A new developed weighting method based on TOPSIS for decision makers in the group decision making process with interval type-2 fuzzy sets

M.H. Haghighi*, S.M. Mousavi, V. Mohagheghi

Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran

* Corresponding author: *E-mail address*: mh.haghighi.ie@gmail.com

Abstract

Fuzzy multiple criteria decision making is one of the most widely used decision methodologies in engineering, technology, science and management and business. In recent years, because of increasing the accuracy and breadth of the opinions, attention to group decision making has increased; so, it is necessary to determine the importance of each decision maker in this process. In this paper, concepts of the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is developed for determining weights of decision makers under group decision environments, in which each individual decision information is expressed by a matrix in interval type-2 fuzzy numbers. In this paper the positive and negative ideal solutions of group decision are defined and are expressed by a matrix, respectively. Finally, for the validation of the method, a numerical example of the previous studies has been presented.

Keywords: Multiple attributes group decision making (MAGDM); Weights of decision makers; TOPSIS; Interval type-2 fuzzy sets (IT2FSs)

9. Introduction

Multiple attributes decision making (MADM) has happen in a variety of actual situations, such as supply chain, strategic planning, management and economic analysis. The increasing complexity of the socioeconomic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem[1]. As a result, many decision making processes, in real world, take place in group settings. For example, consider that these DMs usually come from different specialty fields, and thus, each DM has his/her unique characteristics with regard to knowledge, skills, experience and personality, which implies that each DM usually has different influence in overall decision result; i.e., the weights of DMs are different. Therefore, how to determine weights of the DMs will be an interesting and important research topic[2]. Multi-criteria group decision making (MCGDM) is applied when several DMs might be considered to assessment and solve the problem, and the complexity of decision-making problems is increased.

In the past, methods have been proposed to determine weights of the DMs. French[3] proposed a method to determine the relative importance of the group's members by using the influence relations, which may exist between the members. Martel and Ben Khélifa[4] developed a method to determine the relative importance of group's members by using individual outranking indexes. Xu[5] gave some straightforward formulas to determine weights of the DMs.
One of known classical MADM method is TOPSIS (technique for order of preference by similarity to ideal solution); TOPSIS was first developed by Hwang and Yoon[6] for solving an MADM problem. TOPSIS simultaneously considers the distances to both the positive ideal solution (PIS) and the negative ideal solution (NIS), and a preference order is ranked according to their relative closeness, and a combination of these two distance measures is based on the concept that the chosen alternative should have the shortest distance from the PIS and the farthest from the NIS for solving a multiple criteria decision-making problem. Jahanshahloo et al[7] have extended the concept of TOPSIS to extend a methodology for solving MADM problems with interval numbers. Ye and Li[8] extended the TOPSIS technique for solving MCGDM problems with interval data. Yalcin et al[9] and Chen and Hwang[10] apply fuzzy numbers to establish fuzzy TOPSIS. Yong[11] used fuzzy TOPSIS for plant location selection, and Chen et al[12] applied fuzzy TOPSIS for supplier selection. Kahraman et al[13] utilized fuzzy TOPSIS for industrial robotic system selection.Kaya and Kahraman[14] proposed a modified fuzzy TOPSIS for selection of the best energy technology.

Fuzzy set based approach is one of the well-known theories that help DMs to handle fuzziness uncertainty in decision making and vagueness of information. This paper proposes a new decision making method of Interval Type-2 Fuzzy Sets (IT2FSs) as to deal with uncertainty and vagueness. Developed fuzzy sets have more advantages and are effective and useful with the context of this problem in front of classical fuzzy sets, so that this paper is considering the developed fuzzy sets.

In real-world, determining the numerical values to assign the alternatives is not always possible and is accompanied by uncertainty and ambiguity. In this situation, using the linguistic variables could help the DMs to evaluate the MCDM problems appropriately. We will be able to compare the alternatives by converting the linguistic variables to numerical values.

In this paper an extended TOPSIS technique is presented according to group TOPSIS with interval type-2 fuzzy data for determining weights of DMs. For each DM individual decision matrixes are given, the PIS of group opinion is depicted by a matrix with interval type-2 fuzzy numbers; similarly, the NIS of group opinion is also depicted by a matrix with interval type-2 numbers. Fuzzy developed sets have been used to cope better and more precisely with data uncertainty. The proposed method is suitable for determining the weights of attributes of group decision making when the exchange takes place between corresponding positions of the DMs and attributes in each individual decision matrix.

This paper is organized as follows. Section 2 presents the preliminary knowledge of IT2FSs. In section 3 the proposed IT2F TOPSIS approach is presented and section 4 presents the numerical example of the proposed approach. After that in section 5 conclusion is proposed.

2. Preliminary

Type-2 fuzzy sets are especially useful in uncertain situations where presenting the exact membership function of a fuzzy set gets very complicated. General type-2 fuzzy sets are computationally intensive and this feature has made the application of interval type-2 FSs more common. The main goal of this section is to provide the basic knowledge of interval type-2 fuzzy sets.

