Abstract — In this paper, a metric based symbol predistortion method is introduced for Peak to Average Power Ratio (PAPR) reduction of the Space Frequency Block Coded (SFBC) Orthogonal Frequency Division Multiplexing (OFDM) systems with two transmitter antennas. In the proposed method, a metric is defined which shows the contribution of each frequency domain symbol to the time domain samples with high amplitude at both antennas. Then, the symbols with the highest metric values are chosen and predistorted such that the peak powers are reduced while the SFBC relationship between two antennas holds constant. Three variants of this method will be introduced. Then, this technique is extended to the general SFBC systems with an arbitrary number of antennas and SFBC structures. Simulation results show that the proposed methods converge when a different number of transmitter antennas and different modulation techniques are used. However, the best performance is achieved in the case of two transmitter antennas with QPSK modulation\(^1\).

Index Terms — Orthogonal Frequency Division Multiplexing (OFDM), Peak to Average Power Ratio (PAPR), Space Frequency Block Coding (SFBC).

I. INTRODUCTION

OFDM systems have made it possible to transmit digital data over frequency-selective broadband wireless fading channels. Therefore, it has become the base of some important protocols such as IEEE 802.11 Wireless Local Area Networks (WLAN), IEEE 802.16 and Digital Video Broadcasting - Terrestrial (DVB-T). In OFDM systems, the high rate input bit stream is multiplexed among several narrowband overlapping subcarriers. The channel response can be assumed to be flat over the narrowband sub-channels which makes it possible to use several simple equalizers instead of one complex equalizer [1], [2]. Another improvement in broadband wireless communications has been the idea of space diversity. Based on this idea, the signal is transmitted from several antennas to achieve the diversity. Space Time Codes (STC) introduced in [3] and [4] can achieve full space diversity. By combining the space diversity with the OFDM technique, MIMO-OFDM systems have been introduced. In space diversity OFDM transmitters, SFBC codes can be used. In SFBC systems, the complex symbols are transmitted simultaneously from different antennas and also across several adjacent subcarriers of an OFDM symbol [5].

One of the drawbacks of multicarrier systems such as OFDM is high PAPR, i.e., the time domain signal has large peaks which leads to saturation of high power amplifiers; therefore, high dynamic range amplifiers are needed. To overcome this problem, some baseband algorithms in the single antenna OFDM systems have been proposed [6]. Some of these algorithms are based on the clipping of the time domain signal and the reconstruction of the original signal from the clipped version at the receiver side [7]-[9]. Companding is another method for PAPR reduction [10]-[12], in which a nonlinear transform is applied to the time domain samples to change the distribution of the amplitudes. Some other methods are based on coding, in which only the codewords with low PAPR are transmitted [13], [14]. In multiple signal representation methods, such as Selected Mapping (SLM) [15]-[19] and Partial Transmit Sequence (PTS) [20]-[23], the representation of the data symbols with minimum PAPR is transmitted. One important approach for PAPR reduction is constellation extension [24]. In this approach the constellation points are moved such that the PAPR is minimized while the minimum distance of the constellation points does not decrease. In [25] a metric based constellation extension method has been proposed. In that method, a metric is defined that shows the effect of each data symbol on the reduction of peak powers, and consequently the symbols with the highest metric values are selected and moved.

In this paper, a metric based symbol predistortion method has been proposed to reduce the PAPR in SFBC-OFDM systems with two transmitter antennas. For this purpose, a new metric is defined which shows the effect of each symbol on the peak reduction of both antennas. Then, the complex symbols with the highest metric values are chosen and moved such that the PAPR is reduced, while the SFBC relationship between the signals of antennas remains constant. Three variants of the metric based symbol predistortion, introduced in [25], have been extended to SFBC-OFDM transmitters. Afterward, the proposed methods are extended to the general SFBC-OFDM systems with an arbitrary number of transmitter antennas as well as arbitrary code structure. In simulations, the performance of the proposed methods in two and four antenna SFBC systems has been compared with the performance of the original methods in the single antenna case using QPSK and 16-QAM modulations. It is shown that all the proposed methods converge and properly reduce the PAPR. However, the best performance is achieved in the case of two transmitter antennas with QPSK modulation.

The remainder of the paper is organized as follows. In section II, The system model of an SFBC-OFDM system with

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two transmitter antennas and its PAPR problem are introduced. Then, in section III the metric-based symbol predistortion methods for this system are discussed. Next, in section IV the proposed methods are extended to the general SFBF systems with an arbitrary number of transmitter antennas and arbitrary code lengths. The computational complexity of the proposed methods is discussed in section V. Section VI includes the simulation results.

