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State-feedback control of robot manipulators using polytopic LPV modelling with fuzzy-clustering

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ABSTRACT

This paper proposes a new algorithm to full state systematic feedback control design for robot manipulators based on fuzzy-clustered polytopic model. At first, a linear parameter varying (LPV) representation of the system is generated via linearization of usual Lagrangian equation about a desired state trajectory and a vector of scheduling signals from the desired trajectory information is produced to construct an initial polytopic model. Then a fuzzy-clustering algorithm is introduced to categorize the vertices of the initial polytopic model in several clusters such that a sufficient condition of asymptotic stability of the closed loop models in each cluster is satisfied. Hence, the number of vertices is reduced to the number of clusters and a new reduced polytopic model is generated with the representative models of the clusters. The proposed algorithm is applied to control of a two-degree-of-freedom (DOF) robotic manipulator that illustrates the validity of the proposed scheme.

1. Introduction

Recently, considering the nonlinear dynamical systems using LPV and polytopic linear models has received significant attention [1–9]. The main perturbation in a nonlinear system, coming from the nonlinearity effects due to operating point variation, can be overcome by constructing the polytopic linear model based on the LPV modelling. One alternative that is considered in this study is designing a controller for all the linear models at the related vertices in a linear matrix inequality (LMI)-based framework so that the overall stability is achieved. A survey of experimental results in LPV control is provided by [10]. It concisely reviewed and compared some different LPV controller synthesis methods, and it was shown that how synthesis can be done via LMIs. LPV modelling of a system is representing its dynamical equations as linear state-space model with time-varying parameters dependent matrices [11]. In fact, nonlinearity of the system is described by parameter variation of LPV model. This is a straightforward manner for system descriptions, especially when system variations are state-dependent, like robot manipulator dynamics.

The high performance controller synthesis with a low online computational load for complex nonlinear systems like industrial manipulators is still an open research topic [12]. In Hoffmann et al. [12], a benchmark problem is proposed for a robotic manipulator, where a solution is achieved by considering the LPV controller synthesis based on a reduced parameter set. Cai et al. [13] addressed the gain-scheduled H_2 controller synthesis problem for continuous-time LPV systems and assumed that the values of the parameters are measurable on-line in a polytope space. A review on historical development of gain scheduling schemes can be found in Rugh and Shamma [14]. Xie [15] considered the multi-objective H_2/L_2 performance controller synthesis of LPV systems by a kind of matrix inequalities formulation, where the proposed formulation parameterizes controllers without involving the Lyapunov variables. Robust pole placement control synthesis with a prescribed mixed H_2/H_∞ performance is derived in [16] for robot manipulators by solving a set of LMIs such that the asymptotic stability of the closed-loop uncertain polytopic LPV system is guaranteed against the uncertainties on the vertices.

In our study, the parameterization of the linearized plants corresponded to some sample points on the reference trajectory. It means that the linearization of the robot manipulator is performed about a set of desired trajectory points to produced primary vertices of the polytopic model. Although the resulted linearized models family describes the behavior of the system, but the main problem is the huge number of element in this family. There are many operating points that their corresponding linearized models have the same properties, i.e., there are some unnecessary vertices in initial polytopic model. Therefore, if the vertices of the polytopic

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model are categorized to some clusters so that the elements of each cluster have similar characteristics, then the number of vertices can be reduced to the number of clusters. Now the main question is how we can make this clustering and what criteria should be used to pick out the desired vertices. Our clustering objective is to categorize the vertices models set such that models within the same cluster demonstrate maximum similarity. This similarity may be defined with different criteria such as minimum norm of the distance between the model parameters, open loop poles locations or closed loop poles locations and so on.

Takagi–Sugeno (T–S) modelling have attracted rapidly growing interest to describe the nonlinear systems rules [17,18]. In this case the stability of the T–S fuzzy systems in the sense of Lyapunov are the main problem which are investigated in several situations [19–21]. Numerous clustering methods have been documented in the literature [22,23]. A survey on fuzzy clustering, which has received considerable attention in the clustering literature, can be found in Yang [24]. In fuzzy clustering, the fuzzy c-means (FCM) clustering algorithm is the most well-known and powerful clustering algorithm that has been defined in [25] and generated by Bezdek [26]. Several generalizations and extensions of FCM clustering have been proposed by researchers [27–32].

Our algorithm in this paper is based on FCM clustering with least possible number of cluster such that a given sufficient condition and an LMI condition are satisfied for the models in each cluster. The given sufficient condition guarantees that there is a common state feedback controller for all models in each cluster. Therefore, all models of each cluster can be substituted for the one representative model. The representative model of each cluster is defined as the model with the highest degree of membership. Hence, the primary polytopic model will be reduced to a new polytopic model with the vertices that are the representative models of the clusters. The next step is design a linear controller for the new polytopic model using linear control theory. There are many different schemes for this design process that can be followed in literatures [33–35]. However, these methods involve complexities during the implementation and synthesis [36]. This problem has been extensively studied for robot manipulator control problems in [12,37,38].

This article is organized as follows: In the next section, the problem statement has been stated. Section 3 describes the proposed algorithm for fuzzy-clustering of a primary polytopic model to produce a reduced polytopic model. In Section 4, the proposed control law is presented based on a set of LMIs introduced for a given polytopic model. The Controller implementation for a two-degree-of-freedom robotic manipulator is given in Section 5 and Section 6 concludes the paper.

