State-dependent Multiple Access Relay Channel with Cooperating Transmitters

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***Abstract*— In this paper, we consider a state-dependent multiple access relay channel (MARC) with two-way cooperation at transmitters, in which the transmitters cooperate with each other over two limited-rate channels. We derive an achievable rate region for this channel by using block-Markov superposition coding ,and double binning techniques. In our proposed scheme, the relay decodes only parts of the messages which are shared between the transmitters.**

I. Introduction

State-dependent channels play an important role in analyzing the performance of communication systems since the state is used to model the phenomena such as fading and interference. Shannon introduced a state-dependent channel, in which the channel state information (CSI) is known causally at the transmitter (TX) and he derived the capacity of this channel [1]. The capacity of the discrete memoryless state-dependent channel where the state known non-causally at the TX was characterized by Gel'fand and Pinsker [2]. Furthermore, the capacity of a single user channel, where the partial CSI is known at both TX and receiver (RX), was studied in [3]. Multiuser channels such as multiple access [4], [5], broadcast [6] and relay [7]-[10] channels were widely studied.

The relay channel is a model of communication between TX and RX with the help of one or more transceivers as relays. In 1971, Van der Meulen [11] introduced the relay channel as a special case of a three terminal network. Cover and El Gamal [12] proposed a number of important concepts for the relay channel such as block-Markov superposition coding, random binning and side information coding and by this concepts presented two basic coding schemes: Decode and Forward (DF) and Compress and Forward (CF).

The state-dependent relay channel is a basic model for cooperation in the channels which face fading and interference. Based on the extension of the *Shannon strategy* [1] and DF [12] scheme the capacity of the degraded discrete memoryless state-dependent relay channel was found [7]. In [7], the CSI is available at both the TX and the relay in a causal manner. Some classes of state-dependent relay channel are considered in [13]. The state-dependent relay channel with causal and non-causal CSI was considered in [8], [9]. The authors used CF [12, Theorem 6] technique which is advantageous compared to DF scheme when the channel from the TX to the relay is worse than the channel from the source to the destination. The state-dependent relay channel with conferencing links between the relay and the TX was investigated in [14], where only the relay knows the strictly casual CSI. Moreover, the transmission schemes have been compared in [14]. The relay may be used to help more than one TX in a MARC setup. The Multiple Access Relay Channel (MARC) was considered in [21] and a new strategy for decoding (offset decoding) was proposed for this channel. Furthermore, a number of cooperative strategies for the MARC and some classes of the relay networks have been considered in [22]. A general achievable rate region for a MARC with common message was obtained in [23]. Also, MARC with non-causal channel state information at the relay was considered in [24] and an inner bound as well as an outer bound was achieved for this channel.

One way to schedule the multiple transmitters is to make them cooperate through conferencing links. Initially, Willems [15] introduced Multiple Access Channel (MAC) with cooperating TXs and derived the capacity region. He showed that if the TXs use noise-free limited-rate cooperation links to share parts of their messages, capacity region can be achieved. The coding scheme which has been used for this purpose was introduced by Slepian and Wolf [16] for achieving the capacity region of MAC with correlated TXs. Various models of state-dependent MAC with cooperating TXs are considered in [17], [18]. The state-dependent MAC with two-way cooperation between the TXs has been proposed in [5], where each TX non-casually knows a part of CSI and the RX knows full CSI. It has been shown that by using a new binning technique, named double binning [19], [20], the capacity can be achieved. The question we are interested in is how relaying strategies can be employed with cooperating TXs.

In this paper, we investigate the state-dependent MARC, where each TX has CSI. We propose an achievable scheme that increases the data transmission rate with two classes of cooperation. The first one is among the TXs which is achieved by two limited-rate channels between them. The other one is between the relay and the TXs. The TXs cooperate with each other and share their information about the channel state and parts of their messages. These parts of the messages which are shared by the TXs, are decoded by the relay, that has complete CSI. Our scheme is based on the combination of the double binning and block-Markov superposition coding to

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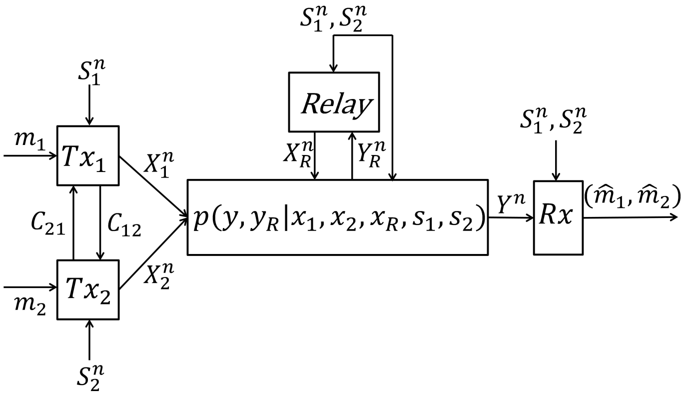

Fig. 1. State-dependent Multiple Access Relay Channel (MARC)

achieve this two classes of cooperation together.

