The choice of sliding surface for robust roll control: Better suppression of high angle of attack/sideslip perturbations

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Abstract
The nominal aerodynamic parameters of aircraft are often approximate and aircraft may experience high value of angle of attack/sideslip perturbations during their manoeuvres. Preventing instability and air crash requires a robust controller capable of containing the dynamic uncertainty and the perturbations. In this respect, the problems of roll control in such a situation are studied and a better choice of sliding surface is proposed. Sliding mode control manages the uncertainty and adaptive fuzzy is employed to shape the transient response. As a result, a setup is formed which outperforms the basic controller both in terms of transient speed, response robustness and control effort. The strength of the method is more appreciated in case of high angle of attack/sideslip perturbed manoeuvres. This is proved theoretically and illustrated by simulations.

Keywords
Flight control, high angle of attack, adaptive fuzzy sliding mode, roll stabilization

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Introduction
The aim of an aircraft control loop is to enforce accurate tracking of the ordered lateral-directional and longitudinal commands in all intended flight manoeuvres over a wide range of working conditions. A well-tuned feedback control law is expected to suppress external disturbances (e.g. wind gusts) and to reduce sensitivity to the aircraft parameters’ variations (robustness against parametric uncertainties).

Design of an aircraft control is tough due to nonlinearity, coupling and time varying characteristics of the system and strong dependency of the system parameters to the working conditions, i.e. angle of attack/sideslip. Nevertheless, gain scheduling (GS) approach is traditionally employed where the system dynamic around each axis is linearized and the proportional-integral-derivative (PID) controller is independently utilized for each axis. Success of the design depends on the precise definition of the type of operation so that the neglected dynamic and cross-couplings among variables to be adequately small. Apart from that, a lot of more sophisticated approaches such as: fuzzy logic (FL), intelligently tuned $\mu$-PID, robust, intelligently tuned fractional order PID, sliding mode, $H_\infty$, fractional order PID, dynamic inversion, a combination of model inversion with an online adaptive neural network, a nonlinear adaptive design based on backstepping, neural networks and RBFNN-based adaptive design have also been studied. The main goal in all of the newly developed methods is flight envelope extension and manoeuvrability improvement.

Sliding mode control (SMC) has been commonly used in aerospace, process control, power control and robotics to administer dynamic and disturbance perturbations. However, SMC results in a high switching gain particularly when the system uncertainty band is large. This may cause degradation of the steady-state performance and system failure due to mechanical
vibrations. The problem has well been investigated,\textsuperscript{9,10} and modifications have been developed, e.g. instead of using fixed switching gain, various adaptive nonlinear gain algorithms have been introduced.\textsuperscript{11,12} By integrating adaptive fuzzy and SMC, various types of interesting methods are formed.\textsuperscript{13} In Amieur et al.,\textsuperscript{14} adaptive fuzzy SMC (AFSMC) alongside with feedback linearization has been explored. Application of AFSMC for a flexible robot\textsuperscript{15} and a hypersonic vehicle control has also been reported.\textsuperscript{16} For a simplified F/A-18 aircraft model, a nonlinear AFSC is designed for trajectory tracking during aircraft manoeuvres.\textsuperscript{17} Application of integral back stepping SMC has also been elaborated in Yang et al.\textsuperscript{20} Adaptive fuzzy multi-surface sliding control (AFMSSC) for trajectory tracking of an MIMO aerial vehicle has been detailed in Norton et al.\textsuperscript{21} In Rubagotti et al.,\textsuperscript{22} integral sliding mode controller for reducing disturbances is the subject of study. A nonlinear sliding mode controller using bifurcation for commanded spin of an aircraft is given in Rao and Sinha.\textsuperscript{23} Employing robust SMC using bifurcation for commanded spin of an aircraft and its negative influences is given in Guruganesh et al.\textsuperscript{24} The performance is more improved when SMC is modified to a special AFSMC configuration. It is shown that the method is capable of providing safe and reliable flight motion in spite of 30\% tolerances in the aircraft roll aerodynamic parameters and against high value of angle of attack/sideslip perturbations. This is proved theoretically and illustrated through extensive simulations.