2. Problem statement

A rigid-body dynamics model of the n-DOF manipulator can be derived through the Euler-Lagrangian approach as compact form:

$$\mathbf{D}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\dot{\mathbf{q}},\mathbf{q}) + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}, \tag{1}$$

where $\boldsymbol{\tau}(t) \in \Re^n$ is the vector of driving joint torques, which are the control inputs. The vectors $\mathbf{q}(t)$ and $\dot{\mathbf{q}}(t)$ are the angular joint positions and velocities respectively which all belong to $\Re^n$ and assumed available by measurement. The vector $\ddot{\mathbf{q}} \in \Re^n$ presents the joint accelerations $\mathbf{D}(\mathbf{q}) \in \Re^{n\times n}$, $\mathbf{D}(\mathbf{q}) = \mathbf{D}(\mathbf{q})^T > 0$ is the link inertia matrix $\mathbf{C}(\dot{\mathbf{q}},\mathbf{q}) \in \Re^n$ expresses a vector of Coriolis and centrifugal torques and $\mathbf{G}(\mathbf{q}) \in \Re^n$ denotes a gravity vector. More details about robot manipulator modelling are referred to [39,40]. In practical implementation point of view, any proposed control law requires consideration of various uncertainties such as modelling errors, unknown loads, computation errors, and specially nonlinearity effects. In order to envelope these uncertainties, we formulate an LPV model of robot dynamics around a specific desired trajectory. The nonlinear dynamics of the robot model (1) can be written as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\boldsymbol{\tau}), \tag{2}$$

where the state vector $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T]^T = [\mathbf{q}^T\ \dot{\mathbf{q}}^T]^T$, and the vector fields function $\mathbf{f}(\mathbf{x},\boldsymbol{\tau})$ is defined as

$$\mathbf{f}(\mathbf{x},\boldsymbol{\tau}) = \begin{bmatrix} \mathbf{x}_2 \\ -\mathbf{D}(\mathbf{x}_1)^{-1}(\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2) + \mathbf{G}(\mathbf{x}_2) - \boldsymbol{\tau}) \end{bmatrix} := \begin{bmatrix} \mathbf{x}_2 \\ \mathbf{g}(\mathbf{x},\boldsymbol{\tau}) \end{bmatrix}. \tag{3}$$

It is assumed that the function $\mathbf{f}(\mathbf{x},\boldsymbol{\tau})$ to be continuously differentiable a sufficient number of time. If we write

$$\begin{aligned} \mathbf{x} &= \bar{\mathbf{x}} + \delta\mathbf{x} \\ \boldsymbol{\tau} &= \bar{\boldsymbol{\tau}} + \delta\boldsymbol{\tau} \end{aligned} \tag{4}$$

where the state vector $\bar{\mathbf{x}} = [\bar{\mathbf{x}}_1^T\ \bar{\mathbf{x}}_2^T]^T = [\bar{\mathbf{q}}^T\ \dot{\bar{\mathbf{q}}}^T]^T$ is defined as a desired manipulator trajectory for the input joint torques $\bar{\boldsymbol{\tau}}$ such that

$$\mathbf{D}(\bar{\mathbf{q}})\,\ddot{\bar{\mathbf{q}}} + \mathbf{C}(\dot{\bar{\mathbf{q}}},\bar{\mathbf{q}}) + \mathbf{G}(\bar{\mathbf{q}}) = \bar{\boldsymbol{\tau}}, \tag{5}$$

then, state Eq. (2) can be approximated by linear Taylor expansion with respect to the components of $\mathbf{x}$ and $\boldsymbol{\tau}$. In fact, the actual manipulator dynamics in the immediate proximity of the desired trajectories are approximated by the first terms of the Taylor series. So, the following LPV model $P(\theta)$ can be introduced for the robot manipulator about the desired trajectory $\bar{\mathbf{x}}$,

$$\delta\dot{\mathbf{x}} = A(\theta(t))\delta\mathbf{x} + B(\theta(t))\delta\boldsymbol{\tau}, \tag{6}$$

where

$$A(\theta(t)) := \left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \boldsymbol{\tau}=\bar{\boldsymbol{\tau}}}}, \quad B(\theta(t)) := \left[\frac{\partial\mathbf{f}}{\partial\boldsymbol{\tau}}\right]_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \boldsymbol{\tau}=\bar{\boldsymbol{\tau}}}}, \tag{7}$$

denote the $2n \times 2n$ and $2n \times n$ Jacobian matrices of $\mathbf{f}(\mathbf{x},\boldsymbol{\tau})$ with respect to $\mathbf{x}$ and $\boldsymbol{\tau}$, respectively, i.e. the ij-th entries of $A(\cdot)$ and $B(\cdot)$ are the partial derivative of the ith component of $\mathbf{f}(\mathbf{x},\boldsymbol{\tau})$ with respect to the jth components of $\mathbf{x}$ and $\boldsymbol{\tau}$, respectively where evaluated along the desired trajectory. The mappings $A(\cdot)$ and $B(\cdot)$ are continuous functions of time-varying scheduling signal vector $\theta(t) \in \Re^{3n}$ which is defined by measurable signal vectors $\bar{\mathbf{x}}$ and $\bar{\boldsymbol{\tau}}$ as

$$\theta(t) := \begin{bmatrix} \bar{\mathbf{x}}(t) \\ \bar{\boldsymbol{\tau}}(t) \end{bmatrix}. \tag{8}$$

Using (3) and (7), the matrices $A(\cdot)$ and $B(\cdot)$ can be formulated as

$$A(\theta(t)) := \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ \frac{\partial\mathbf{g}}{\partial\mathbf{x}_1} & \frac{\partial\mathbf{g}}{\partial\mathbf{x}_2} \end{bmatrix}_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \boldsymbol{\tau}=\bar{\boldsymbol{\tau}}}}, \quad B(\theta(t)) := \begin{bmatrix} 0_{n\times n} \\ \mathbf{D}(\mathbf{x}_1)^{-1} \end{bmatrix}_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \boldsymbol{\tau}=\bar{\boldsymbol{\tau}}}}, \tag{9}$$