II. SYSTEM MODEL AND PAPR PROBLEM

In the SFBF-OFDM systems, the input bit stream is interleaved and encoded by a channel encoder. Then, the coded bits are mapped onto the complex symbols using digital modulation techniques. The sequential symbols are converted to blocks of Nc complex symbols denoted by the vector $S=[S(0), S(1), ..., S(N_c-1)]^T$, where Nc is the number of OFDM subcarriers. Afterward, the SFBF encoder generates the vectors $S^{(i)}=[S^{(i)}(0), S^{(i)}(1), ..., S^{(i)}(N_c-1)]$ and $S^{(2)}=[S^{(2)}(0), S^{(2)}(1), ..., S^{(2)}(N_c-1)]$ from S as follows [5]:

$$
\begin{bmatrix}
  S^{(1)}(2\nu) & S^{(1)}(2\nu+1) \\
  S^{(2)}(2\nu) & S^{(2)}(2\nu+1)
\end{bmatrix}
= 
\begin{bmatrix}
  S(2\nu) & S(2\nu+1) \\
  -S^{*}(2\nu+1) & S^{*}(2\nu)
\end{bmatrix}
$$

$$
\nu = 0, 1, ..., N_c/2 - 1
$$

Then $N_c-N_c$ zeros are added at the end of the frames $S^{(1)}$ and $S^{(2)}$ and they are passed through the IFFT blocks to generate the oversampled time domain signals $s^{(0)}=[s^{(0)}(0), s^{(0)}(1), ..., s^{(0)}(N_c-1)]$ and $s^{(2)}=[s^{(2)}(0), s^{(2)}(1), ..., s^{(2)}(N_c-1)]$ where $N_c-N_c$ is the oversampling ratio. The time domain samples $s^{(i)}(n)$ and $\hat{s}^{(i)}(n)$ can be written as

$$
s^{(i)}(n) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c - 1} S(k) e^{j\frac{2\pi nk}{N_c}}
$$

$$
\hat{s}^{(i)}(n) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c - 1} S^*(k)(-1)^k e^{j\frac{2\pi nk}{N_c}} e^{-j\frac{2\pi nk}{N_c}}
$$

To avoid Inter Symbol Interference (ISI) a Cyclic Prefix (CP) is added to the time domain signals $s^{(0)}(n)$ and $\hat{s}^{(2)}(n)$. Then, the samples are passed through a Digital to Analogue Converter (DAC) to yield the baseband OFDM signals which are then transmitted from two antennas simultaneously (Fig.1). It is noteworthy that if it can be assumed that the channel response is the same at the adjacent subcarriers based on the encoding structure of (1), full diversity can be achieved [5].

![Diagram](image)

Figure 1 Block diagram and frame structure of SFBF-OFDM system with two transmitter antennas.

The matrix form of (2) is as follows:

$$
s^{(1)} = FS,
$$

$$
s^{(2)} = FPAS^T = \Omega S^*,
$$

where $F_{x \times y}$ is the IFFT matrix with entries $f_{x,y} = \left(\frac{1}{\sqrt{N_c}}\right) \exp\left(j \frac{2\pi nk}{N_c}\right)$ and $\Lambda_{N_c-N_c}$ is defined by

$$
\Lambda = \text{diag}\left\{1, -1, 1, -1, ..., 1\right\}_{N_c-N_c}
$$

and the permutation matrix $P_{N_c-N_c}$ is as follows:

$$
P = \frac{I_{N_c-N_c} \otimes}{2}
\begin{bmatrix}
  0 & 1 \\
  1 & 0
\end{bmatrix}
$$

$I$ is the identity matrix and $\otimes$ is the Kronecker product. The PAPR of the $i$th antenna is defined as follows:

$$
PAPR^{(i)} = \frac{\max\left\{E\left[s^{(i)}(n)^2\right]\right\}}{\mathbb{E}\left[E\left[s^{(i)}(n)^2\right]\right]}, \quad i = 1, 2
$$

where $E\left\{|s^{(i)}(n)|^2\right\}$ is the mathematical expectation. The overall PAPR of the SFBF-OFDM system is defined by

$$
PAPR = \max\left\{PAPR^{(i)}\right\}, \quad i = 1, 2
$$

Based on (2), the time domain samples $s^{(0)}(n)$, $i=1,2$; $n=0,1,...,N-1$ are the sum of $N_c$ independent terms. When $N_c$ is large, based on the central limit theorem, the time domain samples have Gaussian distribution. Thus, they may have large amplitudes which in turn leads to a high PAPR.