Definition 1. A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse *X* can be depicted by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$. This set is presented as follows [15].

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) | \forall x \in X, \forall u \in p_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\} \tag{1}$$

Where p_x shows an interval in [0, 1]. $\tilde{\tilde{A}}$ can be denoted as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} \frac{\mu_{\tilde{\tilde{A}}}(x,u)}{(x,u)} \tag{2}$$

Where $J_x \subseteq [0,1]$ and $\iint$ displays union over all admissible *x* and *u*.

Definition 2. If in $\tilde{\tilde{A}}$ all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is known as an interval type-2 fuzzy set. This is a special case of a type-2 fuzzy set that is depicted as follows [15]:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} \frac{1}{(x,u)} \tag{3}$$

Where $J_x \subseteq [0,1]$.

Definition 3. A trapezoidal interval type-2 fuzzy number is an interval type-2 fuzzy number that has trapezoidal fuzzy numbers for its upper membership function (UMF) and lower membership function (LMF). *A* as a trapezoidal interval type-2 fuzzy set can be displayed as follows:

$$A = (A^U, A^L) = \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1(A^U), H_2(A^U)\right), \left(a_1^L, a_2^L, a_3^L, a_4^L; H_1(A^L), H_2(A^L)\right)\right) \tag{4}$$

Where A^U and A^L show the UMF and LMF of *A*, respectively, and $H_j(A^U)$ and $H_j(A^L)$ $(H_j(A^U) \in [0,1], H_j(A^L) \in [0,1], j = 1,2)$ express the membership values of the corresponding elements $a_{j+1}^L$and a_{j+1}^U, respectively. This interval type-2 fuzzy set is displayed in figure 1.

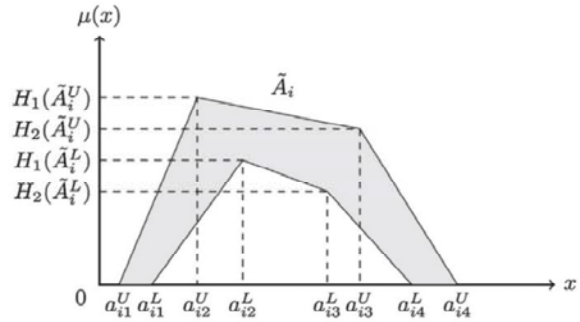

Fig. 1. A trapezoidal interval type-2 fuzzy set

Let $A_1$and A_2 be two trapezoidal interval type-2 fuzzy numbers:

$$A_1 = (A_1^U, A_1^L) = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)\right)\right) \tag{5}$$

$$A_2 = (A_2^U, A_2^L) = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(A_2^U), H_2(A_2^U)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(A_2^L), H_2(A_2^L)\right)\right) \tag{6}$$

The addition operation between them is defined as follows [16]:

$$\begin{aligned} A_1 \oplus A_2 = (A_1^U, A_1^L) + (A_2^U, A_2^L) \\ = \Big[\Big(a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; H_1(\tilde{A}_1^U) + H_1(\tilde{A}_2^U) \\ - H_1(\tilde{A}_1^U). H_1(\tilde{A}_2^U), H_2(\tilde{A}_1^U) + H_2(\tilde{A}_2^U) - H_2(\tilde{A}_1^U). H_2(\tilde{A}_2^U)\Big), \Big(a_{11}^L + a_{21}^L, a_{12}^L \\ + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; H_1(\tilde{A}_1^L) + H_1(\tilde{A}_2^L) - H_1(\tilde{A}_1^L). H_1(\tilde{A}_2^L), H_2(\tilde{A}_1^L) + H_2(\tilde{A}_2^L) \\ - H_2(\tilde{A}_1^L). H_2(\tilde{A}_2^L)\Big)\Big] \end{aligned} \tag{7}$$

The subtraction operation is defined as follows [16]:

$$\begin{aligned} A_1 \ominus A_2 = (A_1^U, A_1^L) - (A_2^U, A_2^L) \\ = \Big[\Big(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; H_1(\tilde{A}_1^U) + H_1(\tilde{A}_2^U) \\ - H_1(\tilde{A}_1^U). H_1(\tilde{A}_2^U), H_2(\tilde{A}_1^U) + H_2(\tilde{A}_2^U) - H_2(\tilde{A}_1^U). H_2(\tilde{A}_2^U)\Big), \Big(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L \\ - a_{22}^L, a_{14}^L - a_{21}^L; H_1(\tilde{A}_1^L) + H_1(\tilde{A}_2^L) - H_1(\tilde{A}_1^L). H_1(\tilde{A}_2^L), H_2(\tilde{A}_1^L) + H_2(\tilde{A}_2^L) \\ - H_2(\tilde{A}_1^L). H_2(\tilde{A}_2^L)\Big)\Big] \end{aligned} \tag{8}$$