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- Conclusion

### Aircraft control

Generally, longitudinal and lateral-directional motion of an aircraft is governed by the three main control surfaces: elevators, rudders and ailerons. Longitudinal or pitch axis control is directed by the symmetric deflection of the elevators. Deflection of the ailerons manages the roll, and the lateral-direction (yaw) motion is managed by the joint rudder-aileron action.

#### Equations of motion

The six-degrees of freedom nine-state mathematical model of an aircraft are expressed in a nonlinear state space form as follows

\[
\dot{x} = f(x, u)
\]

where \(x = [V, \beta, a, p, q, r, \phi, \theta, \psi] \) is a vector of nine states, i.e. speed, sideslip angle, angle of attack, roll rate, pitch rate, yaw rate, roll angle, pitch angle and yaw angle, respectively. The functioning of the control surfaces is expressed in the vehicle dynamics equations by \(u = [\delta_{\text{ail}}, \delta_{r}, \delta_{s}] \) which is a vector of three control surfaces angles. The aircraft force equations\textsuperscript{1} are given by

\[
\dot{V} = \frac{1}{m} \left( -D \cos \beta + Y \sin \beta + T \cos \alpha \cos \beta \right) \\
+ g \left( -\cos \alpha \cos \phi \sin \theta + \sin \beta \cos \beta \sin \phi + \sin \alpha \cos \beta \cos \phi \cos \beta \right) \\
\dot{q} = \frac{1}{m} \cos \beta \left( -L - T \sin x + mg \left( \cos \beta \cos \phi \cos \theta + \sin \alpha \sin \alpha \right) \right) \\
- \tan \beta \left( p \cos \alpha \cos \phi + r \sin \phi \right) \\
\dot{\beta} = \frac{1}{W} \left( Y \cos \beta + D \sin \beta - T \sin \beta \cos \phi \right) \left( p \sin x - r \cos \phi \right) \\
+ \frac{g}{V} \left( \cos \beta \cos \phi \sin \phi + \cos \alphath \sin \theta - \sin \alpha \cos \beta \phi \right) \\
D = C^1_p(\beta, \delta_{\text{ail}}), \quad L = C^1_L(\beta, \delta_{\text{ail}}), \quad \\
Y = C^1_Y(\beta, \delta_{\text{ail}}, \delta_{\text{ail}}) \\
\tag{1}
\]

where \(D, L, Y \) and \(T \) are the drag, lift, side and thrust forces. The Euler angles rates are also
formulated by
\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\
0 & \cos\phi & -\sin\phi \\
0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

The three moment equations which are directly related to the aerodynamic parameters are given below
\[
p = k I_{zz} C_l + k I_{zx} C_n + I_{yy} q r + I_{xz} p q - I_{zz} r q
\]
\[
q = \frac{0.5 \rho V^2 S \tau}{I_{yy}} C_m - I_{xx} p r + I_{zz} r^2 - I_{xz} p^2 + I_{zz} r p
\]
\[
r = k I_{zz} C_l + k I_{zx} C_n + I_{yy} p q - I_{zz} r q - I_{yy} q p
\]
\[
k = \frac{0.5 \rho V^2 S \tau}{(I_{xx} I_{zz} - I_{xz}^2)}.
\]
\[
C = \begin{bmatrix}
C_l^{xy}(\beta, \delta_{ail}, \delta_{rud}, p, r) \\
C_m^{xy}(\delta_{stab}, q) \\
C_n^{xy}(\beta, \delta_{ail}, \delta_{rud}, p, r)
\end{bmatrix}
\]

Some of the system parameters taken from the F/A-18 fighter airplane have been depicted in Table 1. The other parameters involved are $C_r$, $C_m$ and $C_n$ which are aerodynamic roll, pitch and yaw coefficients, respectively. These parameters are nonlinear functions of the system variables and often vary as flight condition changes.