III. SYMBOL PREDISTORTION METHOD FOR SFBF-OFDM SYSTEM WITH TWO TRANSMITTER ANTENNAS

One of the proposed techniques to reduce the PAPR of the single antenna OFDM systems is symbol predistortion. In this method, the complex symbols $S(k), k=0,1,...,N_c-1$ are distorted such that the PAPR is reduced but the minimum distance between the complex symbols of the Constellation points is not decreased. Fig.2 shows the regions in which the constellation points can be moved such that their minimum distance does not decrease. As can be seen from this figure, in QPSK modulation, all of the constellation points can be extended, while in 16-QAM the inner points must remain unchanged.

To reduce the PAPR of the SFBF-OFDM systems, the symbol predistortion must be used to reduce the PAPR of the frames $s^{(1)}$ and $s^{(2)}$ simultaneously, while the SFBF relationship in (1) still holds. It is important to note that based on (3), $s^{(1)}$ and $s^{(2)}$ are not independent and they are generated from a common vector $S$; thus, the distortion of the symbols $S(k), k=0,1,...,N_c-1$ leads to the change of the time domain signals at both antennas. If the vector $S$ is changed to $S+C$, where $C=[C(0), C(1), ..., C(N_c-1)]^T$ is the predistortion vector, based on (3) the terms $FC$ and $FPAC$ are added to the time domain vectors $s^{(1)}$ and $s^{(2)}$, respectively. Therefore, the following optimization problem must be solved:
Figure 2 Symbol predistortion for PAPR reduction of OFDM systems with QPSK (Up) and 16-QAM (Down) modulations

where $F_a$ is the $n$th row of the matrix $F$, $\Psi$ is the acceptable region for constellation predistortion shown in Fig. 2, and $\Delta P$ limits the power increase. In [25], a metric based method has been proposed to find the vector $C$ for PAPR reduction of the single antenna OFDM system. In that method, a metric is defined which shows the effect of the extension of each frequency domain symbol on the peak powers of the time domain signal. Then, the symbols with the largest metric values are chosen and predistorted. This procedure is performed iteratively. Three variants of this method have been discussed in [25]. In this paper, this method is modified to find the suboptimum solution of (8) for a SFBC-OFDM system.

### A. Amplitude predistortion for a SFBC-OFDM system with two transmitter antennas

In metric-based predistortion methods, a metric must be defined which shows the effect of each symbol $S(k)$ on the reduction of the time domain samples with high amplitudes. This metric is denoted by $\mu(k)$. For SFBC-OFDM system with two transmitter antennas, (2) shows that the predistortion of the symbol $S(k)$ has the effect on the time domain samples of both antennas. It must be noted that the effect of $S(k)$ on the time domain sample $s^{(1)}(n)$ occurs via the term $S(k)e^{j2\pi nk/N}$ and its effect on the time domain sample $s^{(2)}(n)$ transpires via the term $S'(k)(-1)^i e^{j2\pi (n-1)^i/N}$. Thus, the effect of the frequency domain symbol $S(k)$ on the time domain samples $s^{(1)}(n)$ and $s^{(2)}(n)$ is evaluated by $f^{(1)}(n,k)$ and $f^{(2)}(n,k)$:

\[
\begin{align*}
    f^{(1)}(n,k) &= -\cos(\phi^{(1)}_a) = -\frac{Re\{s(n)S'(k)e^{-j2\pi nk/N}\}}{|S(k)||s(n)|} \tag{9} \\
    f^{(2)}(n,k) &= -\cos(\phi^{(2)}_a) = -\frac{Re\{s(n)S(k)(-1)^i e^{-j2\pi nk/N}\}}{|S(k)||s(n)|} 
\end{align*}
\]

where $\phi^{(1)}_a$ is the angle between the complex numbers $S(k)e^{j2\pi nk/N}$ and $S^{(1)}(n)$ and $\phi^{(2)}_a$ is the angle between the complex numbers $S'(k)(-1)^i e^{j2\pi (n-1)^i/N}$ and $S^{(2)}(n)$. Based on this definition, $f^{(0)}(n,k)$, $i=1,2$ has its maximum value of $+1$ if these numbers have a phase difference of $\pi$, which shows that the extension of the symbol $S(k)$ reduces the amplitude of the time domain sample $s^{(i)}(n)$. To find the metric $\mu(k)$, the effect of the symbol $S(k)$ on all of the time domain samples of both antennas with the amplitudes higher than a predefined threshold must be considered. Thus, a metric can be defined for SFBC system with two transmitter antennas as follows:

\[
\mu(k) = \frac{2}{M^{(1)} + M^{(2)}} \times \left\{ \sum_{m=1}^{M^{(1)}} w^{(1)}(n) f^{(1)}(n,k) + \sum_{m=1}^{M^{(2)}} w^{(2)}(n) f^{(2)}(n,k) \right\} \tag{10}
\]

where $A^{(i)}$, $i=1,2$ is the set of the of the time domain samples of the $i$th antenna with power higher than a predetermined threshold $\gamma$; and $M^{(i)}$ is the number of points in set $A^{(i)}$; and $w^{(i)}(n)$ is a weighting function. The time domain samples with higher amplitudes must have bigger weights. Thus, the weight $w^{(i)}(n)$ is defined by $|s^{(i)}(n)|^p$, where $p$ is a positive number. Combining (9) and (10), the metric $\mu(k)$ can be written as

\[
\mu(k) = \frac{-2}{|S(k)|[M^{(1)} + M^{(2)}]} \times \left\{ \sum_{m=1}^{M^{(1)}} |s^{(1)}(n)|^p Re\{s^{(1)}(n)S'(k)e^{-j2\pi nk/N}\} \right. \\
&+ \left. \sum_{m=1}^{M^{(2)}} |s^{(2)}(n)|^p Re\{s^{(2)}(n)(-1)^i S(k)e^{-j2\pi nk/N}\} \right\} \tag{11}
\]

After the calculation of the metric, the $L$ symbol indices with the highest positive metric values are selected in set $A_L$ and their corresponding constellation points are extended. In the amplitude predistortion method, the extension is performed by multiplication of the symbol $S(k)$ by a real number $k|k|>1$. Then, the vectors $S^{(1)}$ and $S^{(2)}$ are regenerated from the predistorted symbols $\bar{S}$ using (1). It can be easily shown that this is equivalent to the update of $s^{(1)}(n)$ and $s^{(2)}(n)$ as follows:
\[ \bar{s}^{(1)}(n) = s^{(1)}(n) + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} S(k) (d(k) - 1) e^{\frac{2\pi i n^2}{N}} \]  
\[ \bar{s}^{(2)}(n) = s^{(2)}(n) + \frac{1}{\sqrt{N}} \sum_{i=1}^{N} S^*(k) (d(k) - 1) (-1)^i e^{\frac{2\pi i n^2}{N}} e^{\frac{2\pi i n n}{N}} \]  
(12)

\[ \bar{s}^{(1)}(n) = s^{(1)}(n) + \frac{\alpha - 1}{\sqrt{N}} \sum_{i=1}^{N} S(k) e^{\frac{2\pi i n^2}{N}} \]  
\[ \bar{s}^{(2)}(n) = s^{(2)}(n) + \frac{\alpha - 1}{\sqrt{N}} \sum_{i=1}^{N} S^*(k) (-1)^i e^{\frac{2\pi i n^2}{N}} e^{\frac{2\pi i n n}{N}} \]  
(13)

The metric calculation procedure and updating of the time domain samples are performed iteratively to achieve the target PAPR, or to stop after a predetermined number of iterations in the case that target PAPR was not reached. It must be noted that by updating the time domain samples based on equations (13) and (14), the PAPR is reduced while the SFBC structure of the signals shown in (1) still holds. Therefore, at the receiver side full diversity can still be achieved. The parameters \{γ, p\} and \{α, β, L\} must be optimized by simulation to achieve the maximum PAPR reduction (considering the power increase due to the extension of the constellation points).

B. Complex predistortion for SFBC-OFDM system with two transmitter antennas

In MBAP methods, the phase of the symbols in the frequency domain does not change. In [25] also a complex predistortion technique has been introduced for the single antenna OFDM systems. In that technique, the amplitude and the phase of the complex symbols may be changed, and because of the higher degree of freedom, it has better performance than the amplitude predistortion technique. In the current section, this technique is applied to SFBC-OFDM systems with two transmitter antennas.