Sampling the desired trajectory $\bar{\mathbf{x}}$ to produce N distinct gridding points from the scheduling signal. The compact set $\mathrm{P}_\theta \subset \Re^{3n} : \theta \in \mathrm{P}_\theta, \forall t > 0$ is considered as a polytopic system defined by the convex hull

$$\mathrm{P}_\theta := \mathrm{Co}\{\theta_{v1},\ \theta_{v2},\ \ldots,\ \theta_{vN}\}, \tag{10}$$

where N is the number of vertices and it is determined according to the sampling time and trajectory duration. It follows as stated in [41] that the system can be represented by a linear combination of LTI models at the vertices; this is called a polytopic LPV system

$$P(\theta) \in Co\{P(\theta_{v1}),\ P(\theta_{v2}),\ \ldots,\ P(\theta_{vN})\} = \sum_{i=1}^{N} \alpha_i P(\theta_{vi}), \tag{11}$$

where $\sum_{i=1}^{N} \alpha_i = 1$ and $\alpha_i \geqslant 0$ are the convex coordinates. The ith-vertex of this convex polytope is defined by $P_i := (A_i, B_i)$ for all $i \in \mathbf{N}_0 := \{1, 2, ..., N\}$, where each of these matrices is constant.

The main objective is to find a constant gain matrix $K \in \Re^{n\times 2n}$ such that, the feedback control input

$$\boldsymbol{\delta\tau} = -K\,\boldsymbol{\delta x}, \tag{12}$$

into the system (6) results in an asymptotic stable closed loop system. By considering (4), it implies that the nonlinear robot dynamics (2) tracks the desired trajectory $\bar{\mathbf{x}}$ asymptotically when the control input torque

$$\boldsymbol{\tau} = \bar{\boldsymbol{\tau}} - K\boldsymbol{\delta x}, \tag{13}$$

is implemented to the robot manipulator.

As stated before, when the number of vertices is large, a critical problem may be occurred for any control synthesis. For this reason, an algorithm is presented in the next section to eliminate the superfluous vertices.

3. Clustering algorithm

The objective of this section is to propose an algorithm based on FCM clustering such that a new polytopic model with the least number of vertex is introduced.

3.1. FCM clustering

Let a set of N data point as the polytopic model (11) with the N vertices models $P_i := (A_i, B_i)$ are given, and c be a positive integer greater than one. For computational simplicity, the FCM clustering algorithm is applied only to the varying parameters of each vertex model by defining the $X_i := Vec(A_i, B_i)$ as a vector of varying elements of (A_i, B_i). Then the FCM clustering objective function [24,26] is expressed as:

$$J_m(\mu, a) = \sum_{k=1}^{N} \sum_{i=1}^{c} \mu_{ik}^m \| X_k - a_i \|^2, \tag{14}$$

where $m > 1$ is the weighting exponent, $a = \{a_1, ..., a_c\}$ is the set of cluster centers, c is the number of clusters, and μ_{ik} represents the membership degree of data point X_k to the cluster i with

$$\begin{aligned}\mu &= [\mu_{ik}]_{c\times N} \in M_{fcm} \\ &= \left\{ \mu = [\mu_{ik}]_{c\times N} \middle| \sum_{i=1}^{c} \mu_{ik} = 1,\ \mu_{ik} \geqslant 0,\ 0 < \sum_{k=1}^{N} \mu_{ik} < N \right\}.\end{aligned} \tag{15}$$

The FCM algorithm goal is to find a partition matrix $\mu = [\mu_{ik}]_{c\times N}$ and a set $a = \{a_1, ..., a_c\}$of cluster centers to minimize the objective function $J_m(\mu, a)$. The following updating equations are found as the necessary conditions for the minimum of $J_m(\mu, a)$.

$$a_i = \frac{\sum_{k=1}^{N} \mu_{ik}^m X_k}{\sum_{k=1}^{N} \mu_{ik}^m} \tag{16}$$

$$\mu_{ik} = \frac{\| X_k - a_i \|^{\frac{-2}{m-1}}}{\sum_{j=1}^{c} \| X_k - a_j \|^{\frac{-2}{m-1}}} \tag{17}$$

Using (16) and (17), the following FCM clustering algorithm can be introduced to obtain a partition matrix $\mu = [\mu_{ik}]_{c\times N}$ and cluster centers $a = \{a_1, ..., a_c\}$.

FCM Algorithm

Step 1: Fix $2 \leqslant c \leqslant N$ and fix any $\varepsilon > 0$. Give an initial $a^{(0)}$ and let $t = 0$.
Step 2: Using (17) to compute the membership $\mu^{(t+1)}$ with $a^{(t)}$.
Step 3: Using (16) to update the cluster center $a^{(t+1)}$ with $\mu^{(t+1)}$.
Step 4: Compare $a^{(t+1)}$ to $a^{(t)}$ in a convenient norm $\|\cdot\|$. IF $\|a^{(t+1)} - a^{(t)}\| < \varepsilon$, stop the algorithm, otherwise $t = t + 1$ and return to step 2.

3.2. Proposed algorithm

With the FCM clustering algorithm, we can categorize the LPV models into c clusters that their models have the minimum Euclidian error norm with respect to their center models. However, it doesn't give us an idea to select the optimal value for c or a proper value for ε. Therefore, the aim of this section is to propose a complete algorithm that gives the best clustering relative to the desired control objectives.

As stated before, the main control objective is to design a full state-feedback control gain matrix to have an asymptotically stable closed loop system. Hence, the condition for some vertices to consist a cluster is that there exists a common state-feedback control gain matrix that asymptotically stabilize those vertices. To introduce the mentioned condition, first the following problem should be solved.

