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Location of cross-docking centers and vehicle routing scheduling under uncertainty: A fuzzy possibilistic–stochastic programming model

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ABSTRACT

The location of multiple cross-docking centers (CDCs) and vehicle routing scheduling are two crucial choices to be made in strategic/tactical and operational decision levels for logistics companies. The choices lead to more realistic problem under uncertainty by covering the decision levels in cross-docking distribution networks. This paper introduces two novel deterministic mixed-integer linear programming (MILP) models that are integrated for the location of CDCs and the scheduling of vehicle routing problem with multiple CDCs. Moreover, this paper proposes a hybrid fuzzy possibilistic–stochastic programming solution approach in attempting to incorporate two kinds of uncertainties into mathematical programming models. The proposed solving approach can explicitly tackle uncertainties and complexities by transforming the mathematical model with uncertain information into a deterministic model. m' imprecise constraints are converted into $2Rm'$ precise inclusive constraints that agree with $R\alpha$-cut levels, along with the concept of feasibility degree in the objective functions based on expected interval and expected value of fuzzy numbers. Finally, several test problems are generated to appraise the applicability and suitability of the proposed new two-phase MILP model that is solved by the developed hybrid solution approach involving a variety of uncertainties and complexities.

1. Introduction

In an increasingly competitive market, logistics companies are recently tending to well-performed distribution networks in supply chain management. Cross-docking distribution networks with minimal storage and retrieval functions are introduced to have vital benefits, such as reducing handling costs, operating costs, transportation costs, the storage of inventory, space requirement and delivery lead time as well as no or less inventory. In this distribution network, products arriving at cross-docking center (CDC) as key components from manufacturers or suppliers are carried to trucks serving the customers or retailers, and then are delivered to the retailers without being stored as inventory at the CDC by spending very little time. The functions at the CDC usually perform less than 48 h, sometimes less than 12 h [1].

Regarding the recent development of the cross-docking distribution networks, Chen et al. [2] consider a cross-docking distribution network with delivery and pickup time windows, warehouse capacities and inventory-handling costs. Then,

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local search approaches are extended and applied based on simulated annealing (SA) and tabu search (TS) algorithms. Lee et al. [3] address the scheduling problem of inbound and outbound trucks with the cross-docking. The mathematical model to maximize the number of product and the genetic algorithms are presented to solve the problem. Yeung and Lee [4] take the scheduling problem of delivery into consideration, where products are transported within time windows. The objective is to minimize the inventory, transportation and penalty cost. Then, the genetic algorithm is applied to solve the problem. Cóccola et al. [5] concentrate on the integration of the production and distribution management with cross-docking in the supply chain. To formulate the problem, the integrated mixed-integer linear programming (MILP)-based framework is presented for multi-site systems in the supply chain. Konur and Golias [6] investigate the truck scheduling problem at inbound doors of a cross-docking facility and regard the uncertainty in truck arrival times for operations. Then, the optimization approach is presented for the stable scheduling problem, and it is solved by the genetic algorithm. Liao et al. [7] focus on the simultaneous dock assignment and sequencing of inbound trucks for a multi-door cross-docking operation, and the objective is to minimize total weighted tardiness under a fixed outbound truck departure schedule. Then, six different meta-heuristic algorithms are presented to solve the problem, including SA, TS, ant colony optimization, differential evolution and two hybrid differential-evolution algorithms. Joo and Kim [8] study the truck scheduling problem for three types of trucks in a multi-door cross-docking facility. A mathematical model is proposed to determine the door assignments and the docking sequences of trucks, and then genetic and self-evolution algorithms are presented to solve the model.

In cross-docking distribution networks, finding the best locations for CDCs and the best vehicle routing scheduling with minimizing inventory and minimizing transportation cost has recently attracted the attention of some researchers. The integration of CDCs locating and vehicle routing scheduling can be introduced to solve two important issues related to cross-docking distribution networks concurrently. It assists logistics companies to take interactions between the strategic/tactical and operational decision levels into consideration with effects on long-term and short-term planning. The models are particularly necessary for the distribution networks where location costs can be compared with vehicle routing costs, for instance, distribution in perishable and agricultural products.