In the Metric Based Complex Predistortion (MBCP) method, the real and imaginary parts of the symbols are distorted independently. The amount of extension for real and imaginary parts of the symbol \(S(k)\) is determined by the metrics \(\mu^R(k)\) and \(\mu^I(k)\), respectively. To extend the complex predistortion to the SFBC case, the metrics \(\mu^R(k)\) and \(\mu^I(k)\) are defined as follows:

\[ \mu^R(k) = \mu^R_{(1)}(k) + \mu^R_{(2)}(k) \]  
\[ \mu^I(k) = \mu^I_{(1)}(k) + \mu^I_{(2)}(k) \]  
(15)

where \(\mu^R_{(1)}(k)\) and \(\mu^R_{(2)}(k)\) are the metrics which show the effect of the real part of the symbol \(S(k)\) on the peaks of the frames \(s^{(1)}\) and \(s^{(2)}\), respectively. Similarly, \(\mu^I_{(1)}(k)\) and \(\mu^I_{(2)}(k)\) are the metrics for the imaginary part of \(S(k)\). From (2), it can be seen that the proper definitions for these parameters are

\[ \mu^R_{(1)}(k) = -\frac{1}{M} \left| \text{Re}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(1)}(n) \right| \]  
\[ \mu^I_{(1)}(k) = -\frac{1}{M} \left| \text{Im}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(1)}(n) \right| \]  
(16)

\[ \mu^R_{(2)}(k) = -\frac{1}{M} \left| \text{Re}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(2)}(n) \right| \]  
\[ \mu^I_{(2)}(k) = -\frac{1}{M} \left| \text{Im}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(2)}(n) \right| \]  
(17)

In the Metric Based Complex Predistortion (MBCP) method, the real and imaginary parts of the symbol \(S(k)\) are distorted independently. The amount of extension for real and imaginary parts of the symbol \(S(k)\) is determined by the metrics \(\mu^R(k)\) and \(\mu^I(k)\), respectively. To extend the complex predistortion to the SFBC case, the metrics \(\mu^R(k)\) and \(\mu^I(k)\) are defined as follows:

\[ \mu^R(k) = \mu^R_{(1)}(k) + \mu^R_{(2)}(k) \]  
\[ \mu^I(k) = \mu^I_{(1)}(k) + \mu^I_{(2)}(k) \]  
(18)

where \(\mu^R_{(1)}(k)\) and \(\mu^R_{(2)}(k)\) are the metrics which show the effect of the real part of the symbol \(S(k)\) on the peaks of the frames \(s^{(1)}\) and \(s^{(2)}\), respectively. Similarly, \(\mu^I_{(1)}(k)\) and \(\mu^I_{(2)}(k)\) are the metrics for the imaginary part of \(S(k)\). From (2), it can be seen that the proper definitions for these parameters are

\[ \mu^I_{(1)}(k) = -\frac{1}{M} \left| \text{Re}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(1)}(n) \right| \]  
\[ \mu^I_{(2)}(k) = -\frac{1}{M} \left| \text{Im}\{S(k)\} \right| \times \sum_{n=1}^{N} \left| s^{(2)}(n) \right| \]  
(19)
where \( n = 0,1,2,\ldots, N_s / 4 \). In this case the combination and permutation matrices are defined by

\[
\Gamma^{(i)} = \Gamma^{(0)} = \mathbf{1}_{N_s \times N_s}, \quad \Gamma^{(D)} = \Gamma^{(0)} = 0_{N_s \times N_s}.
\]

\[
\mathbf{A}^{(i)} = \mathbf{A}^{(0)} = 0_{N_s \times N_s}, \quad \mathbf{A}^{(D)} = \text{diag} \{ [1, -1, 1, -1, \ldots, 1, -1] \}_{N_s}.
\]

Using (19), the symbol predistortion optimization problem can be written as

\[
\underset{\mathbf{C} \in \Psi}{\text{argmin}} \left\{ \max_{i,n} \{ |s^{(i)}(n) + \sum_{k=0}^{N_s-1} \alpha^{(k)}_{\psi d} \mathbf{C}(k) + \sum_{k=0}^{N_s-1} \alpha^{(k)}_{d} \mathbf{C}^{*}(k) | \} \right\}
\]

Subject to \( \forall k \ S(k) + C(k) \in \Psi \) and \( \| \mathbf{C} \| \leq \Delta P \).