Problem 3.1. Suppose that the FCM clustering algorithm is applied to the given polytopic model (11) with N vertices for an arbitrary $2 \leqslant c \leqslant N$. Let Π_j be the set of models in the jth-cluster and $\mathbf{N}_j$ denote their indices, i.e., for $j = 1, 2, ..., c$, we have:

$$\Pi_j := \left\{ (A_i, B_i) \middle| i \in \mathbf{N}_0,\ \max_k \mu_{ki} = \mu_{ji} \right\}, \tag{18}$$

$$\mathbf{N}_j := \{ i \in \mathbf{N}_0 | (A_i, B_i) \in \Pi_j \}. \tag{19}$$

Find a full state-feedback control gain matrix K_j that stabilizes the following systems for $j = 1, 2, ..., c$.

$$\begin{aligned}\dot{x} &= A_i x + B_i u \\ u &= -K_j x\end{aligned}, \tag{20}$$

$$\begin{aligned}\dot{x} &= A_{0j} x + B_{0j} u \\ u &= -K_j x\end{aligned}, \tag{21}$$

for all $(A_i, B_i) \in \mathbf{\Pi}_j$ and $i \in \mathbf{N}_j$, where (A_{0j}, B_{0j}) is the representative model of jth-cluster (the model with the highest degree of membership in jth-cluster).

Solution: Consider the following uncertain system.

$$\begin{aligned}\dot{x} &= (A_{0j} + \Delta A_{ij})x + (B_{0j} + \Delta B_{ij})u \\ u &= -K_j x\end{aligned}, \tag{22}$$

where $\Delta A_{ij} := A_i - A_{0j}$ and $\Delta B_{ij} := B_i - B_{0j}$ for $i \in \mathbf{N}_j$ can be considered as parametric uncertainties about the representative model of jth-cluster. Then the closed-loop system dynamics of (22) can be described by

$$\dot{x} = (A_{0j} - B_{0j}K_j)x + \Delta_{ij} x, \tag{23}$$

where

$$\Delta_{ij} := \Delta A_{ij} - \Delta B_{ij} K_j, \tag{24}$$

for $i \in \mathbf{N}_j$ is the closed loop parametric uncertainties. Now, let $V = x^T P x$ be a quadratic Lyapunov candidate function, where

$P = P^T > 0$ is a symmetric positive definite matrix. Evaluating the time derivative of V along (23) implies that

$$\dot{V} = x^T(A_{0j}^T - K_j^T B_{0j}^T)Px + x^T P(A_{0j} - B_{0j}K_j)x + 2x^T \Delta_{ij}^T Px. \tag{25}$$

Replacing $K_j = M_j X^{-1}$ and $P = X^{-1}$, it follows

$$\dot{V} = x^T P\left[XA_{0j}^T - M_j^T B_{0j}^T + A_{0j}X - B_{0j}M_j\right]Px + 2x^T \Delta_{ij}^T Px. \tag{26}$$

If the following LMI is feasible,

$$XA_{0j}^T - M_j^T B_{0j}^T + A_{0j}X - B_{0j}M_j < -\gamma_j X \tag{27}$$

for sufficiently large scalar $\gamma_j > 0$, then from (26) it will be observed that

$$\begin{aligned}\dot{V} &< -\gamma_j x^T PXPx + 2x^T \Delta_{ij}^T Px \\ &= x^T\left[-\gamma_j I + 2\Delta_{ij}^T\right]Px.\end{aligned} \tag{28}$$

It is obvious that $\dot{V} < 0$ can be concluded if the following condition is satisfied,

$$\max_{i \in \mathbb{N}_j} \| \Delta_{ij} \| < \frac{1}{2}\gamma_j. \tag{29}$$

It implies that if the LMI (27) and condition (29) are satisfied, then the state-feedback control gain $K_j = M_j X^{-1}$ stabilizes all models of jth-cluster.

Definition 3.1. Such a cluster that the LMI (27) and condition (29) are satisfied for its models, is called as feasible-cluster.

In other word, if the jth-cluster is a *feasible-cluster*, then all its vertices models can be substituted for one vertex model that is the representative model (A_{0j}, B_{0j}). Therefore, by this strategy we can search for *feasible-clusters* and construct a new polytopic model with vertices of all representative models (A_{0j}, B_{0j}). Now the question that may be arisen is how the parameters c and γ_j for $j = 1, 2, \ldots, c$ should be obtained. The following search algorithm responds this question.

Proposed Algorithm

Step 1: Fix $c = 2$ and construct a set of given primary models namely *models set*.

Step 2: Using FCM algorithm for the *models set* to compute (A_{0j}, B_{0j}), $\mathbf{N}_j$ and Π_j for $j = 1, 2, \ldots, c$.

Step 3: Fix $j = 1$ and $\gamma_j = \gamma_0$ (a lower bound, defined by designer, here we adopt $\gamma_0 = 1$).

Step 4: Solving the LMI (27) to compute $K_j = M_j X^{-1}$ and using (24) to compute Δ_{ij} and check the condition (29) for all $j = 1, 2, \ldots, c$. If the condition (29) isn't satisfied for any value of j, then go to Step 9.

Step 5: Extracting the *feasible-clusters* for $j = 1, 2, \ldots, c$ and remove their models from the *models set*. Then set $c = c - n_f$, where n_f is the number of *feasible-clusters* in this step.

Step 6: If the *models set* is empty or $c = 0$, stop the algorithm.

Step 7: If the *models set* has only one model and $c = 1$, then stop the algorithm and the remained model in the *models set* is the last *feasible-cluster*.

Step 8: If $c = 1$, then set $c = c + 1$. Go to Step 2.