The rest of the paper is organized as follows. In Section II, we present the discrete memoryless state-dependent MARC and the related definitions. In Section III, the main results are presented. Finally, the paper is concluded in Section IV.

Notation: we use capital letters to show random variables (e.g., X); lower case letters for realization of random variables (e.g., x) and $p(x)$ to show the probability mass function (p.m.f) of random variable X on set $\mathcal{X}$. Also, we define $X_{i,j}^n$ as an n-sequence X_i^n specifically in block j. Also the set of ϵ-strongly joint-typical of n-sequences X^n and Y^n with p.m.f $p(x, y)$, is shown by $T_\epsilon^n(X, Y)$. The two-user discrete memoryless state-dependent MARC, as shown in Fig. 1, is defined with the alphabet sets $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_R, \mathcal{Y}, \mathcal{Y}_R, \mathcal{S}_1, \mathcal{S}_2)$ and a probability transition function $p(y, y_R|x_1, x_2, x_R, s_1, s_2)$ which is defined for all $(x_1, x_2, x_R, s_1, s_2, y, y_R) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_R \times \mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{Y} \times \mathcal{Y}_R$, where X_1^n, X_2^n are the TXs inputs and X_R^n is the relay input; Y^n, Y_R^n are RX and relay outputs, respectively; S_1^n, S_2^n are channel state sequences which are independent and identically distributed (i.i.d.) according to p.m.f $p(s_1, s_2)$. Each TX knows partial CSI (transmitters 1 and 2, know S_1^n and S_2^n, respectively) and the relay and the RX know full CSI in a non-causal manner, i.e., (S_1^n, S_2^n). Each TX wishes to transmit a message $m_l, l \in \{1, 2\}$ which is uniformly distributed over message set $\mathcal{M}_l$ with cardinality M_l. We define $\frac{\log M_l}{n}$ as the transmission rate R_l. It is neede to say that both TXs talk to each other over two transmission links with the limitted capacities C_{12} and C_{21}. TX_1 starts this process.

II. Problem Setup

Definition: A $(2^{nR_1}, 2^{nR_2}, 2^{nC_{12}}, 2^{nC_{21}}, n)$ code for this model consists of the following encoding functions:
at TX_1

$$\begin{aligned} &f_{12} : \{1...2^{nR_1}\} \times \mathcal{S}_1^n \to \{1...2^{nC_{12}}\}, \\ &f_1 : \{1...2^{nR_1}\} \times \{1...2^{nC_{21}}\} \times \mathcal{S}_1^n \to \mathcal{X}_1^n, \end{aligned} \tag{1}$$

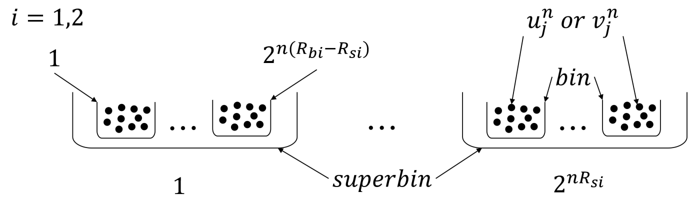

Fig. 2. Double-binning consists of superbins contain bins and bins contain codewords

at TX_2

$$\begin{aligned} &f_{21} : \{1...2^{nR_2}\} \times \{1...2^{nC_{12}}\} \times \mathcal{S}_2^n \to \{1...2^{nC_{21}}\}, \\ &f_2 : \{1...2^{nR_2}\} \times \{1...2^{nC_{12}}\} \times \mathcal{S}_2^n \to \mathcal{X}_2^n \end{aligned} \tag{2}$$

and a relay encoder:

$$f_{R,i} : \mathcal{S}_1^n \times \mathcal{S}_2^n \times \mathcal{Y}_R^{i-1} \to \mathcal{X}_R^i \tag{3}$$

and a decoding function:

$$g : \mathcal{S}_1^n \times \mathcal{S}_2^n \times \mathcal{Y}^n \to \{1...2^{nR_1}\} \times \{1...2^{nR_2}\} \tag{4}$$

The average probability of error for a given code is

$$\begin{aligned} P_\epsilon^{(n)} = \frac{1}{2^{n(R_1+R_2)}} \sum_{m_1, m_2} Pr\{g(S_1^n, S_2^n, Y^n) \neq (m_1, m_2)| \\ (m_1, m_2) \text{sent}\}. \end{aligned} \tag{5}$$

A rate pair (R_1, R_2) is achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, 2^{nC_{12}}, 2^{nC_{21}}, n)$ codes with $P_\epsilon^{(n)} \to 0$ as $n \to \infty$. The capacity region is the closure of all achievable rates.