(22)

To extend the metric-based predistortion methods to the generalized SFBC codes, a metric must be defined which shows the effect of the symbol \( S(k) \) on the peak reduction of all antennas. From (19), it can be seen that the effect of the symbol \( S(k) \) on the \( \ell \)th time domain sample of the \( \ell \)th antenna transpires via the term \( \theta^{(i)}_{\psi d} S(k) + \alpha^{(i)}_{d} S^{*}(k) \). Thus, the proper definition of the metric is

\[
\mu(k) = \frac{1}{\mathbf{M}_{s}} \sum_{i=0}^{\mathbf{M}_{s}} |s^{(i)}(n)|^4 \text{Re} \left\{ \left| \theta^{(i)}_{\psi d} S(k) + \alpha^{(i)}_{d} S^{*}(k) \right| \right\}
\]

(23)

\[
\mu^{(i)}(k) = \frac{1}{\mathbf{M}} \sum_{s=0}^{\mathbf{M}_{s}} |s^{(i)}(n)|^4 \text{Re} \left\{ s^{(i)}(n) \left[ \theta^{(i)}_{\psi d} S(k) + \alpha^{(i)}_{d} S^{*}(k) \right] \right\}
\]

\[
\mathbf{M} = \frac{1}{N_{t}} \sum_{i=0}^{\mathbf{M}_{s}} M^{(i)}
\]

where \( \mathbf{A}^{(i)} \) is the set of the points in which the amplitude of the signal \( s^{(i)}(n) \) is higher than a predetermined threshold, and \( \mathbf{M}^{(i)} \) is the number of these points. If \( \mathbf{A}_{\psi} \) is the set of the symbol indices with the highest positive metric values, then, based on (19), the predistortion procedure of the \( i \)th antenna signal for MBAP1 method is defined by

\[
\overline{s}^{(i)}(n) = s^{(i)}(n) + (\alpha - 1) \sum_{k=\ell_{i}}^{\mathbf{A}_{\psi}} \theta^{(i)}_{\psi d} S(k) + \alpha^{(i)}_{d} S^{*}(k),
\]

(24)

\[
\overline{s}^{(i)}(n) = s^{(i)}(n) + \sum_{k=\ell_{i}}^{\mathbf{A}_{\psi}} \left[ \theta^{(i)}_{\psi d} \sqrt{\mu(k)} S(k) + \alpha^{(i)}_{d} \sqrt{\mu(k)} S^{*}(k) \right],
\]

(25)

Similar predistortion can be defined for MBAP2 method as follows:

\[
\overline{s}^{(i)}(n) = s^{(i)}(n) + \beta \sum_{k=\ell_{i}}^{\mathbf{A}_{\psi}} \left[ \theta^{(i)}_{\psi d} \sqrt{\mu(k)} S(k) + \alpha^{(i)}_{d} \sqrt{\mu(k)} S^{*}(k) \right],
\]

\[
\overline{s}^{(i)}(n) = s^{(i)}(n) + \sum_{k=\ell_{i}}^{\mathbf{A}_{\psi}} \left[ \theta^{(i)}_{\psi d} \sqrt{\mu(k)} S(k) + \alpha^{(i)}_{d} \sqrt{\mu(k)} S^{*}(k) \right],
\]
For the MBCP method, based on (19), the proper definitions for the real and imaginary metrics are:

\[
\mu_k(k) = \sum_{i=1}^{N_t} \mu_k^{(i)}(k),
\]

(26)

\[
\mu_k^{(i)}(k) = \frac{-1}{M |\text{Re}\{S(k)\}|} \times \sum_{n \in d^{(i)}} |s^{(i)}(n)|^2 \text{Re}\left\{s^{(i)}(n)\right\} \text{Re}\left\{\theta_{n,k}^{(i)}S(k) + \omega_{n,k}^{(i)}S^{*}(k)\right\}
\]

\[
\mu_i(k) = \sum_{j=1}^{N_t} \mu_i^{(i)}(k),
\]

(27)

\[
\mu_i^{(i)}(k) = \frac{-1}{M |\text{Re}\{S(k)\}|} \times \sum_{n \in d^{(i)}} |s^{(i)}(n)|^2 \text{Im}\left\{s^{(i)}(n)\right\} \text{Im}\left\{\theta_{n,k}^{(i)}S(k) + \omega_{n,k}^{(i)}S^{*}(k)\right\}
\]

If the real and imaginary parts of the symbols \(S(k), k \in A\), are extended by the factors \(1+\beta\sqrt{\mu_k(k)}\) and \(1+\beta\sqrt{\mu_i(k)}\), respectively; then, based on (19), the signal of the \(i\)th antenna is disturbed as follows:

\[
\bar{s}^{(i)}(n) = s^{(i)}(n) + \beta \sum_{k=1}^{K} \sqrt{\mu_k(k)} \left[\theta_{n,k}^{(i)} + \omega_{n,k}^{(i)}\right] \text{Re}\left\{S(k)\right\} + j \sqrt{\mu_i(k)} \left[\theta_{n,k}^{(i)} - \omega_{n,k}^{(i)}\right] \text{Im}\left\{S(k)\right\}
\]