Step 9: Increasing γ_j and if $\gamma_j \leqslant \bar{\gamma}$ (an upper bound, defined by designer, here we adopt $\bar{\gamma} = 1000$), then go to Step 4. The step size for increasing γ_j is defined by designer. This paper applies the following increasing procedure.

$$\gamma = \begin{cases} \gamma + 1 & for \quad \gamma < 10 \\ \gamma + 10 & for \quad 10 \leqslant \gamma < 100 \\ \gamma + 100 & for \quad 100 \leqslant \gamma < 1000 \end{cases} \tag{30}$$

Step 10: Set $c = c + 1$ and go to Step 2,

Implementing the above algorithm to the polytopic model (11), results in the new reduced polytopic LPV system

$$P(\theta) \in Co\{P(\theta_{v01}),\ P(\theta_{v02}),\ \ldots,\ P(\theta_{v0N^-})\} = \sum_{j=1}^{N^-} \alpha_{0j} P(\theta_{v0j}), \tag{31}$$

where $\sum_{j=1}^{\bar{N}} \alpha_{0j} = 1$ and $\alpha_{0j} \geqslant 0$ are the convex coordinates, and $\bar{N}$ is the number of *feasible-clusters*. The jth-vertex of this convex polytope, $P_{0j} := (A_{0j}, B_{0j})$ for $j \in \{1, 2, \ldots, \bar{N}\}$, is the representative model of the jth-*feasible-cluster*.

4. Control design

The main control objective is to design the control law (12) for the LPV model (6), that is equivalent to design the control law (13) for the nonlinear dynamics model (1). As stated before, according to the algorithm presented in Section 3, the initial polytopic model (11) with N vertices can be reduced to a new reduced polytopic model (31) with $\bar{N}$ vertices.

Now, the objective is to find a constant matrix $K \in \Re^{n \times 2n}$, such that, the closed-loop system given by (6) and (12) is asymptotically stable. Using (12) and LPV formulation (31), the closed-loop system can be represented by

$$\begin{aligned}\delta\dot{\mathbf{x}} &= [A(\theta(t)) - B(\theta(t))K]\delta\mathbf{x} \\ &= \textstyle\sum_{j=1}^{\bar{N}} \alpha_{0j}(A_{0j} - B_{0j}K)\delta\mathbf{x} = \sum_{j=1}^{\bar{N}} \alpha_{0j} A_{clj}\delta\mathbf{x}\end{aligned} \tag{32}$$

where A_{clj} is the close loop system matrix of the jth model.

With this information, in the following theorem, the sufficient condition for asymptotic stability of the system (32) is proposed.

Theorem 4.1. *The polytopic LPV system (32) is asymptotically stable if there exist common symmetric positive definite matrices $X = X^T > 0$ and Msuch that for all $j \in \{1, 2, \ldots, \bar{N}\}$ the following LMIs are satisfied.*

$$XA_{0j}^T + A_{0j}X - M^T B_{0j}^T - B_{0j}M < -\gamma_j X, \tag{33}$$

where γ_j is that one resulted in the proposed algorithm for jth-feasible-cluster. If there exists such a solution, the controller gain is given by

$$K = MX^{-1}. \tag{34}$$

Proof. *Evaluating the time derivative of a quadratic Lyapunov candidate function $V = \delta\mathbf{x}^T P\delta\mathbf{x}$ along (32) implies that*

$$\begin{aligned}\dot{V} &= \textstyle\sum_{j=1}^{\bar{N}} \alpha_{0j}\delta\mathbf{x}^T A_{clj}^T P\delta\mathbf{x} + \delta\mathbf{x}^T P \sum_{j=1}^{\bar{N}} \alpha_{0j} A_{clj}\delta\mathbf{x} \\ &= \textstyle\sum_{j=1}^{\bar{N}} \alpha_{0j}\delta\mathbf{x}^T\left(A_{clj}^T P + PA_{clj}\right)\delta\mathbf{x} \\ &= \textstyle\sum_{j=1}^{\bar{N}} \alpha_{0j}\delta\mathbf{x}^T\left[\left(A_{0j}^T - K^T B_{0j}^T\right)P + P(A_{0j} - B_{0j}K)\right]\delta\mathbf{x}\end{aligned} \tag{35}$$

Replacing $K = MX^{-1}$ and $P = X^{-1}$ in (35), it follows

$$\dot{V} = \sum_{j=1}^{N^-} \alpha_{0j}\delta\mathbf{x}^T X^{-1}\left[XA_{0j}^T - M^T B_{0j}^T + A_{0j}X - B_{0j}M\right]X^{-1}\delta\mathbf{x} \tag{36}$$

Using (33), it will be observed that

$$\dot{V} < \sum_{j=1}^{\bar{N}} \alpha_{0j} \delta\mathbf{x}^T \left(-\gamma_j P\right) \delta\mathbf{x} \leqslant -\left(\sum_{j=1}^{\bar{N}} \alpha_{0j}\gamma_j\right) P \| \delta\mathbf{x} \|^2 . \tag{37}$$

It follows that $\dot{V} < 0$ and the proof is concluded. □

This theorem implies that if the set of LMIs (33) are feasible for all representative models, then there exists a symmetric matrix $P = X^{-1} > 0$ satisfying the condition of Lyapunov. $\dot{V} < 0$ for the system (32). Thus, system (32) considering the control gain (34) is asymptotically stable.

5. Simulation results

A two-link planar rotary robot manipulator, shown in Fig. 1, is considered for illustrating the proposed clustering algorithm, implementing the proposed controller and investigating the stability and performance behavior of the close loop system.