III. Main Results

In this section, based on block-Markov coding and double binning techniques (Fig. 2), we establish a lower bound for the state-dependent MARC with cooperating TXs. These two techniques provide two classes of cooperation where the first one is between the TXs and the other one is between the relay on one side and the TXs on the other side. The common messages shared between the TXs play important roles in achieving these two cooperation strategies together.

Theorem 1: For a two-user discrete memoryless state-dependent MARC with two way cooperation between transmitters, the following rate region is achievable:

$$\begin{aligned} &C_{12} \geq I(U; S_1|S_2, Z), \\ &C_{21} \geq I(V; S_2|S_1, Z, U), \\ &R_1 \leq C_{12} - I(U; S_1|S_2, Z) \\ &\qquad + I(X_1; Y|S_1, S_2, Z, U, V, X_R, X_2), \\ &R_2 \leq C_{21} - I(V; S_2|S_1, Z, U) \\ &\qquad + I(X_2; Y|S_1, S_2, Z, U, V, X_R, X_1), \\ &R_1 + R_2 \leq I(Z, U, V, X_R, X_1, X_2; Y|S_1, S_2), \\ &R_1 + R_2 \leq I(U, V; Y_R|S_1, S_2, Z, X_R) \\ &\qquad + I(X_1, X_2; Y|S_1, S_2, Z, U, V, X_R), \\ &R_1 + R_2 \leq C_{12} - I(U; S_1|S_2, Z) + C_{21} - I(V; S_2|S_1, Z, U) \\ &\qquad + I(X_1, X_2; Y|S_1, S_2, Z, U, V, X_R), \end{aligned} \tag{6}$$

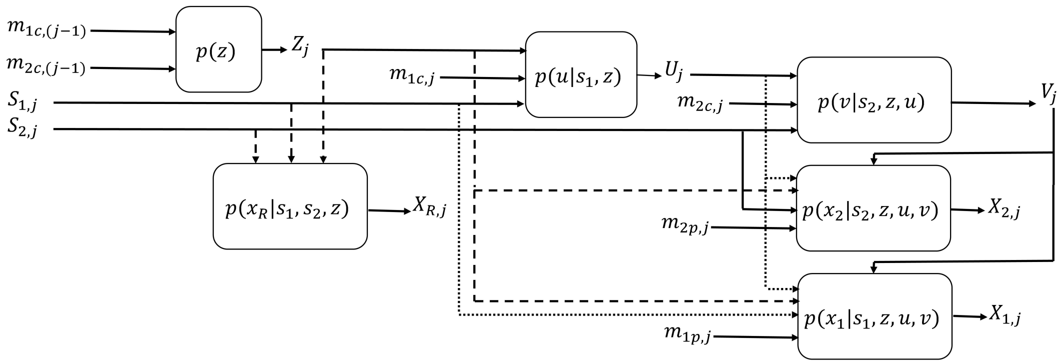

Fig. 3. Generation of random variables in block j

for some joint distribution that can be factored in the form of

$$\begin{gathered} p(s_1,s_2,z,u,v,x_R,x_1,x_2,y_R,y)= \\ p(s_1,s_2)p(z)p(u|s_1,z)p(v|s_2,z,u) \\ \times p(x_R|s_1,s_2,z)p(x_1|s_1,z,u,v)p(x_2|s_2,z,u,v) \\ \times p(y_R,y|s_1,s_2,x_R,x_1,x_2) \end{gathered} \tag{7}$$

where Z,U,V are auxiliary random variables.

Remark 1: The inner bound region of state-dependent MAC with cooperating encoders [5] is obtained by setting $Z = X_R = Y_R = \emptyset$ in (6).

Proof: In the proof, we use block-Markov superposition encoding and double binning techniques [5], [19], [20]. We use double binning because the first layer of binning is needed to produce a common message between the transmitters and the other layer is needed to originate an empirical state coordination between the transmitters. Also we use block-Markov superposition encoding technique for the relay. The transmitters share parts of their messages, which are known to both of them at the end of each block. These parts, referred to them as common messages, generate the basis of cooperation at the next block between the relay and transmitters. Each transmitter transmits $B-1$ message at B block of transmission.

Codebook generation:

For each block, randomly and independently we generate a codebook. We illustrate the codebook generation for block $j \in \{1...B\}$.