(28)

V. COMPUTATIONAL COMPLEXITY

The modified metric based predistortion algorithms include two steps: metric calculation and updating procedures. From (23) and (26), it can be seen that the metric for the SFBC case is the sum of \(N_t\) submetrics. The complexity of each submetric is proportional to \(KN_t\) submetrics, where \(K\) is the number of the time samples with higher amplitude than the threshold. For QPSK modulation, the metric calculation involves \(7KN_t\) real multiplications, \(3KN_t\) additions and \(N_tN\) divisions. The updating procedure involves \(5KN_tL\) real multiplications and \((2L+1)N_tN\) additions. Simply, it can be seen that the complexity of the modified methods in the SFBC case is \(N_t\) times greater than the complexity of the original methods in the single antenna case. Significantly, the parameters \(\gamma, \alpha\) (or \(\beta\)), \(p\) and \(L\) must be determined by offline optimization to yield the best performance, thus, the calculation complexity of this optimization is not added to the complexity of the proposed algorithms.
VI. SIMULATION RESULTS

In the simulations, OFDM frames with $N_c=256$ subcarriers have been considered. The PAPR reduction algorithms are applied on the frames whose PAPR is larger than 6dB. For each method, two scenarios have been simulated: one is based on the application of the original metric-based symbol predistortion methods on the single antenna case [25], while the other is based on the application of the proposed methods for SFBC-OFDM systems with two or four transmitter antennas. The performances of these methods are measured based on the Complementary Cumulative Density Function (CCDF) of the PAPR which is defined by

$$CCDF\{\text{PAPR} \geq \text{PAPR}_o\}$$

(29)

All of these methods are applied to the samples with an oversampling ratio of 2; however, for PAPR estimation of the analogue signal, the oversampling ratio of 4 is used. Importantly, the PAPR has been defined as the ratio of the peak power to the initial average power (before predistortion). Therefore, the effect of the power increase is considered in the PAPR calculation.

Figure 3 shows the CCDF of the PAPR in different numbers of iterations of the MBAP1 method when QPSK modulation is used. The CCDF of the single antenna and two antenna SFBC cases are plotted by dashed and solid lines, respectively. In [25], the parameters have been optimized. The same parameters are used for both cases. The optimum parameters for the MBAP1 method are $\alpha=1.5$, $L=28$ and $p=6$, and the clipping threshold is $3.9\text{dB}$ above the average power [25]. As can be seen, the initial PAPR of the two antenna system is about $0.2\text{dB}$ higher than the initial PAPR of the single antenna case. It can be observed in this figure that the modified MBAP1 method converges and, similar to the single antenna case, the PAPR is reduced. Table 1 shows the values of PAPR reduction (at the probability of $10^{-4}$) and the power increase in three methods versus the number of iterations. As can be seen, for the MBAP1 method, the values of PAPR reduction in the SFBC case are very close to those of the single antenna case. Using only one iteration, a PAPR reduction of about $2.05\text{dB}$ and $1.8\text{dB}$ can be achieved for single antenna and two antenna cases, respectively. It must be noted that in the SFBC system, there is an additional restriction in optimization of the predistortion vector, which is the restriction of holding the SFBC structure in the frequency domain. Thus, it can be seen that there is only $0.2\text{dB}$ performance degradation due to this additional restriction. From Table 1, it can be seen that in the MBAP1 method the value of power increase in the SFBC case is about $0.15\text{dB}$ less than the single antenna case at the third iteration.

Figure 4 shows the CCDF using a different number of iterations of the MBAP2 method. The optimum parameters are $\beta=0.26$, $L=36$ and $p=5$. The clipping threshold is $4.7\text{dB}$ above the average power [25]. In this case, the PAPR reduction after one and two iterations is greater than that of the MBAP1...
method, but the MBAP1 outperforms the MBAP2 algorithm at the third iteration. Table 1 shows that the power increase in both scenarios is higher than in the MBAP1 method and the two antennas case has about 0.25 dB greater power increase than that of the single antenna case.

Figure 5 shows the simulation results for MBCP method. The optimum parameters are $\beta=0.19$, $L=45$, and $p=5$, and the clipping threshold is at the same level as that of MBAP2 method [25]. This method outperforms both MBAP methods in all iterations and it achieves about 3.9 dB PAPR reduction in the two antenna case at the third iteration. Table 1 shows that the values of the power increase is also less than those of the MBAP methods.