Through the Euler-Lagrangian approach, the manipulator dynamics equation is derived as compact form (1) with

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \mathbf{D}(\mathbf{q}) = \begin{bmatrix} D_{11}(\mathbf{q}) & D_{12}(\mathbf{q}) \\ D_{21}(\mathbf{q}) & D_{22}(\mathbf{q}) \end{bmatrix},$$

$$D_{11}(\mathbf{q}) = (M_1 + M_2)L_1^2 + M_2 L_2^2 + 2M_2 L_1 L_2 \cos q_2 ,$$

$$D_{12}(\mathbf{q}) = D_{21}(\mathbf{q}) = M_2 L_2^2 + M_2 L_1 L_2 \cos q_2 ,$$

$$D_{22}(\mathbf{q}) = M_2 L_2^2 ,$$

$$C(\dot{\mathbf{q}}, \mathbf{q}) = \begin{bmatrix} -M_2 L_1 L_2 (2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) \sin q_2 \\ M_2 L_1 L_2 \dot{q}_1^2 \sin q_2 \end{bmatrix},$$

$$G(\mathbf{q}) = \begin{bmatrix} G_1(\mathbf{q}) \\ G_2(\mathbf{q}) \end{bmatrix},$$

$$G_1(\mathbf{q}) = -(M_1 + M_2) L_1 g \sin q_1 - M_2 L_2 g \sin(q_1 + q_2),$$

$$G_2(\mathbf{q}) = -M_2 L_2 g \sin(q_1 + q_2),$$

The physical parameters of the robot manipulator are defined as follows: $L_1 = L_2 = 0.26\,(\mathrm{m})$, $M_1 = 6.5225\,(\mathrm{kg})$, $M_2 = 2.0458\,(\mathrm{kg})$. For testing the performance of the proposed control, the following reference trajectories are selected in joint space [42].

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{q}_1 \\ \bar{q}_2 \end{bmatrix} = \begin{bmatrix} b_1(1 - e^{-2t^3}) + c_1(1 - e^{-2t^3}) \sin(\omega_1 t) \\ b_2(1 - e^{-2t^3}) + c_2(1 - e^{-2t^3}) \sin(\omega_2 t) \end{bmatrix} \tag{38}$$

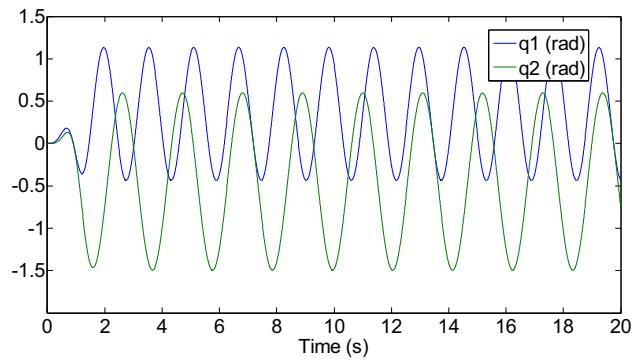

Fig. 2. Desired reference trajectories profiles.

where $b_1 = \pi/9$[rad], $c_1 = \pi/4$[rad] and $\omega_1 = 4$[rad/s], are parameters of the first joint reference and $b_2 = -\pi/7$[rad], $c_2 = \pi/3$[rad] and $\omega_2 = 3$[rad/s], correspond to parameters of the second joint trajectory. Fig. 2 shows graphs of these reference trajectories.

Using the desired trajectory information and sampling scheduling signal vector $\theta(t)$ with sampling time 0.1 s for a total time of 20 s to construct $N = 201$ primary vertices (A_i, B_i) by (9). In the first step the behavior of the conventional FCM clustering algorithm, is simulated. The primary LPV models can be categorized into c clusters such that the models of each cluster have the minimum Euclidian error norm relative to the center model. If ε_j denotes the maximum error norm between the models of jth-cluster and jth-center model, then the maximum error for all cluster can be computed as

$$\bar{\varepsilon} = \max_{j=1,2,\ldots,c} \varepsilon_j . \tag{39}$$

Fig. 3 shows the deviation of $\bar{\varepsilon}$ against different values of c. It is clear that for decreasing $\bar{\varepsilon}$, we should increase c, but there is no sense of the best value for $\bar{\varepsilon}$.

Now, implementing the proposed algorithm presented in Section 3.2, the resulted information is shown in Table 1. The number of generated clusters is 78, the indices of the models inside the clusters and the related γ that satisfy (33) are indicated in the table for some sample of clusters. The bolded and underlined indices show the representative model of each cluster.

For more illustration the effectiveness of the proposed algorithm, the poles of all primary models and representative models are indicated in Fig. 4. As we can see in this figure, the non-effective poles are removed.

Although the main objective of this paper is to introduce a new methodology for constructing a reduced polytopic model for a non-

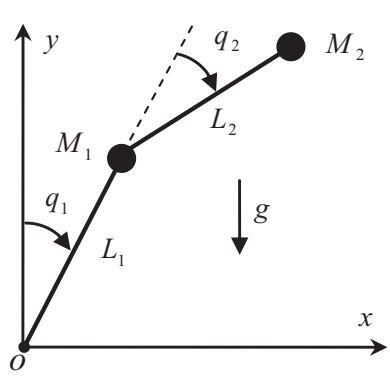

Fig. 1. Two-link planar rotary robot manipulator.

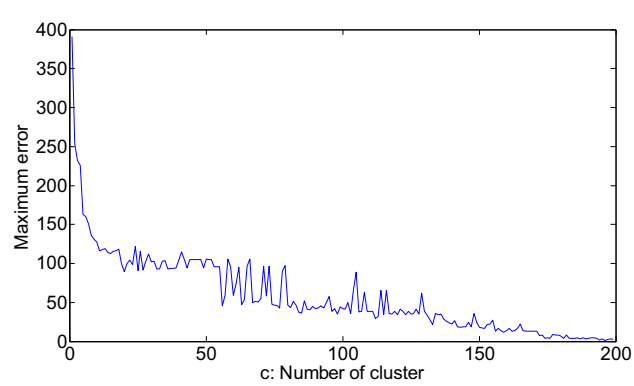

Fig. 3. Maximum error between models and centers for all clusters.