For any joint p.m.f defined in (7):

1) Generate $2^{n(R_{s1}+R_{s2})}$ i.i.d. codewords Z_j^n with p.m.f $p(z)$. For each Z_j^n, generate 2^{nR_u} i.i.d. codewords U_j^n with p.m.f $p(u|z)$. Also, consider $2^{nR_{s1}}$ superbins and $2^{nR_{b1}}$ bins (Fig. 2). Partition 2^{nR_u} codewords U_j^n into $2^{nR_{s1}}$ equal-size superbins. Thus each superbin contains $2^{n(R_u-R_{s1})}$ codewords U_j^n. In the same way, partition 2^{nR_u} codewords U_j^n into $2^{nR_{b1}}$ equal-size bins .Therefore, each bin contains $2^{n(R_u-R_{b1})}$ codewords U_j^n and each superbin contains $2^{n(R_{b1}-R_{s1})}$ bins. Superbins represent the common messages. The bin number has information about the channel state and the common message sent from TX_1 to TX_2. It is good to notice that, because of the limited capacity of the cooperation link from TX_1 to TX_2, the total number of bins cannot exceed $2^{nC_{12}}$.
2) For each pair of (Z_j^n,U_j^n), generate 2^{nR_v} i.i.d. codewords V_j^n with p.m.f $p(v|z,u)$. Also, generate $2^{nR_{s2}}$ superbins and $2^{nR_{b2}}$ bins (Fig. 2). Partition 2^{nR_v} codewords V_j^n into $2^{nR_{s2}}$ equal-size superbins. Thus each superbin contains $2^{n(R_v-R_{s2})}$ codewords V_j^n. In the same way, partition 2^{nR_v} codewords V_j^n into $2^{nR_{b2}}$ equal-size bins. Therefore, each bin contains $2^{n(R_v-R_{b2})}$ codewords V_j^n and each superbin contains $2^{n(R_{b2}-R_{s2})}$ bins. The bin number sent from TX_2 to TX_1. Since the capacity of the cooperation link from TX_2 to TX_1 is limited, the total number of bins cannot exceed $2^{nC_{21}}$.
3) For each Z_j^n and for each pair of $(S_{1,j}^n,S_{2,j}^n)$, generate an i.i.d. codeword $X_{R,j}^n$ according to p.m.f $p(x_R|s_1,s_2,z)$.
4) For each triple (Z_j^n,U_j^n,V_j^n) and for each $S_{1,j}^n$, generate $2^{n(R_1-R_{s1})}$ i.i.d. codewords $X_{1,j}^n$ with p.m.f $p(x_1|s_1,z,u,v)$ and similarly, for each triple (Z_j^n,U_j^n,V_j^n) and for each $S_{2,j}^n$, generate $2^{n(R_2-R_{S2})}$ i.i.d. codewords $X_{2,j}^n$ with p.m.f $p(x_2|s_2,z,u,v)$.

Encoding at the beginning of block j:

Generation of all codes needed to prove the theorem is shown in Fig. 3. Split $m_1 \in \{1...2^{nR_1}\}$ into two messages $m_{1c} \in \{1...2^{nR_{s1}}\}$ which represents the common message for TX_1 and $m_{1p} \in \{1...2^{n(R_1-R_{s1})}\}$ which represents the private message for TX_1. Similarly split $m_2 \in \{1...2^{nR_2}\}$ into common message $m_{2c} \in \{1...2^{nR_{s2}}\}$ and private message $m_{2p} \in \{1...2^{n(R_2-R_{s2})}\}$. Since the TXs know the common messages of the block $j-1$, i.e., $(m_{1c,(j-1)},m_{2c,(j-1)})$, a codeword, denoted by $Z_j^n(m_{1c,(j-1)},m_{2c,(j-1)})$ is chosen at both TXs. For each Z_j^n, there are $2^{nR_{s1}}$ superbins. Associate each $m_{1c,j}$ with a superbin. Thus a superbin is chosen and a codeword is searched in the superbin, denoted by $U_j^n(m_{1c,j},s_{1,j}^n,z_j^n)$, that is jointly typical with $(S_{1,j}^n,Z_j^n)$. Then, TX_1 sends its bin number to TX_2. If such a codeword U_j^n does not exist in the superbin, chooses an arbitrary U_j^n from the superbin and if more than one such codewords U_j^n are found, chooses the smallest one. TX_2 receives the bin number from TX_1 and looks for a codeword U_j^n that is jointly typical with $(S_{2,j}^n,Z_j^n)$. If such a codeword U_j^n does not exist in the bin, an arbitrary

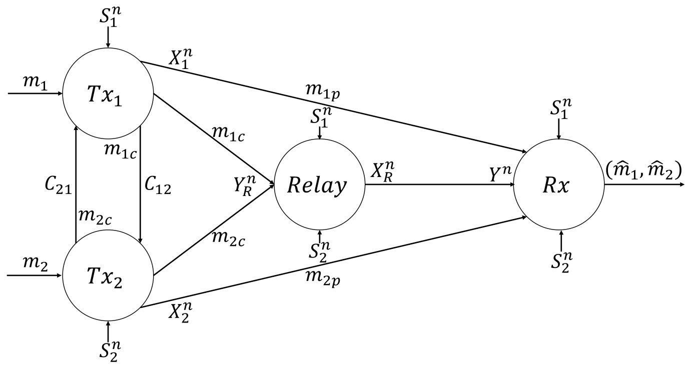

Fig. 4. State-dependent MARC

U_j^n from the bin is chosen.