The multiplication operation is defined as follows [16]:

$$A_1 \otimes A_2 = (A_1^U, A_1^L) \otimes (A_2^U, A_2^L) = \begin{pmatrix} \left(x_{11}^U, x_{12}^U, x_{13}^U, x_{14}^U; H_1(A_1^U). H_1(A_2^U), H_2(A_1^U). H_2(A_2^U)\right), \\ \left(x_{11}^L, x_{12}^L, x_{13}^L, x_{14}^L; H_1(A_1^L). H_1(A_2^L), H_2(A_1^L). H_2(A_2^L)\right) \end{pmatrix} \tag{9}$$

Where$x_{1i}^T = \min\left(a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right), T \in \{U, L\}, i \epsilon \{1,2\}$

$$x_{1j}^T = \max\left(a_{1(5-j)}^T a_{2(5-j)}^T, a_{1(5-j)}^T a_{2j}^T, a_{1j}^T a_{2(5-j)}^T, a_{1j}^T a_{2j}^T\right), T \in \{U, L\}, j \epsilon \{3,4\}$$

The multiplication operation between a crisp value (λ) andA_1 is defined as follows [16]:

$$\lambda A_1 = \left[\left((\lambda a_{11}^U, \lambda a_{12}^U, \lambda a_{13}^U, \lambda a_{14}^U);\ 1 - \left(1 - H_1(\tilde{A}_1^U)\right)^\lambda, 1 - \left(1 - H_2(\tilde{A}_1^U)\right)^\lambda\right), \left((\lambda a_{11}^L, \lambda a_{12}^L, \lambda a_{13}^L, \lambda a_{14}^L);\ 1 - \left(1 - H_1(\tilde{A}_1^L)\right)^\lambda, 1 - \left(1 - H_2(\tilde{A}_1^L)\right)^\lambda\right)\right] \tag{10}$$

The division operation is depicted as follows;

$$A_1 \oslash A_2 = (A_1^U, A_1^L) \oslash (A_2^U, A_2^L) = \begin{pmatrix} \left(Y_{11}^U, Y_{12}^U, Y_{13}^U, Y_{14}^U; H_1(A_1^U).H_1(A_2^U), H_2(A_1^U).H_2(A_2^U)\right), \\ \left(Y_{11}^L, Y_{12}^L, Y_{13}^L, Y_{14}^L; H_1(A_1^L).H_1(A_2^L), H_2(A_1^L).H_2(A_2^L)\right) \end{pmatrix} \tag{11}$$

Where $Y_{1i}^T = \min\left(a_{1i}^T/a_{2i}^T, a_{1i}^T/a_{2(5-i)}^T, a_{1(5-i)}^T/a_{2i}^T, a_{1(5-i)}^T/a_{2(5-i)}^T\right), T \in \{U, L\}, i \epsilon \{1,2\}$

$x_{1j}^T = \max\left(a_{1(5-j)}^T/a_{2(5-j)}^T, a_{1(5-j)}^T/a_{2j}^T, a_{1j}^T/a_{2(5-j)}^T, a_{1j}^T/a_{2j}^T\right), T \in \{U, L\}, j \epsilon \{3,4\}$

Degree of similarity between interval type-2 fuzzy numbers:

$$d(A_1, A_2) = \sqrt{\sum_{b=1}^{4}(a_{1b}^U - a_{2b}^U)^2 + \sum_{b=1}^{4}(a_{11}^L - a_{21}^L)^2 + \sum_{b=1}^{2}(H_b(A_1^U) - H_b(A_2^U))^2 + \sum_{b=1}^{2}(H_b(A_1^L) - H_b(A_2^L))^2} \tag{12}$$

3. Methodology

In this paper, TOPSIS method is developed for determining weights of experts or DMs under group decision environments, in which each individual decision information and PIS and NIS is expressed by a matrix with interval type-2 fuzzy numbers. The PIS is expressed by the average matrix of group decision and the NIS is maximum and minimum separation from PIS. According to the relative closeness, we determine weights of DMs in accordance with the values of the relative closeness; figure 2 is a framework of the proposed method:

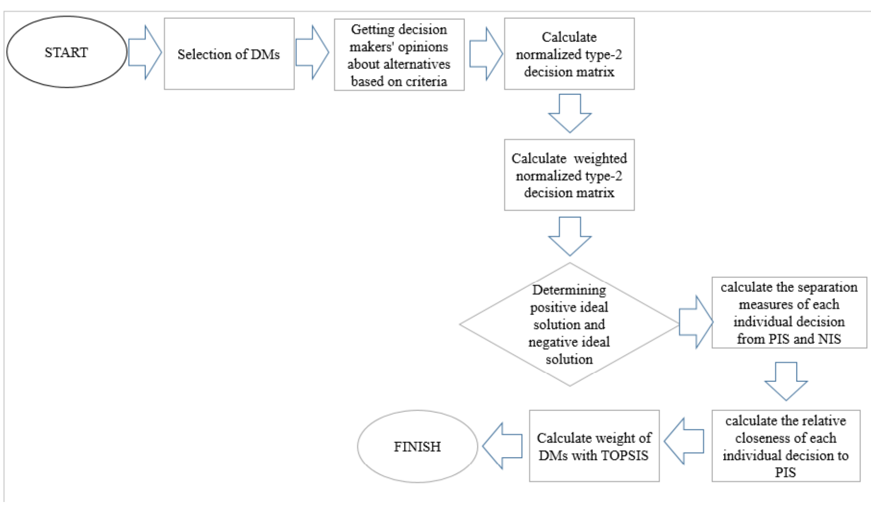

Fig. 2- General framework

Step 1. Gather the judgments of each DM and form the initial decision matrices:

$$\widetilde{\widetilde{DM}}_K = \left(\widetilde{\widetilde{DM}}_{ij}^K\right)_{m\times n} = \begin{bmatrix} \widetilde{\widetilde{DM}}_{11}^K & \cdots & \widetilde{\widetilde{DM}}_{1n}^K \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{DM}}_{m1}^K & \cdots & \widetilde{\widetilde{DM}}_{mn}^K \end{bmatrix} \tag{13}$$

$$\tilde{\tilde{W}}_j = \left(\tilde{\tilde{w}}_1{}^k, \tilde{\tilde{w}}_2{}^k, \dots, \tilde{\tilde{w}}_n{}^k\right), K \in T \tag{14}$$

Where $\widetilde{DM}_K$ shows the decision matrix and $\tilde{W}_j$ shows the weight of criteria, n denotes the number of criteria, m shows number of compared alternatives and T denotes the group of DMs.

Step 2. Compute the normalized decision matrix ($\widetilde{\widetilde{NDM}}^K{}_{ij}$) by the means of the following Eqs.:

$$\widetilde{\widetilde{NDM}}^K{}_{ij} = \begin{bmatrix} \widetilde{\widetilde{NDM}}^K{}_{11} & \cdots & \widetilde{\widetilde{NDM}}^K{}_{1n} \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{NDM}}^K{}_{m1} & \cdots & \widetilde{\widetilde{NDM}}^K{}_{mn} \end{bmatrix} \tag{15}$$

$$\widetilde{\widetilde{NDM}}^K{}_{ij} = \left(\left(\frac{d_{ij_1}{}^U}{d^\star}, \frac{d_{ij_2}{}^U}{d^\star}, \frac{d_{ij_3}{}^U}{d^\star}, \frac{d_{ij_4}{}^U}{d^\star}; H_1\left(\tilde{D}_{ij}^U\right), H_2\left(\tilde{D}_{ij}^U\right)\right), \left(\frac{d_{ij_1}{}^L}{d^\star}, \frac{d_{ij_2}{}^L}{d^\star}, \frac{d_{ij_3}{}^L}{d^\star}, \frac{d_{ij_4}{}^L}{d^\star}; H_1\left(\tilde{D}_{ij}^L\right), H_2\left(\tilde{D}_{ij}^L\right)\right)\right) \tag{16}$$

$i = 1,2, \dots, n, j \epsilon B$

$$\widetilde{\widetilde{NDM}}^K{}_{ij} = \begin{pmatrix} \left(Y_{11}^U, Y_{12}^U, Y_{13}^U, Y_{14}^U; H_1(A_1^U).H_1(A_2^U), H_2(A_1^U).H_2(A_2^U)\right), \\ \left(X_{11}^L, X_{12}^L, X_{13}^L, X_{14}^L; H_1(A_1^L).H_1(A_2^L), H_2(A_1^L).H_2(A_2^L)\right) \end{pmatrix} \tag{17}$$

Where $Y_{1i}^T = \min\left(d^-/d_{ijl}^T, d^-/d_{ij(5-l)}^T, d^-/d_{ijl}^T, d^-/d_{ij(5-l)}^T\right), T \in \{U, L\}, l \epsilon \{1,2\}$

$x_{1j}^T = \max\left(d^-/d_{2(5-v)}^T, d^-/d_{2v}^T, d^-/d_{2(5-v)}^T, d^-/d_{2v}^T\right), T \in \{U, L\}, v \epsilon \{3,4\}$