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Table 1
Some sample of clusters, the models indices, and related γ.

Cluster No.	Model index	γ
1	72, 73, 135, 136, **198**, 199	400
2	53, 71, **116**, 134, 179, 197	400
3	5, **6**, 7	50
⋮	⋮	⋮
32	34, **35**, 97, 98, 160, 161	300
33	29, **92**, 155	20
34	30, **93**	10
⋮	⋮	⋮
77	**62**	1
78	**80**	1

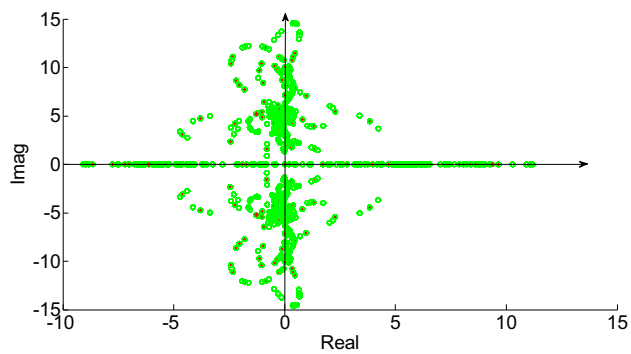

Fig. 4. The poles of the primary models and representative models.

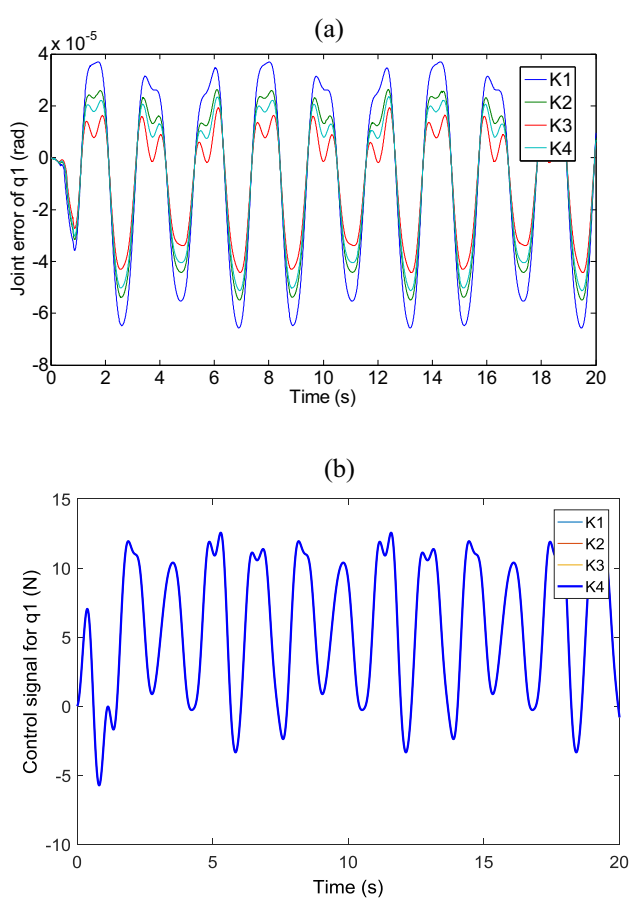

Fig. 5. (a) Tracking error of the first joint with control gains K_1, K_2, K_3 and K_4; (b) Control signal of the first joint.

linear system, the effectiveness of the proposed polytopic model is also compared with some different design control strategies. We have considered four control plans to design the control gain (34) by solving the set of LMIs (33).

In first controller K_1, the conventional FCM according to presented algorithm in Section 3.1 is used to construct a polytopic model with $c = 78$ vertices (is considered equal to the number of clusters in the proposed algorithm). Then the LMIs (33) are solved for the centers of the clusters with $\gamma_j = \gamma_{\max}$, $j = 1, 2, \ldots, c$, where $\gamma_{\max} = 500$ is chosen as the maximum value of γ that is obtained by our proposed algorithm shown in Table 1. The resulted control gain is computed as

$$K_1 = \begin{bmatrix} 8.2086 & 1.3787 & 0.2569 & 0.0640 \\ 1.4040 & 3.7722 & 0.0655 & 0.0457 \end{bmatrix} \times 10^3 . \tag{40}$$

The second controller K_2 is obtained by implementing the proposed algorithm presented in Section 3.2 and solving the LMIs (33) for the representative model of the clusters (specified in Table 1) with fixed $\gamma_{\max}$.

$$K_2 = \begin{bmatrix} 8.5435 & 1.8702 & 0.2690 & 0.0675 \\ 1.8867 & 2.6575 & 0.0681 & 0.0554 \end{bmatrix} \times 10^3 . \tag{41}$$

The next controller K_3, the proposed controller of this paper, is computed similar to the strategy of designing K_2 with this difference that using γ_j, $j = 1, 2, \ldots, c$ as described in Table 1 instead of using a fixed $\gamma_{\max}$ for solving the LMIs (33). It will imply that

$$K_3 = \begin{bmatrix} 8.5231 & 2.2679 & 0.2542 & 0.0677 \\ 1.9907 & 2.1801 & 0.0651 & 0.0457 \end{bmatrix} \times 10^3 . \tag{42}$$

Finally, the controller K_4 is designed without clustering. It is obtained by solving the LMIs (33) for whole of the primary models with fixed $\gamma_{\max}$.