For each pair of (Z_j^n, U_j^n), there are $2^{nR_{s2}}$ superbins. Associate each $m_{2c,j}$ with a superbin. Thus, a superbin is chosen and a codeword is searched in the superbin, denoted by $V_j^n(m_{2c,j}, s_{2,j}^n, z_j^n, u_j^n)$, that is jointly typical with $(S_{2,j}^n, Z_j^n, U_j^n)$. Then TX_2 sends its bin number to TX_1. If such a codeword V_j^n does not exist in the superbin, chooses an arbitrary V_j^n from the superbin and if more than one such codewords V_j^n are found, chooses the smallest one. TX_1 receives bin number from TX_2 and looks for a codeword V_j^n that is jointly typical with $(S_{1,j}^n, Z_j^n, U_j^n(m_{1c,j}, S_{1,j}^n, Z_j^n))$. If such a codeword V_j^n does not exist in the bin, an arbitrary V_j^n from the bin is chosen.

At the end of block $j-1$, assume that the relay decoded the common messages ($m_{1c,(j-1)}$, $m_{2c,(j-1)}$) correctly otherwise error will be declared (Fig. 4). Thus, the relay knows $Z_j^n(m_{1c,(j-1)}, m_{2c,(j-1)})$ and transmits $X_{R,j}^n(s_{1,j}^n, s_{2,j}^n, z_j^n)$.

In block j, TX_1 and TX_2 send $X_{1,j}^n(m_{1p,j}, s_{1,j}^n, z_j^n, u_j^n, v_j^n)$ and $X_{2,j}^n(m_{2p,j}, s_{2,j}^n, z_j^n, u_j^n, v_j^n)$, respectively. The codewords of the bock j and $j+1$ are given in Table 1.

Before starting proof of the theorem, it is reasonable to examine the roles of the auxiliary random variables (rv) introduced before. The rv Z serves as cooperating rv between TXs and Relay. Each rv U and V plays double role. They not generate common messages between TXs, but coordinate among them regarding the CSI.

Decoding:

In addition to the decoding at the relay and at the RX, each TX has a decoding process.

At the beginning of block j:

1) TX_1 searches in the superbin associated with $m_{1c,j}$ for a codeword U_j^n such that

$$(u_j^n, z_j^n, s_{1,j}^n) \in T_\epsilon^{(n)}(U, Z, S_1)$$

According to covering lemma [26], there exists a codeword U_j^n with high probability, if $n \to \infty$ and

$$R_u - R_{s1} > I(U; S_1|Z). \tag{8}$$

2) TX_2 searches for a codeword U_j^n in the bin sent from TX_1 such that

$$(u_j^n, z_j^n, s_{2,j}^n) \in T_\epsilon^{(n)}(U, Z, S_2)$$

According to joint typicality lemma [26], there exists a unique codeword U_j^n with high probability, if $n \to \infty$ and

$$R_u - R_{b1} < I(U; S_2|Z) \tag{9}$$

Also there is a capacity constraint for the cooperation channel from TX_1 to TX_2. Since the bin number should be sent from TX_1 to TX_2, and there are totally $2^{nR_{b1}}$ bins and finite capacity of this channel is C_{12}, we obtain:

$$R_{b1} < C_{12}. \tag{10}$$

3) TX_2 searches in the superbin associated with $m_{2c,j}$ for a codeword V_j^n such that

$$(v_j^n, u_j^n, z_j^n, s_{2,j}^n) \in T_\epsilon^{(n)}(V, U, Z, S_2)$$

According to covering lemma [26], there exists a codeword V_j^n with high probability, if $n \to \infty$ and

$$R_v - R_{s2} > I(V; S_2|Z, U). \tag{11}$$

4) TX_1 searches for a codeword V_j^n in the bin sent from TX_2 such that

$$(v_j^n, u_j^n, z_j^n, s_{1,j}^n) \in T_\epsilon^{(n)}(V, U, Z, S_1).$$

According to joint typicality lemma [26], there exists a unique codeword V_j^n with high probability, if $n \to \infty$ and

$$R_v - R_{b2} < I(V; S_1|Z, U). \tag{12}$$

Also there is a capacity constraint for the cooperation channel from TX_2 to TX_1. Since the bin number should be sent from TX_2 to TX_1, and there are totally $2^{nR_{b2}}$ bins and finite capacity of this channel is C_{21}, we obtain:

$$R_{b2} < C_{21}. \tag{13}$$

At the end of block j, the relay searches for indices $\hat{m}_{1c,j}$ and $\hat{m}_{2c,j}$ such that

$$\begin{aligned}(u_j^n(\hat{m}_{1c,j}, s_{1,j}^n, z_j^n), v_j^n(\hat{m}_{2c,j}, s_{2,j}^n, z_j^n, u_j^n), z_j^n, x_{R,j}^n, s_{1,j}^n,\\ s_{2,j}^n, y_{R,j}^n) \in T_\epsilon^{(n)}(U, V, Z, X_R, S_1, S_2, Y_R)\end{aligned} \tag{14}$$