Figures 6, 7 and 8 show the simulation results for the above three methods when 16-QAM modulation is employed. The optimum parameters are the same as those of the previous cases. As can be seen from these figures, the values of PAPR reduction in the single antenna case after 3 iterations are 2.6 dB, 2.1 dB and 2.6 dB for the MBAP1, MBAP2 and MBCP methods, respectively. These values for the proposed methods in SFBC case are 1.8 dB, 1.2 dB and 1.85 dB, respectively. It is clear that when 16-QAM modulation is used, the PAPR reductions in the single antenna and the SFBC cases are less than those in the QPSK modulation, because in 16-QAM modulation the inner constellation points can not be distorted (see Fig.1). Notably, the convergence rate of the MBCP method is higher than that of the other two methods. Figures 9, 10 and 11 show the performance of three metric-based predistortion methods for the case of four transmitter antenna SFBC-OFDM systems. The space frequency block length is $D=4$ and encoding is done based on (20). The parameters have been optimized to result in more effective PAPR reduction. While many of the parameters are the same as those in [25] for single antenna case, the exponent $p$ is modified to 5.75 for MBAP1 and 4.25 for both MBAP2 and MBCP methods. Consequently, after three iterations, the PAPR reductions (in probability of $10^{-4}$) are 1.7 dB, 1.85 dB and 3 dB, respectively, for the MBAP1, MBAP2 and MBCP methods.

VII. CONCLUSION

In this paper, a metric-based symbol predistortion method has been proposed to reduce the PAPR of the SFBC-OFDM system with two transmitter antennas. The metric definition has been modified such that it shows the effect of each data symbol on the peak reduction of both antennas. Furthermore, the predistortion process has been defined such that the PAPRs of both antennas are reduced while the SFBC structure still holds. Three variants of this algorithm have been discussed. In two of them, only the amplitude of the symbols are predistorted, while in the third algorithm the amplitude and the phase of each data symbol can be changed. Afterward, these methods have been extended to the general SFBC case with an arbitrary number of transmitter antennas and various SFBC coding schemes. Simulation results show that the modified methods converge when different numbers of antennas and different modulations.
are used. For the case of QPSK modulation with two transmitter antennas, the proposed methods in the SFBC case have very impressive performance which is close to the performance of the metric-based methods in the single antenna case. In this case, the complex predistortion method has the best performance among all the variants.

**Table 1** The values of (PAPR reductions \((dB)\), power increase \((dB)\)) for single antenna and two antenna cases with SFBC using QPSK modulation.

<table>
<thead>
<tr>
<th></th>
<th>1 Iteration</th>
<th>2 Iterations</th>
<th>3 Iterations</th>
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<tbody>
<tr>
<td>MBAP1 Single antenna</td>
<td>(2.05,0.56)</td>
<td>(3.15,1.01)</td>
<td>(3.40,1.22)</td>
</tr>
<tr>
<td>MBAP1 two antennas</td>
<td>(1.80,0.56)</td>
<td>(2.90,0.93)</td>
<td>(3.25,1.07)</td>
</tr>
<tr>
<td>MBAP2 Single antenna</td>
<td>(2.70,0.82)</td>
<td>(3.50,1.10)</td>
<td>(3.60,1.20)</td>
</tr>
<tr>
<td>MBAP2 two antennas</td>
<td>(2.20,1.11)</td>
<td>(3.10,1.40)</td>
<td>(3.10,1.46)</td>
</tr>
<tr>
<td>MBCP Single antenna</td>
<td>(3.30,0.53)</td>
<td>(4.10,0.77)</td>
<td>(4.30,0.95)</td>
</tr>
<tr>
<td>MBCP two antennas</td>
<td>(2.90,0.63)</td>
<td>(3.80,0.88)</td>
<td>(3.90,1.04)</td>
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</table>

**REFERENCES**


**BIOGRAPHIES**

Mahmoud Ferdosizadeh Naeiny was born in Iran in 1979. He received B.Sc. (with highest honors) form Amirkabir university of technology (Tehran Polytechnic) in 2002 and M.Sc. from Sharif university of technology in 2004. He is now the Ph.D. student in electrical engineering department of Sharif university of technology under the supervision of professor Farrokh Marvasti. He is also the member of Advance Communication Research Institute (ACRI) in Sharif university of technology from 2006. His research interests are design and implementation of OFDM systems including the synchronization issues, software defined radio, MIMO systems and the application of signal processing techniques in communications.

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