$i = 1,2, \dots, n, j \epsilon C$

Where B denotes the set of benefit criteria and C represents the set of cost criteria. $d^\star$ and d^- are also obtained as follows:

$$d^\star = \max_i (d_{ij}^k)_4^U \tag{18}$$

$$d^- = \min_i (d_{ij}^k)_1^U \tag{19}$$

Step 3. Calculate weighted normalized decision matrix $\widetilde{\widetilde{V^K}}_{ij}$:

$$\widetilde{\widetilde{V^K}}_{ij} = \begin{pmatrix} \widetilde{\widetilde{V^K}}_{11} & \cdots & \widetilde{\widetilde{V^K}}_{1n} \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{V^K}}_{m1} & \cdots & \widetilde{\widetilde{V^K}}_{mn} \end{pmatrix} \text{ for all } k \in T \tag{23}$$

$$\widetilde{\widetilde{V^K}}_{ij} = \left(\left(w_{j1}^{K(U)}, w_{j2}^{K(U)}, w_{j3}^{K(U)}, w_{j4}^{K(U)}; H_1\left(w_j^{K(U)}\right), H_2\left(w_j^{K(U)}\right)\right), \left(w_{j1}^{K(L)}, w_{j2}^{K(L)}, w_{j3}^{K(L)}, w_{j4}^{K(L)}; H_1\left(w_j^{K(L)}\right), H_2\left(w_j^{K(L)}\right)\right)\right) \otimes \left(\left(NM_{ij1}^{K(U)}, NM_{ij2}^{K(U)}, NM_{ij3}^{K(U)}, NM_{ij4}^{K(U)}; H_1\left(NM_{ij}^{K(U)}\right), H_2\left(NM_{ij}^{K(U)}\right)\right), \left(NM_{ij1}^{K(L)}, NM_{ij2}^{K(L)}, NM_{ij3}^{K(L)}, NM_{ij4}^{K(L)}; H_1\left(NM_{ij}^{K(L)}\right), H_2\left(NM_{ij}^{K(L)}\right)\right)\right) \tag{24}$$

Step 4. Determine PIS and NIS.

$\widetilde{\widetilde{A^+}}$ is the PIS of all individual decision.

$$\widetilde{\widetilde{A^+}} = \begin{pmatrix} \widetilde{\widetilde{V^+}}_{11} & \cdots & \widetilde{\widetilde{V^+}}_{1n} \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{V^+}}_{m1} & \cdots & \widetilde{\widetilde{V^+}}_{mn} \end{pmatrix} \tag{25}$$

$$\widetilde{\widetilde{V^+}}_{ij} = \frac{1}{T}\sum_{k=1}^{T}\left(\left(V_{ij1}^{K(U)}, V_{ij2}^{K(U)}, V_{ij3}^{K(U)}, V_{ij4}^{K(U)}; H_1\left(V_{ij}^{K(U)}\right), H_2\left(V_{ij}^{K(U)}\right)\right), \left(V_{ij1}^{K(L)}, V_{ij2}^{K(L)}, V_{ij3}^{K(L)}, V_{ij4}^{K(L)}; H_1\left(V_{ij}^{K(L)}\right), H_2\left(V_{ij}^{K(L)}\right)\right)\right) \tag{26}$$

$\widetilde{\widetilde{A^-}}$ is the NIS of all individual decision.

$$\widetilde{\widetilde{A^{-l}}} = \begin{pmatrix} \widetilde{\widetilde{V^{-l}}}_{11} & \cdots & \widetilde{\widetilde{V^{-l}}}_{1n} \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{V^{-l}}}_{m1} & \cdots & \widetilde{\widetilde{V^{-l}}}_{mn} \end{pmatrix} \quad \widetilde{\widetilde{A^{-r}}} = \begin{pmatrix} \widetilde{\widetilde{V^{-r}}}_{11} & \cdots & \widetilde{\widetilde{V^{-r}}}_{1n} \\ \vdots & \ddots & \vdots \\ \widetilde{\widetilde{V^{-r}}}_{m1} & \cdots & \widetilde{\widetilde{V^{-r}}}_{mn} \end{pmatrix} \tag{27}$$

$$\widetilde{\widetilde{V^{-l}}}_{ij} = \min_{1 \le k \le t}\left\{\left(\left(V_{ij1}^{K(U)}, V_{ij2}^{K(U)}, V_{ij3}^{K(U)}, V_{ij4}^{K(U)}; H_1\left(V_{ij}^{K(U)}\right), H_2\left(V_{ij}^{K(U)}\right)\right), \left(V_{ij1}^{K(L)}, V_{ij2}^{K(L)}, V_{ij3}^{K(L)}, V_{ij4}^{K(L)}; H_1\left(V_{ij}^{K(L)}\right), H_2\left(V_{ij}^{K(L)}\right)\right)\right)\right\} \tag{28}$$

$$\widetilde{\widetilde{V^{-r}}}_{ij} = \max_{1 \le k \le t}\left\{\left(\left(V_{ij1}^{K(U)}, V_{ij2}^{K(U)}, V_{ij3}^{K(U)}, V_{ij4}^{K(U)}; H_1\left(V_{ij}^{K(U)}\right), H_2\left(V_{ij}^{K(U)}\right)\right), \left(V_{ij1}^{K(L)}, V_{ij2}^{K(L)}, V_{ij3}^{K(L)}, V_{ij4}^{K(L)}; H_1\left(V_{ij}^{K(L)}\right), H_2\left(V_{ij}^{K(L)}\right)\right)\right)\right\}$$