Now, we analyze the probability of error at the relay. Assume $(m_{1c,j}, m_{2c,j}) = (1, 1)$. We define the event:

$$\begin{aligned}E_{a,b} \triangleq \{(u_j^n(a, s_{1,j}^n, z_j^n), v_j^n(b, s_{2,j}^n, z_j^n, u_j^n), z_j^n, x_{R,j}^n, s_{1,j}^n,\\ s_{2,j}^n, y_{R,j}^n) \in T_\epsilon^{(n)}(U, V, Z, X_R, S_1, S_2, Y_R)\}\end{aligned} \tag{15}$$

Using the union bound the probability of error at the relay at the end of block j is derived as

$$\begin{aligned}P_e^{(n)} \leq Pr(E_{1,1}^c) + \textstyle\sum_{a=1,b\neq 1} Pr(E_{a,b}) + \sum_{a\neq 1,b=1} Pr(E_{a,b})\\ + \textstyle\sum_{a\neq 1,b\neq 1} Pr(E_{a,b})\end{aligned} \tag{16}$$

The first term goes to zero by Asymptotic Equipartition Property (AEP) theorem. According to joint typicality lemma [26] the other terms go to zero, if $n \to \infty$ and

$$\begin{aligned}R_{s2} &< I(V; Y_R|S_1, S_2, Z, U, X_R), \\ R_{s1} + R_{s2} &< I(U, V; Y_R|S_1, S_2, Z, X_R).\end{aligned} \tag{17}$$

TABLE I

CODEWORDS OF BLOCKS J AND J+1

block j	block j+1
$Z_j^n(m_{1c,(j-1)},m_{2c,(j-1)})$	$Z_{j+1}^n(m_{1c,j},m_{2c,j})$
$U_j^n(m_{1c,j},s_{1,j}^n,z_j^n)$	$U_{j+1}^n(m_{1c,(j+1)},s_{1,(j+1)}^n,z_{j+1}^n)$
$V_j^n(m_{2c,j},s_{2,j}^n,z_j^n,u_j^n)$	$V_{j+1}^n(m_{2c,(j+1)},s_{2,(j+1)}^n,z_{j+1}^n,u_{j+1}^n)$
$X_{R,j}^n(s_{1,j}^n,s_{2,j}^n,z_j^n)$	$X_{R,(j+1)}^n(s_{1,(j+1)}^n,s_{2,(j+1)}^n,z_{j+1}^n)$
$X_{1,j}^n(m_{1p,j},s_{1,j}^n,z_j^n,u_j^n,v_j^n)$	$X_{1,(j+1)}^n(m_{1p,(j+1)},s_{1,(j+1)}^n,z_{j+1}^n,u_{j+1}^n,v_{j+1}^n)$
$X_{2,j}^n(m_{2p,j},s_{2,j}^n,z_j^n,u_j^n,v_j^n)$	$X_{2,(j+1)}^n(m_{2p,(j+1)},s_{2,(j+1)}^n,z_{j+1}^n,u_{j+1}^n,v_{j+1}^n)$

We also mention that the constraint caused by the case $(a \neq 1, b = 1)$ is redundant.

At the RX, we use backward decoding technique [26] and utilize the channel outputs at the block j and $j+1$ to decode $((m_{1c,j}, m_{1p,j}), (m_{2c,j}, m_{2p,j}))$ .In backward decoding scheme, RX waits until all blocks are received and then starts decoding process by using two last blocks (instead of two first block).

At the end of block $j+1$, at first, the RX searches for indices $\hat{m}_{1c,j}$ and $\hat{m}_{2c,j}$ such that

$$\begin{gathered}(z_{j+1}^n(\hat{m}_{1c,j},\hat{m}_{2c,j}), x_{R,j+1}^n(s_{1,j+1}^n, s_{2,j+1}^n, z_{j+1}^n(\hat{m}_{1c,j},\hat{m}_{2c,j})),\\ u_{j+1}^n(\hat{m}_{1c,j+1}, s_{1,j+1}^n, z_{j+1}^n), v_{j+1}^n(\hat{m}_{2c,j+1}, s_{2,j+1}^n, z_{j+1}^n, u_{j+1}^n),\\ s_{1,j+1}^n, s_{2,j+1}^n, y_{j+1}^n \in T_\epsilon^{(n)}(Z, X_R, U, V, S_1, S_2, Y)\end{gathered} \tag{18}$$