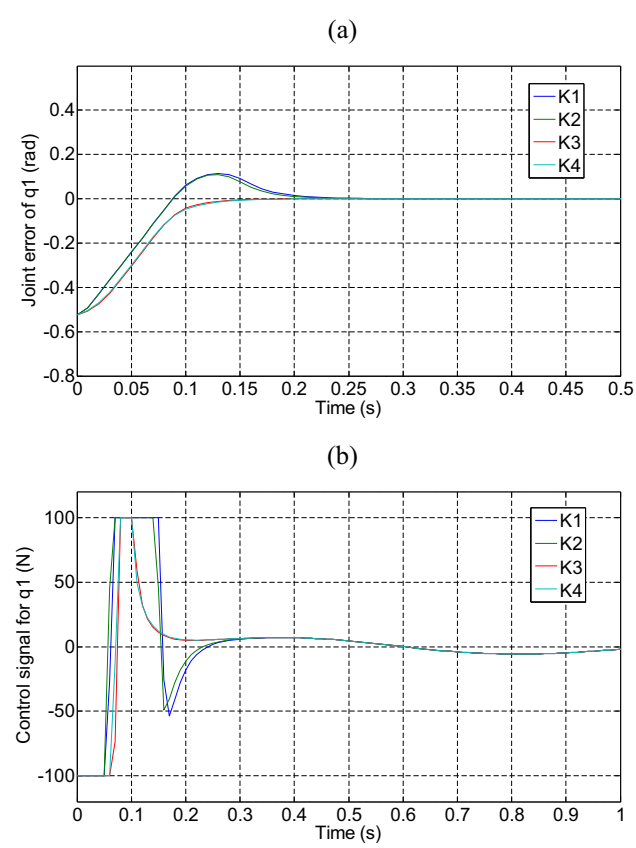

Fig. 6. (a) Tracking error of the first joint with control gains K_1, K_2, K_3 and K_4, and initial condition error; (b) Control signal of the first joint.

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Table 2
Computation time and performance comparison of the controllers.

Control methods	Proposed method		Conventional LMI-based	
	With optimum γ (K_3)	With fixed γ (K_2)	With conventional FCM (K_1)	Without FCM (K_4)
Computation time (s)	0.580	0.723	0.899	1.752
2-norm of tracking error	0.0040	0.0032	0.0026	0.0035

$$K_4 = \begin{bmatrix} 8.5709 & 1.9728 & 0.2680 & 0.0675 \\ 1.9941 & 2.3751 & 0.0682 & 0.0550 \end{bmatrix} \times 10^3 . \tag{43}$$

Applying the above control gains to manipulator for tracking the reference trajectories (38), the tracking error of manipulator's first joint is shown in Fig. 5a. The tracking error of the second joint has also the same behavior and therefore it is ignored to show. As can be seen in this figure, all the tracking errors belong to an acceptable range. Note that the difference between the errors are not very considerable and it cannot be a proper criterion for comparing the responses because it is obvious that by increasing γ, the tracking errors can be reduced to whatever is wanted, provided that the related LMIs are feasible and moreover, the control efforts is in an applicable scope. However, this figures show that the proposed controller (K_3) has a satisfactory performance close to the ideal case (K_4), beside it has a lower consuming time because of its reduced polytopic model. The number of LMIs is reduced from 201 for computing K_4 to 78 in the proposed controller and it effectively causes that the computation time of the proposed controller to be reduced. The control signals for all controllers are shown in Fig. 5b. Because the difference between the signals are very small, this figure shows that all the controllers have approximately equal control effort. This is because of the low amplitude of the error signals. So in next simulation the amplitude of error signals is increased by considering an arbitrary initial condition for each joint.

For more evaluating the performance of the controllers, the ability of the controllers to put the manipulator on the desired trajectory is investigated by applying a large distance error for each joint. This error may be occurred by a large external disturbance or an initial condition error. Considering the initial condition $q_{10} = \pi/6$ and $q_{20} = -\pi/6$, the tracking error of the first joint is shown in Fig. 6a. This figure shows that the controllers with gains K_3 and K_4 have approximately similar performance and suitable responses without any overshoot. In the other word, our proposed controller with a reduced polytopic model has presented a satisfactory performance comparable with the ideal case that a primary polytopic model without any clustering is used. In this case the control efforts are also shown in Fig. 6b. It can be seen in this figure that in the cases of K_3 and K_4, the control actuators have lower saturation time relative to the cases of K_1 and K_2.

Finally, the computation time and output performance of the proposed controller are compared with the conventional LMI controller in different cases as shown in Table 2. The 2-norm of the joints tracking error is chosen as a performance criterion for comparing the controllers. It can be seen that the proposed method has a lower computation time than others. However, the performance of the all controllers have an acceptable index and it shows that without any more performance distortion of the closed loop system, the computational time is impressively diminished.

6. Conclusion

In this paper, a new algorithm was addressed for LPV modelling of nonlinear systems in the purpose of state-feedback control design. The desired trajectory data is used to produce primary LPV models and then a Fuzzy-clustering algorithm is introduced to reduce the vertices models. A sufficient condition for asymptotic stability of the closed loop models is extracted as a criterion for categorizing the models. Since all control computations are based on trajectory data, they could be carried out off-line once the desired trajectory has been defined and hence, there is no restriction for on-line implementation of the proposed controller. It is cleared that there is a weighty control calculation for the primary polytopic model due to the huge number of vertex and LMI, however using the proposed algorithm in this paper, a new reduced polytopic model with a lower number of vertex is produced that highly decreases the computational time of controller synthesis. Using heuristic optimization approaches instead of Fuzzy-clustering can be also suggested for expanding the proposed method in future works. A 2-DOF robotic manipulator is considered as a numerical example. The proposed scheme is applied to generate a reduced polytopic model for the manipulator. The reduced polytopic model is used to design a full-state feedback controller for tracking of a desired trajectory, and it is shown that its performance is comparable with the ideal case, while its computational time is reduced to the about 33% of the primary controller where a polytopic model without any clustering is used.

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