Step 5. Utilize Eqs. (23)-(25) to calculate the degree of similarity of each individual decision from PIS and NIS, respectively:

$$D_k^{+}=\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\begin{array}{c}\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(U)}}}_{ijb}-V_{ijb}^{+(U)})^2+\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(L)}}}_{ijb}-V_{ijb}^{+(L)})^2+\sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(U)}}}))-H_b(V_{ij}^{+(U)}))^2+\\ \sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(L)}}})-H_b(V_{ij}^{+(L)}))^2\end{array}\right)}\qquad k\in T \tag{29}$$

Similarly, the separation from the NIS is given as:

$$D_k^{-l}=\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\begin{array}{c}\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(U)}}}_{ijb}-V_{ijb}^{-l(U)})^2+\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(L)}}}_{ijb}-V_{ijb}^{-l(L)})^2+\sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(U)}}}))-H_b(V_{ij}^{-l(U)}))^2+\\ \sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(L)}}})-H_b(V_{ij}^{-l(L)}))^2\end{array}\right)} \tag{30}$$

$$D_k^{-r}=\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\begin{array}{c}\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(U)}}}_{ijb}-V_{ijb}^{-r(U)})^2+\sum_{b=1}^{4}(\widetilde{\widetilde{NM^{K(L)}}}_{ijb}-V_{ijb}^{-r(L)})^2+\sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(U)}}}))-H_b(V_{ij}^{-r(U)}))^2+\\ \sum_{b=1}^{2}(H_b(\widetilde{\widetilde{NM^{K(L)}}})-H_b(V_{ij}^{-r(L)}))^2\end{array}\right)} \tag{31}$$

$k\in T$

Step 6. Calculate the distance for each DMs:

$$RC_k=\frac{S_k^{-r}+S_k^{-l}}{S_k^{+}+S_k^{-r}+S_k^{-l}}\qquad k\in T$$

$$\lambda_k=\frac{R_k}{\sum_{k=1}^{T}R_k}\quad k\in T$$

4. Numerical example

To illustrate the proposed method, a numerical example of Bashiri et al[17] is used. Suppose that a company desires to select the most appropriate maintenance strategy that requires different alternatives to be assessed for a range of criteria. There is a variety of strategies in maintenance management systems depending on the type of applied industry
in a company. Strategies would be five custom policies of corrective maintenance (CM), preventive maintenance (PM), time-based maintenance (TBM), condition-based maintenance (CBM) and predictive maintenance (PDM). A committee of three maintenance experts, D_1, D_2 and D_3, has been formed to conduct the evaluation process and to select the most suitable maintenance strategy. Each of DMs will evaluate five custom policies CM, PM, TBM, CBM and PDM based on criteria (each of criteria is benefit) with linguistic variables

Table 1. Weights of each criterion

Criteria	DM1	DM2	DM3
C1	VH	H	H
C2	H	MH	MH
C3	M	MH	H
C4	VH	VH	MH

Table 2. Decision matrix

Criteria	Alternatives	DMs		
		DM1	DM2	DM3
C1	A1	MG	VG	G
	A2	G	G	MG
	A3	MG	G	MG
	A4	VG	VG	MG
	A5	G	F	MG
C2	A1	G	VG	F
	A2	VG	MG	G
	A3	G	MG	G

	A4	VG	MG	VG
	A5	VG	F	G
C3	A1	G	F	MG
	A2	MG	G	MG
	A3	G	G	MG
	A4	F	F	G
	A5	G	MG	VG
C4	A1	MG	G	VG
	A2	VG	F	G
	A3	G	MG	G
	A4	G	VG	MG
	A5	MG	G	MG

Table3. Linguistic terms for weights of each criterion

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0,0,0,1;1,1),(0,0,0,0.5;0.9,0.9))
Low (L)	((0,1,1,3;1,1),(0.5,1,1,2;0.9,0.9))
Medium Low (ML)	((1,3,3,5;1,1),(2,3,3,4;0.9,0.9))
Medium (M)	((3,5,5,7;1,1),(4,5,5,6;0.9,0.9))
Medium High (MH)	((5,7,7,9;1,1),(6,7,7,8;0.9,0.9))
High (H)	((7,9,9,10;1,1),(8,9,9,9.5;0.9,0.9))
Very High (VH)	((9,10,10,10;1,1),(9.5,10,10,10;0.9,0.9))

Table 4. Linguistic terms for rating of alternatives

Linguistic terms	Interval type-2 fuzzy sets
Very Poor (VP)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Poor (P)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Poor (MP)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Fair (F)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium Good (MG)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very Good (VG)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

In order to reduce space, all tables are presented for the first decision maker:

Table 5. Weighted normalized for DM1

	C_1	C_2	C_3	C_4
A_1	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((2.1,4.5,4.5,7;1,1),(3.2,4.5,4.5,5.7;0.81,0.81))	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))
A_2	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81, 0.81))	((1.5,3.5,3.5,6.3;1,1),(2.4,3.5,3.5, 4.8;0.81,0.81))	((8.1,10,10,10;1,1),(9.025,10,10,10;0.81,0.81))
A_3	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((2.1,4.5,4.5,7;1,1),(3.2,4.5,4.5,5.7;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))
A_4	((8.1,10,10,10;1,1),(9.025,10,10,10;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81, 0.81))	((0.9,2.5,2.5,4.9;1,1),(1.6,2.5,2.5, 3.6;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))
A_5	((6.3,9,9,10;1,1), (7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81, 0.81))	((2.1,4.5,4.5,7;1,1),(3 .2,4.5,4.5,5.7;0.81,0.81))	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))

Table 6. Overall positive ideal

	C_1	C_2	C_3	C_4
A_1	((5.233,8.033,8.033,9.667;1,1),(4.925,6.025,6.025,6.631;0.993,0.993))	((3.633,6.2,6.2,8.433;1,1),(4.833, 6.2,6.2,7.275;0.993,0.993))	((2.367,4.767,4.767,7.433;1,1),(3.467,4.767,4.767,6.033;0.993,0.993))	((5.1,7.667,7.667,9.333;1,1),(6.333,7.667,7.667,8.5;0.993,0.993))
A_2	((4.9,7.8,7.8,9.667;1,1),(4.7,5.85, 5.85,6.531;0.993,0.993))	((4.1,6.733,6.733,9.033;1,1),(5.333,6.733,6.733,7.833;0.993,0.993))	((2.833,5.367,5.367,8.1;1,1),(4,5.367,5.367,6.667;0.993,0.993))	((4.767,7.1,7. 1,8.667;1,1),(5.875,7.1,7.1,7.867;0.993,0.993))
A_3	((4.3,7.133,7.133,9.333;1,1),(4.225,5.35,5.35,6.156;0.993,0.993))	((3.633,6.433,6.433,9.033;1,1),(4.933,6.433,6.433,7.675;0.993,0.993))	((3.033,5.7,5.7,8.333;1,1),(4.267, 5.7,5.7,6.967;0.993,0.993))	((4.767,7.433,7.433,9.333;1,1),(6.033,7.433,7.433,8.367;0.993,0.993))
A_4	((5.967,8.433,8.433,9.667;1,1),(5.356,6.325,6.325,6.775;0.993,0.993))	((4.433,6.967,6.967,9.033;1,1),(5.633,6.967,6.967,7.967;0.993,0.993))	((2.433,4.7,4.7,7.067;1,1),(3.467, 4.7,4.7,5.808;0.993,0.993))	((5.633,7.967,7.967,9.367;1,1),(6.742,7.967,7.967,8.633;0.993,0.993))

A_5	((2.975,4.95,4.95,6.5;1,1),(3.9,4.95,4.95,5.7;0.993,0.993))	((3.767,6.267,6.267,8.433;1,1),(4.9333,6.267,6.267,7.3;0.993,0.993))	((3.633,6.133,6.133,8.367;1,1),(4.8,6.133,6.133,7.2;0.993,0.993))	((4.433,6.967,6.967,9.333;1,1),(5.633,6.967,6.967,7.967;0.993,0.993))

Table 7. Overall left negative ideal

	C_1	C_2	C_3	C_4
A_1	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))	((1.5,3.5,3.5,6.3;1,1),(2.4,3.5,3.5,4.8;0.81,0.81))	((1.5,3.5,3.5,6.3;1,1),(2.4,3.5,3.5,4.8;0.81,0.81))	((4.5,7,7,9;1,1),(5.7,7,7,8;0.81,0.81))
A_2	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((2.5,4.9,4.9,8.1;1,1),(3.6,4.9,4.9,6.4;0.81,0.81))	((1.5,3.5,3.5,6.3;1,1),(2.4,3.5,3.5,4.8;0.81,0.81))	((2.7,5,5,7;1,1),(3.8,5,5,6;0.81,0.81))
A_3	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((2.5,4.9,4.9,8.1;1,1),(3.6,4.9,4.9,6.4;0.81,0.81))	((2.1,4.5,4.5,7;1,1),(3.2,4.5,4.5,5.7;0.81,0.81))	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))
A_4	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((2.5,4.9,4.9,8.1;1,1),(3.6,4.9,4.9,6.4;0.81,0.81))	((0.9,2.5,2.5,4.9;1,1),(1.6,2.5,2.5,3.6;0.81,0.81))	((2.5,4.9,4.9,8.1;1,1),(3.6,4.9,4.9,6.4;0.81,0.81))
A_5	((2.1,4.5,4.5,7;1,1),(3.2,4.5,4.5,5.7;0.81,0.81))	((1.5,3.5,3.5,6.3;1,1),(2.4,3.5,3.5,4.8;0.81,0.81))	((2.1,4.5,4.5,7;1,1),(3.2,4.5,4.5,5.7;0.81,0.81))	((2.5,4.9,4.9,8.1;1,1),(3.6,4.9,4.9,6.4;0.81,0.81))