Table 1. Aircraft parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing area</td>
<td>$S$</td>
<td>37.16 m$^2$</td>
</tr>
<tr>
<td>Wing span</td>
<td>$b$</td>
<td>11.4 m</td>
</tr>
<tr>
<td>Mean aerodynamic chord</td>
<td>$\bar{c}$</td>
<td>3.51 m</td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>15,097.4 kg</td>
</tr>
<tr>
<td>Roll axis moment of inertia</td>
<td>$I_{xx}$</td>
<td>31,183 kg·m$^2$</td>
</tr>
<tr>
<td>Pitch axis moment of inertia</td>
<td>$I_{yy}$</td>
<td>205,125 kg·m$^2$</td>
</tr>
<tr>
<td>Yaw axis moment of inertia</td>
<td>$I_{zz}$</td>
<td>230,414 kg·m$^2$</td>
</tr>
<tr>
<td>Cross product of inertia about y axis</td>
<td>$I_{xz}$</td>
<td>$-4028$ kg·m$^2$</td>
</tr>
<tr>
<td>gravitational constant</td>
<td>$g$</td>
<td>9.8 m/s$^2$</td>
</tr>
<tr>
<td>Air density at 25°C and 7625 m</td>
<td>$\rho$</td>
<td>0.55 kg/m$^3$</td>
</tr>
<tr>
<td>Thrust</td>
<td>$T$</td>
<td>645,000 N</td>
</tr>
<tr>
<td>Air speed</td>
<td>$V$</td>
<td>106.5 m/s</td>
</tr>
</tbody>
</table>

in terms of approximate polynomial equations such as the one for $C_l$ as below
\[
C_l = (-1.619x^4 + 2.3843x^3 - 0.3620x^2 - 0.4153x - 0.0556)\beta
+ (0.9189x^3 - 0.2646x^2 - 0.0516x + 0.1424)\delta_{ail}
+ (-0.027x^3 + 0.0083x^2 + 0.00416x + 0.0129)\delta_{rud}
+ \frac{37.42}{2V}(-1.0871x^2 + 0.7804x + 0.1983)r
+ \frac{37.42}{2V}(0.2377x - 0.3540)p
\]

By substituting $C_l$ in $\dot{p}$ (equation (1)), the roll rate nominal equation is obtained as follows
\[
\dot{p} = (0.0317q + V(0.0016x - 0.0023))p
+ (-0.8151q + V(-0.0071x^2 + 0.0051x + 0.0013))r
+ 10^{-4}p^2 (-5.6310x^4 + 8.2890x^3 - 1.2350x^2 \beta
- 1.4460x - 0.19800)
+ 10^{-5}p^2 (6.7510x^3 - 8.9920x^2 - 1.8360x
+ 49510)\delta_{ail}
+ 10^{-6}p^2 (-2.37x^4 - 4.068x^3 - 0.497x^2 + 0.594x
+ 4.959)\delta_{rud}
\]

(4)

\[
\dot{p} = 0.0317qp - 0.8151qr + K_p(x)p
+ K_r(x)r + K_{p}(x)\beta + K_{ail}(x)\delta_{ail} + K_{rud}(x)\delta_{rud}
\]

(5)

This is one of the nine equations stating the nonlinear, coupled and time varying (due to change in mass and $V$) behaviour of the simulated system. Any designed controller is expected to be able to offer stable and safe flight considering the family of the equations similar to equation (5).

Aircraft control

The simplified basic flight control of the F/A-18 aircraft is as follows
\[
\delta_{elev} = (8q + 0.8x)G_A(s)
\]
\[
\delta_{rud} = \left(0.5a_q + \frac{1.1a_x + 6}{s + 1}r\right)G_A(s)
\]
\[
\delta_{ail} = (-0.8p - 0.5\beta - 2\beta)G_A(s)
\]

where the actuator, $G_A(s)$ is modelled by a first-order lag. For the longitudinal control, $x$ and $q$ are measured.
and used for stabilization. The directional control consists of feedback from yaw rate, $r$, and lateral acceleration $x_f = f(C_y)$ to the rudder actuator. The lateral acceleration feedback aims at reducing sideslip during coordinated turn. $^2\_7$ Initially, for the roll control just a $p$ feedback, $\delta_{ail} = -0.5p$, was used, and the values of the controller parameters were validated by performing numerous simulations. $^1$