According to joint typicality lemma [26], such indices are found with approximately zero probability of error if

$$R_{s1} + R_{s2} < I(U, V, Z, X_R; Y|S_1, S_2) \tag{19}$$

Based on indices $\hat{m}_{1c,j}$ and $\hat{m}_{2c,j}$ found for all blocks, the RX searches for indices $\hat{m}_{1p,j}$ and $\hat{m}_{2p,j}$ at the end of block j such that

$$\begin{gathered}(u_j^n(\hat{m}_{1c,j}, s_{1,j}^n, z_j^n), v_j^n(\hat{m}_{2c,j}, s_{2,j}^n, z_j^n, u_j^n(\hat{m}_{1c,j}, s_{1,j}^n, z_j^n)),\\ x_{1,j}^n(\hat{m}_{1p,j}, s_{1,j}^n, z_j^n, u_j^n(\hat{m}_{1c,j}, s_{1,j}^n, z_j^n), v_j^n(\hat{m}_{2c,j}, s_{2,j}^n, z_j^n, u_j^n)),\\ x_{2,j}^n(\hat{m}_{2p,j}, s_{2,j}^n, z_j^n, u_j^n(\hat{m}_{1c,j}, s_{1,j}^n, z_j^n), v_j^n(\hat{m}_{2c,j}, s_{2,j}^n, z_j^n, u_j^n),\\ z_j^n(m_{1c,j-1}, m_{2c,j-1}), x_{R,j}^n(s_{1,j}^n, s_{2,j}^n, z_j^n), s_{1,j}^n, s_{2,j}^n, y_j^n)) \in\\ T_\epsilon^{(n)}(U, V, X_1, X_2, Z, X_R, S_1, S_2, Y)\end{gathered} \tag{20}$$

Now, we analyze the probability of error. Assume $((m_{1c,j}, m_{1p,j}), (m_{2c,j}, m_{2p,j})) = ((1, 1), (1, 1))$. We define the event:

$$\begin{gathered}E_{a,b} \triangleq \{(u_j^n(1, s_{1,j}^n, z_j^n), v_j^n(1, s_{2,j}^n, z_j^n, u_j^n(1, s_{1,j}^n, z_j^n)),\\ x_{1,j}^n(a, s_{1,j}^n, z_j^n, u_j^n(1, s_{1,j}^n, z_j^n), v_j^n(1, s_{2,j}^n, z_j^n, u_j^n)),\\ x_{2,j}^n(b, s_{2,j}^n, z_j^n, u_j^n(1, s_{1,j}^n, z_j^n), v_j^n(1, s_{2,j}^n, z_j^n, u_j^n)),\\ z_j^n(m_{1c,j-1}, m_{2c,j-1}), x_{R,j}^n(s_{1,j}^n, s_{2,j}^n, z_j^n), s_{1,j}^n, s_{2,j}^n, y_j^n) \in\\ T_\epsilon^{(n)}(U, V, X_1, X_2, Z, X_R, S_1, S_2, Y)\}\end{gathered} \tag{21}$$

Then we bound the union of probability of error, as:

$$\begin{aligned}P_e^{(n)} &\leq Pr(E_{1,1}^c) + \sum_{a\neq 1, b=1} Pr(E_{a,b})\\ \sum_{a=1, b\neq 1} Pr(E_{a,b}) &+ \sum_{a\neq 1, b\neq 1} Pr(E_{a,b})\end{aligned} \tag{22}$$

The first term goes to zero by AEP. According to joint typicality lemma [26] as $n \to \infty$ the other terms go to zero respectively if

$$\begin{aligned}R_1 - R_{s1} &< I(X_1; Y|S_1, S_2, Z, U, V, X_R, X_2),\\ R_2 - R_{s2} &< I(X_2; Y|S_1, S_2, Z, U, V, X_R, X_1),\\ R_1 + R_2 - R_{s1} - R_{s2} &< I(X_1, X_2; Y|S_1, S_2, Z, U, V, X_R).\end{aligned} \tag{23}$$

Finally, by combining (8)-(13), (17), (19) and (23) and applying the Fourier-Motzkin elimination [26] we see that if (R_1, R_2) satisfies (6), when $n \to \infty$, $P_\epsilon^{(n)}$ goes to zero.

IV. CONCLUSION

In this paper, we investigated a discrete memoryless state-dependent MARC with two way cooperation links between the TXs, in which the TXs know the partial CSI and the relay and the RX know full CSI. We used a compound cooperation strategy for this channel by using the block-Markov superposition coding and double binning techniques. Double binning was used for sharing the CSI and parts of the messages between the TXs. Based on the messages shared between the TXs and by using block-Markov superposition coding, TXs can cooperate with the relay. By using this two cooperation strategy, we derived an achievable rate region for this channel. In our future work, we intend to utilize further rate-splitting to enable the relay to perform partial decoding on the common messages which are shared between the TXs.

REFERENCES

[1] C. E. Shannon,"Channels with side information at the transmitter," *IBM Journal Res. Develop.*, pp. 289–293, 1958.

[2] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters,"*Prob. Contr. Info. Theory*, vol. 9, no. 1, pp. 19–31, 1980.

[3] G. Caire and S. Shamai, "On the capacity of some channels with channel state information," *IEEE Trans. Info. Theory*, vol. 45, no. 6, pp. 20072019, Sept. 1999.