Table 8. Overall right negative ideal

	C_1	C_2	C_3	C_4
A_1	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))
A_2	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((8.1,10,10,10;1,1),(9.025,10,10,10;0.81,0.81))
A_3	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((3.5,6.3,6.3,9;1,1),(4.8,6.3,6.3,7.6;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))
A_4	((8.1,10,10,10;1,1),(9.025,10,10,10;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((4.9,8.1,8.1,10;1,1),(6.4,8.1,8.1,9.025;0.81,0.81))	((8.1,10,10,10;1,1),(9.025,10,10,10;0.81,0.81))
A_5	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))	((6.3,9,9,10;1,1),(7.6,9,9,9.5;0.81,0.81))

Table 9. Relative closeness, weights and ranking

	RC_k	λ_k	Ranking
D1	11.345	0.360	2
D2	11.444	0.363	1
D3	8.717	0.277	3

5. Conclusion

The multiple-criteria group decision-making provides approaches for finding the best state with taking into account decision makers among the possible alternatives based on the evaluation of multiple criteria so that importance of opinion DMs is needed to determine. This paper proposed developed TOPSIS for determining weights of DMs, in which individual decision information and positive ideal solution and negative ideal solution were expressed matrix with interval type-2 fuzzy numbers. In order to more accurately take into account, the indeterminate numbers in this paper developed fuzzy sets with linguistic variables are used. A numerical example according to the previous studies for validation of method is solved. For the future studies, using this method for determining weights of criteria in multi criteria group decision making from this method and applying other methods of decision making weights of decision makers are recommended.

References

[1] S.H. Kim, B.S. Ahn, 1999. Interactive group decision making procedure under incomplete information, European Journal of Operational Research 116 498–507.

[2] Yue, Z. 2011. An extended TOPSIS for determining weights of decision makers with interval numbers. *Knowledge-Based Systems*, *24*(1), 146-153.

[3] J.R.P. French, 1956. A formal theory of social power, Psychological Review 63 (3)181–194.

[4] J.-M. Martel, S. Ben Khélifa, 2000. Deux propositions d'aide multicritère à la décision de groupe, in: Ben Abdelaziz, Haouari et Mellouli (Eds.), Optimisation et Décision, Centre de Publication Universitaire, Tunis, pp. 213–228.

[5] Z.S. Xu, 2008 Group decision making based on multiple types of linguistic preference relations, Information Sciences 178 452–467.
[6] C.L. Hwang, K. Yoon, 1981. Multiple Attribute Decision Making: Methods and Applications, Springer, Berlin, Heidelberg,. G.R. [7] Jahanshahloo, F. Hosseinzadeh Lotfi, M. Izadikhah, 2006. An algorithmic method to extend TOPSIS with interval data, Applied Mathematics and Computation 175 1375–1384.
[8] F. Ye, Y.N. Li, 2009. Group multi-attribute decision model to partner selection in the formation of virtual enterprise under incomplete information, Expert Systems with Applications 36 9350–9357.
[9] Yalcin, N., Bayrakdaroglu, A., Kahraman, C., 2009. Fuzzy performance evaluation in Turkish banking sector using analytic hierarchy process and TOPSIS. Expert Systems with Application 36 (9), 11699–11709.
[10] Chen, S.J., Hwang, C.L., 1992. Fuzzy Multi Attribute Decision Making. Lecture Notes in Economics and Mathematical System Series, vol. 375. Springer-Verlag, New York.
[11] Yong, D., 2006. Plant location selection based on Fuzzy TOPSIS. International Journal of Advanced Manufacturing Technologies 28 (7–8), 839–844.
[12] Chen, C.T., Lin, C.T., Huang, S.F., 2006. A fuzzy approach for supplier evaluation and selection in supply chain management. International Journal of Production Economics 102 (2), 289–301.
[13] Kahraman, C., Cevik, S., Ates, N.Y., Gulbay, M., 2007. Fuzzy multi-criteria evaluation of industrial robotic systems. Computers and Industrial Engineering 52 (4), 414–433.
[14] Kaya, T., Kahraman, C., 2011. Multi criteria decision making in energy planning using a modified fuzzy TOPSIS methodology. Expert Systems with Applications 38 (6), 6577–6585.
[15] Mendel, J.M., John, R.I., Liu, F.L., 2006. Interval type-2 fuzzy logical systems made simple. IEEE Transactions on Fuzzy Systems 14 (6), 808–821.
[16] Hu, J., Zhang, Y., Chen, X., & Liu, Y. 2013. Multi-criteria decision making method based on possibility degree of interval type-2 fuzzy number. *Knowledge-Based Systems*, *43*, 21-29.
[17] Bashiri, M., Badri, H., & Hejazi, T. H. 2011. Selecting optimum maintenance strategy by fuzzy interactive linear assignment method. *Applied Mathematical Modelling*, *35*(1), 152-164.