Later on, due to several high angle of attack perturbation related air crashes, it is realized that the controller is not qualified for stopping the so-called unstable falling leaf motion, a type of out of control action with substantial fluctuations in $x$ and $\beta$. $^1$ During large angle-of-attack/sideslip motion, the dihedral effect (roll caused by sideslip) of the aircraft wings becomes extremely large and the directional stability becomes unstable. $^1$

Therefore, to manage the instability, the basic control law

$$\delta_{ail} = -0.8p - 0.5\beta - 2\dot{\beta}$$

was suggested and employed that augments the roll feedback with two extra $\beta$ and $\dot{\beta}$ loops. By this, the control action is calculated in a way that not only $p$ is stabilized, but also to damp both $\beta$ and $\dot{\beta}$.

Further investigations in improving the response of the aircraft in facing falling leaf, led to the control signal

$$U_{ail} = -1.4p - 18\beta - 8\dot{\beta} - \frac{0.584}{s + 1}r$$

containing an extra heading rate loop that it is shown to be more effective than the basic one. $^2\_7$

Roll aerodynamic uncertainty

It is natural to assume approximation in the values of the parameters of equation (1). For roll control, the engaged equation is

$$\dot{p} = k_{Izz}C'_f + k_{Ixz}C'_n + I_{yy}qr + I_{xz}pq - I_{zz}rq$$

where $C'_f$ and $C'_n$ are the uncertain $C_f$ and $C_n$. The equation may be short written as follows

$$\dot{p} = K_p(x)p + K_{\beta}(x)\beta + K_{ail}(x)\delta_{ail} + d(.)$$

$$d(.) = 0.0317qp - 0.8151qr + +K_r(x)r + K_{rud}(x)\delta_{rud}$$

(6)

To model the uncertainty for the simulation purpose, a family of random systems may be generated using either of the following two approaches. In the first method, randomness is applied to the aerodynamic parameters of equation (3) (and similarly to the parameters of $C_j$) which consequently renders a set of random $p$ (6) equations. The effect of $C_m$ uncertainty in $p$ is around 2% when compared with $C_j$ uncertainty.

To illustrate the effect of uncertainty on the parameters of equation (6), $K_{ail}$ and $K_{p}$ versus $x$ have been presented in Figure 1(a) and (b) where $x$ ranges from $-0.5$ to 1.5 radians. To produce realistic random systems, $k_{ail}>0$ and $K_{p}<0$ constraints have also been applied. As the figures indicate, 10% variation in the coefficients leads to drastic large tolerances at higher $x$ values. Alternatively, the uncertainty may be applied directly to the parameters of equation (6) instead of equation (3) which yield more realistic random systems. The result of applying 30% uncertainty to the $K$ parameters of equation (6) has been shown in Figure 1(c) and (d) which looks more appropriate for the uncertainty evaluations.

Now, the prime question here is that how any randomness in the parameters of equation (6) alters the roll response and flight performance. As the figures indicate, in low angle of attack, there might be no significant operation degradation, but it may not be true when the aircraft is entangled with a high angle of attack perturbations.

Robust roll FSMC control

In our simulations, it is shown that the basic control law fails in securing flight performance in face of tolerances in the $p$ parameters (equation (6)) when the vehicle experiences some high angle of attack/sideslip perturbations. Hence, modifications to the controller or a new controller are required. For combating dynamic uncertainty, SMC is a controller of choice that cannot be ignored particularly when the size of ambiguity is supposedly unknown and large.

![Figure 1](image.png)

Figure 1. Uncertain $K_p$ and $K_{ail}$ parameters of $p$ (6).
(a) $K'\beta_1$, (b) $K'da_1$, (c) $K'\beta$ and (d) $K'da$. 
The block diagram of the proposed adaptive fuzzy sliding mode controller is shown in Figure 2, which consists of the system, sliding mode controller, fuzzy system and adaptive algorithm.