[4] S. Kotagiri and J. N. Laneman, "Multiaccess channels with state known to one encoder: a case of degraded message sets," in Proc. *IEEE Int. Symp. Information Theory (ISIT 07)*, Nice, France, June 2007.

[5] H. Permuter, S. Shamai, and A. Somekh-Baruch. "Message and state cooperation in multiple access channels," *IEEE Trans. Info. Theory*, vol. 57, no. 10, pp. 6379-6396, Oct. 2011.

[6] Y. Steinberg, "Coding for the degraded broadcast channel with random parameters, with causal and non-causal side information," *IEEE Trans. Info. Theory*, vol. 51, no. 8, pp. 28672877, Aug. 2005.

[7] S. Sigurjonsson and Y. H. Kim, "On multiple user channels with state information at the transmitters," in Proc. *IEEE Int. Symp. Info. Theory* (ISIT), Adelaide, Sept. 2005, pp. 7276.

[8] B. Akhbari, M. Mirmohseni, and M. R. Aref, "State-dependent relay channel with private messages with partial causal and non-causal channel state information," in Proc. of *IEEE Int. Symp. Information Theory*, June 2010.

[9] M. Mirmohseni, B. Akhbari, and M. R. Aref, "Compress-and-forward strategy for the relay channel with causal state information," in Proceedings of *IEEE Inf. Theory Workshop*, Oct. 2009.

[10] A. Zaidi, L. Vandendorpe and P. Duhamel, "Lower bounds on the capacity regions of the relay channel and the cooperative relay-broadcast channel with non-causal side information," in Proc. *IEEE communication conference* (ICC), 2007, pp. 60056011.

[11] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob*., vol. 3, pp.120154, 1971.

[12] T. M. Cover and A. El Gamal, "Capacity theorems for relay channels," *IEEE Trans. Info. Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[13] M. N. Khormuji and M. Skoglund, "The relay channel with partial causal state information," in Proc. *IEEE Int. Symp. Info. Theory and its application* (ISITA), New Zealand, Dec. 2008.

[14] M. Li, O. Simeone and A. Yener,"Message and state cooperation in a Relay Channel When Only the Relay Knows the State,"2011, available online at http://arxiv.org/abs/1102.0768v1

[15] F. M. J. Willems. "The discrete memoryless multiple channel with partially cooperating encoders," *IEEE Trans. Info. Theory*, , vol. 29, no. 3, pp. 441-445, May 1983.

[16] D. Slepian and J. K. Wolf. "A coding theorem for multiple access channel with correlated sources," *Bell Syst. Tech. J.* , Vol. 52, no. 7, pp. 1037 - 1076, Sept. 1973.

[17] O. Simeone, D. Gunduz, V. H. Poor, A. J. Goldsmith, and S. Shamai."Compound multiple access channels with partial cooperation". *IEEE Trans. Inf. Theory*, Vol. 55, no. 6, pp. 2425 - 2441, June 2009.

[18] A. Haghi, R. Khosravi-Farsani, M. R. Aref, and F. Marvasti. "The capacity region of fading multiple access channels with cooperative encoders and partial CSIT," in Proc. *IEEE Int. Symp. Information Theory (ISIT 2010)*, Austin, Texas, U.S.A., pp. 485-489, Jun. 2010.

[19] R. Liu, I. Maric, P. Spasojevic, and R. D. Yates. "Discrete memoryless interference and broadcast channels with confidential messages: Secrecy capacity regions," *IEEE Trans. Inf. Theory*, 54(6):24932507, 2008.

[20] R. Liu and V. Poor. "Secrecy capacity region of a multi-antenna gaussian broadcast channel with confidential messages," *IEEE Trans. Inf. Theory*, 55(3):12351249, 2009.

[21] L. Sankaranarayanan, G. Kramer, and N. B. Mandayam, "Capacity theorems for the multiple-access relay channel," in Proceedings of *Allerton Conference on Communication, Control and Computing*, Monticello, IL,September 29-October 1 2004, pp. 1782-1791.

[22] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Info. Theory*, vol. 51, no. 9, pp. 30373063, Sept. 2005.

[23] M. Osmani-Bojd and G. A. Hodtani, "On multiple access relay channel with common message,"Transactions on Emerging Telecommunications Technologies, vol. 27, no. 8, April. 2014.

[24] M. Osmani-Bojd and G. A. Hodtani, "Multipleaccess relay channels with non-causal channel state information at the relay,"Transactions on Emerging Telecommunications Technologies, vol. 28, no. 1, Nov. 2014.

[25] A. B. Carleial, "Multiple-access channels with different generalized feedback signals," *IEEE Trans. Info. Theory*, vol. 28, no. 6, pp. 841850, Nov. 1982.

[26] A. El Gamal and Y.-H. Kim, "Network Information Theory," Cambridge University Press, 2012.