoff of all network providers does not decrease. So the result of DMBAP is a Pareto allocation. \square

VII. CONCLUSION

In this paper we present a repeated game solution for resource allocation and pricing in heterogeneous wireless networks. There is a tree-stage game in iteration of the repeated game. There are three groups of players in our proposed game: Network providers, Clients and the Regulatory agent. In the first stage, all network providers should offer the price of a unit of bandwidth simultaneously; in the second stage the regulatory agent verifies the offers prices and considers a value of penalty or reward for the network providers to enforce the network providers to be honest in their offers. Finally in the third stage, clients evaluate and rank the current available networks and then prepare a request bundle that determines the amount of bandwidth to receive from each network provider. We proved that the violation from the proposed strategy in DMBAP is not profitable for any individual and also we proved that the allocation result is Pareto efficient allocation. According to the second welfare theorem in micro economics [8], the social welfare meets its maximum value when the allocation is Pareto efficient.

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