**SMC design**

To start with SMC control design, the past investigations leading to the basic control law is appreciated and a new variable, \( z \) is introduced by a weighted combination of \( p \) and \( \beta \) as follows

\[
z = p + w\beta
\]

(7)

The dynamic equation of the new variable is composed by employing \( p_\dot{} \) from equation (6) and \( \beta_\dot{} \) from equation (1). Considering the uncertainty in equation (6) and cross-coupling among variables, the dynamic of \( z \) is given by

\[
\dot{z} = az + b\delta_{all} + \Delta + d(\cdot), \quad |d(\cdot)| < \delta
\]

(8)

where \( a, \ b \) and \( \Delta \) are the nominal values and \( \delta \) is an scalar sufficiently large lumped bound for the dynamic uncertainty, \( d(\cdot) \).

**Remarks**

The uncertainty is only applied to the roll-related aerodynamic parameters \( C_l \) and \( C_n \) and the other aerodynamic parameters attain their nominal values.

Assumptions:

1. The basic controller stabilizes the states with nominal parameters including sideslip.
2. The initial conditions are within the region of attraction of the basic control law.

Other than the \( gw\beta \) term, the rest of the \( w\beta \) dynamics are collected in \( \Delta \) since uncertainty is only going to be applied to the roll parameters (Remark 1), otherwise the rudder and elevator controls have to be modified similarly.

Note that the stability of \( \beta \) directed by the nominal system rudder closed loop control (Remark 2); the asymptotical stability of \( p \) is definite if a control action forces to zero the tracking error function

\[
e = z - z_0 = p - p_{\text{trim}} + w(\beta - \beta_{\text{trim}})
\]

(9)
in response to any perturbation. In this respect, the following integral sliding surface

\[
s = e + \lambda \int e(t)dt
\]

(10)
is introduced. The system enjoys asymptotic stability if the control action, \( \delta_{all} \) makes the derivative of the quadratic Lyapunov function \( V(s) = s^2/2 \) negative definite as below

\[
\dot{V}(s) = ss'< 0
\]

To develop such a stabilizer, under constant trim condition, \( s \) is calculated

\[
\dot{s} = \dot{e} + \lambda e = 0 \quad \text{for} \quad t > 0
\]

(11)

The aileron control signal stabilizing the nominal system using feedback linearization is obtained as follows

\[
u_{eq} = -b^{-1}\left( ae + \Delta + \dot{d} + \lambda e \right)
\]

(12)

if the bound on the aileron deflection is not violated. \( \dot{d} \) is an ad hoc estimate of \( d \) if it is available, otherwise \( \dot{d} = 0 \). In case of parameter uncertainty and inaccurate estimate of \( d \), equation (12) is not adequate and as usual of SMC a switching control is needed to handle the parameter uncertainty and to push \( s \) toward the sliding surface. It is enforced by

\[
u_{sw} = -kh^{-1}\text{sgn}(s)
\]

(13)

Applying both continuous and switching control law to the system results in

\[
\dot{V} = ss' = s(\dot{e} + bu_{eq} + bu_{sw} + \Delta + \dot{d} + \lambda e)
\]

\[
\leq s(-kh^{-1}\text{sgn}(s) + \dot{d} - \dot{d}) < 0
\]

(14)

Equation (14) is negative definite and asymptotical stability is ensured for adequately large \( k \) if the
following conditions are met

\[ b \pm \delta_b > 0, \quad \delta_{ail-LB} < u_{eq} + u_{sw} < \delta_{ail-UB} \]
\[ k > |d - \hat{d}| + \eta \]

where \( \eta \) is a positive constant. It is a common practice to replace the switching operator of the discontinuous control action with the saturation function to mitigate its adverse effects as follows

\[ \delta_{ail} = u_{eq} + u_{sw} \]
\[ = -b^{-1} \left( \alpha + \Delta + \hat{d} + \lambda e + k \times \text{sat}(\hat{s}) \right) \]  \( (16) \)

here \( \hat{d} \) is an appropriate small constant.

**Adaptive k using adaptive fuzzy algorithm**

The problem of SMC chattering is relatively mitigated by using saturation instead of switching (equation (16)). However, better performance is also expected if adaptive k is used by estimating \( \hat{d} \), alternatively. The latter can be applied using fuzzy techniques.

The inputs to the fuzzy system are sliding surface, \( s \) and its derivative, \( \dot{s} \). Each of the inputs is fuzzified through a three-membership function fuzzifier, Figure 3(a). The output, defuzzifier, is composed of a nine-membership function set with adaptable membership functions centres, \( c_1 \) to \( c_9 \), Figure 3(b).

A set of nine rules composes the fuzzy inference rule bank, shown in Table 2. They are formed using the following abbreviations:

- N: Negative; Z: Zero and P: Positive
- H: Huge; B: Big; M: medium; S: Small

The \( k_1 \), \( k_2 \) and \( K_\gamma \) parameters are the fuzzy scaling factors, Figure 3. The defuzzification process is conducted using the centre of area method (COA) which is expressed by the following equation

\[ \hat{v} = \left[ \hat{c}_1 \ldots \hat{c}_9 \right], W = \frac{1}{\sum_{i=1}^{9} w_i} \left[ w_1 \ldots w_9 \right] \Rightarrow \hat{d} = \frac{\sum_{i=1}^{9} w_i c_i}{\sum_{i=1}^{9} w_i} \]

where \( \hat{c}_1 \) to \( \hat{c}_9 \) are the estimates of the centre of \( \hat{d} \) membership functions and \( W \) is a vector of degree of membership to each of the nine \( \hat{d} \) membership functions.

Let \( \hat{d}^* = \hat{v}^* W \) be an optimum value for the uncertainty, \( d \), where \( \hat{v}^* \) is its associated optimal parameter. The adaptive fuzzy estimator is expected to minimize \( \hat{d} - \hat{d}^* \) distance or alternatively \( e = \hat{v} - \hat{v}^* \) adopting an appropriate adaptation law, while the overall system stability is also observed. This is ensured by employing the following two section positive definite Lyapunov functions

\[ V_T = \frac{1}{2} s^2 + \frac{1}{2\gamma} e^2, \quad e = \hat{v} - \hat{v}^* \]

where \( \gamma \) is a positive constant. Taking the time derivative of \( V_T \) and using equation (14) yields

\[ V_T = ss + \frac{1}{\gamma} \dot{e}e = \]
\[ = s\left( ae + bu_{eq} + bu_{sw} + \Delta + \hat{d} - \hat{d}^* + d - \hat{d}^* + \dot{\hat{d}} + \dot{\hat{d}}^* \right) + \frac{1}{\gamma} \dot{e}e \]

Now, by using equations (12) and (13), the following results are obtained

\[ V_T = s(-bb \hat{v}^{-1} \text{sgn}(s) + d - \hat{d}^* + \hat{d} - \hat{d}^*) + \frac{1}{\gamma} \dot{e}e \]
\[ = s(-bb \hat{v}^{-1} \text{sgn}(s) + d - \hat{d}^*) + esW(s, \dot{s}) + \frac{1}{\gamma} \dot{e}e \]
\[ = s(-bb \hat{v}^{-1} \text{sgn}(s) + \rho) + \frac{1}{\gamma} \dot{e}(\dot{e} + \gamma eW(s, \dot{s})) \]

(17)

Next, by adopting the adaptation law

\[ \dot{e} = -\gamma sW(s, \dot{s}) \Rightarrow \dot{v} = -\gamma sW(s, \dot{s}) \]
the second part of equation (17) is crossed out and it is reduced to

\[ \dot{V}_T = s(-b k b^{-1} \text{sgn}(s) + \rho) \leq -\eta |s| \]

where more mild condition for \( k \) than equation (15) is reached as below

\[ |d - \hat{d}| + \eta > k > \rho + \eta > |d - \hat{d}^*| + \eta \]

The \( \gamma \) parameter is related to the adaptation rate. By this provision, the level of switch control is no longer constant but depends on the error values defined by \( s \) and \( \hat{s} \).

**Simulations**

The system (equation (1)) is simulated using parameters depicted in Table 1 and aerodynamic parameters reported in Chakraborty et al.\(^1\). Apart from the nominal system, a family of 50 randomly generated systems with 30\% tolerance in the \( K^* \) parameters of equation (6) is produced.

The performance of the basic and the proposed AFSM controllers are evaluated in maintaining reliable and secure flight operation in spite of the aerodynamic parameter uncertainty under high angle of attack/side-slip perturbations. The basic controller can handle system uncertainty well under low angle of attack flight running. However, in facing a type of manoeuvre reported in Chakraborty et al.,\(^1\) which may lead to falling leaf instability, serious problems occur. One of such operations occurs under the following working point (trim values)

\[
x = [V \; \beta \; \alpha \; p \; q \; r \; \phi \; \theta \; \psi]^T
\]

\[
x_{\text{Trim}} = [106.5 \; 0 \; 20 \; -1 \; 2 \; 2.5 \; 35 \; 19 \; 0]^T
\]

Four sets of initial condition perturbations are considered (Remark 3). The first one is a high angle of attack and sideslip perturbation given by the following vector

\[
x_{\text{Initial}} = [106.5 \; 50 \; 80 \; 24 \; 2 \; 3 \; 60 \; 19 \; 0]^T
\]

First, a nonrealistic simulation with no bound on the aileron deflection angle is performed. The responses of the two controllers have been presented in Figure 4. The AFSMC parameters are \( w = 4 \) and \( \lambda = 2 \). As it is noted from the graphs, the proposed controller stables the system faster with lower control effort than what the basic controller demands.

Imposing a bound on the aileron deflection (\(-25^\circ\) to \(45^\circ\))\(^1\) and repeating the test under similar conditions render performances portrayed in Figure 5. Faster response and more coherent behavior, which is expected from a well-designed robust controller, is achieved For the sake of conciseness, the other stable states are not shown.

The superiority of using \( z \) instead of \( p \) in forming the sliding surface has been illustrated in Figure 6. By using \( w = 0 \), the sliding surface based on \( z = p \) is formed. The response of the system with \( \lambda = 2 \) has been depicted in Figure 6 (left). This shows how the system behaviour ends up in instability and flight failure. The same test is repeated with \( \lambda = 0.1 \) (slower convergence) by which the system stability is restored. Therefore, the
sliding surface based on z, yields faster transient response without stability compromise.

In the second test, the initial condition

\[ x_{\text{Initial}} = [106.5 \ 20 \ 80 \ 9 \ 2 \ 8 \ 35 \ 19 \ 0]^T \]

is used in which the angle of attack initial condition is high, while the sideslip initial condition is medium. The performances of the AFSMC and the basic controller have been shown in Figure 7. The difference in coherency in the performance (robustness) is obvious from the figures plus faster transient response, the two advantages of the suggested controller against the basic one.

The third test is conducted under the following initial condition

\[ x_{\text{Initial}} = [106.5 \ 60 \ 30 \ 25 \ 25 \ 20 \ 25 \ 0 \ 0]^T \]

The results of the medium angle of attack and high sideslip perturbation are portrayed in Figure 8. Similar conclusions, i.e. faster transient and performance that is more robust are certified.

Eventually, the last test is conducted under the initial condition of

\[ x_{\text{Initial}} = [106.5 \ 20 \ 30 \ 30 \ 20 \ 3 \ 35 \ 10 \ 0]^T \]

with medium angle of attack and sideslip angle perturbations. The results have been exhibited in Figure 9. Improvement in the response is projected in lower control effort needed for providing asymptotical stability.
The performance of the algorithm under other operating conditions within the region of attraction confirms the validity of the findings.

Conclusion
In this paper, the sensitivity of an aircraft roll control to the approximation in its aerodynamic parameters under high value of angle of attack and or sideslip perturbations are discussed. It is shown that at low values of perturbations, the performance of the already designed controller can be viewed acceptable and the flight performance is not degraded substantially. However, under high angle of attack/sideslip disturbances, like those that may end up in the falling leaf instability, the effect of approximation and uncertainty is substantial to the extent that it may cause air crash. In this respect, a novel sliding surface by a weighted combination of \( p \) and \( \beta \) is suggested. SMC is designed to contain the adverse effect of randomness in the system dynamic. The smoothness of the behaviour is managed by an adaptive fuzzy section that estimates the level of uncertainty and applies nonlinear switching control gain. As a result, the proposed controller enjoys performance that is more robust and delivers faster transient response with respect to the basic controller under various initial condition perturbations. This is proved theoretically and verified through extensive simulations.

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