One of the first decisions that should be considered during the long-term planning is the location of one or more CDCs as a main component of the design of cross-docking distribution networks. Sung and Song [9] are the first researchers that focus on the CDCs location problem. In the study, products should be transferred from suppliers to retailers with a cross-docking distribution center. The cross-docking distribution center can be selected among possible locations regarding fixed costs. An integer programming model is proposed, similar to the model introduced by Donaldson et al. [10] and Musa et al. [11]. In [12], the authors develop their previous work. They present some improvements to the TS algorithm as the solving approach, and consider a branch-and-price algorithm to determine exact solutions. Gümüs and Bookbinder [13] take account of a similar cross-docking network design problem. Multiple product types are studied by allowing direct shipments. Jayaraman and Ross [14] present a different approach for the CDC location problem. A multi-echelon problem is considered where multiple products should be moved from a manufacturing plant to one or more distribution centers. They provide an integer programming model that attempts to minimize the fixed costs of distribution centers and CDCs and the different transportation costs. Ross and Jayaraman [15] also develop two hybrid meta-heuristics to deal with the location problem. Both hybrid meta-heuristics are on the basis of the SA algorithm; however, they utilize an extra mechanism to avoid locally optimal solutions. The first meta-heuristic applies a tabu list, and the second meta-heuristic regards a re-scaling of the system temperature. Bachlaus et al. [16] study a multi-echelon distribution network. The network contains suppliers, plants, distribution centers, CDCs and customers. A multi-objective optimization model is developed that aims to minimize the total costs and to maximize the plant and volume flexibility.

One of the important decisions in the distribution networks that should be considered during the short-term planning is the vehicle routing scheduling with the cross-docking. The delivery from one or more CDCs to customers or retailers can be managed by vehicle routing problem, and consolidating the items through CDCs can be done in the distribution networks. The pickup and delivery processes can be regarded as a vehicle routing problem. Several studies take account of the cross-docking and vehicle routing concurrently in recent years. Lee et al. [17] are the first researchers that study the vehicle routing scheduling problem with the cross-docking. They try to obtain an optimal routing schedule for pickup and delivery processes during the planning horizon. The model minimizes the transportation costs and fixed costs of the vehicles. The solving approach based on a TS algorithm is developed to provide near optimal solutions. Liao et al. [18] introduce another TS algorithm to solve the vehicle routing scheduling problem presented in [17]. Wen et al. [19] incorporate the vehicle routing problem in cross-docking systems. In the problem, orders from suppliers should be picked up by a homogeneous fleet of available vehicles.

The review of the literature indicates that there are a few papers studied the cross-docking location and vehicle routing scheduling problems. There are many opportunities to improve and develop the mathematical models reported in the literature for both location and vehicle routing scheduling problems. In the cross-docking location problem, for instance, the restrictions regarding time window are not taken into consideration in the pickup and delivery processes. Furthermore, inventories costs and restrictions regarding the capacity of multiple products (multi-commodities) at CDCs are not considered in previous works. For the vehicle routing scheduling problem in previous works, the routing schedule is formulated only with one CDC. Incorporating the vehicle routing scheduling with multiple CDCs along with multiple products can be considered to have a more realistic modeling approach in the cross-docking distribution networks.

Two major drawbacks in the previous studies, firstly, the cross-docking location problem and vehicle routing scheduling problem in the distribution networks is considered separately; however, the integration of two problems has a strong influence on the cross-docking distribution networks. In fact, these decisions are often inter-dependent in practice. The integration makes the problem more realistic by addressing the strategic/tactical and operational decisions simultaneously, and then it leads to the results be near to real-life situations. Secondly, critical parameters, such as products demands, supplies quantities in the pickup and delivery processes, volume capacity of vehicles, time for each vehicle to move between nodes, and transportation and operating costs, are deterministic. Values of these parameters may change in the distribution environment not only in long-term planning but also in short-term planning. Hence, the cross-docking distribution networks should be designed in a way that it can cope with the uncertainties in the important parameters.

To address the above-mentioned gaps in the literature, this paper designs a novel two-phase mathematical programming model, in which two MILP models are formulated and then they are integrated for the CDCs location and vehicle routing scheduling problems in the distribution networks. Also, to tackle the uncertainties in critical parameters, this paper introduces a new solving approach in fuzzy possibilistic–stochastic programming (FPSP) that has merits and advantages of the previously well-known developed approaches. By the proposed FPSP solution approach, some uncertainties in parameters can be quantified as probability density functions (PDFs) and the others can be expressed as fuzzy membership functions. In the proposed solution, both fuzzy possibilistic programming and stochastic programming are taken into account and are hybridized within an effective framework to handle the various uncertainties of input data.

In sum, this paper introduces a new two-phase fuzzy possibilistic–stochastic MILP model for the cross-docking location and routing problem that is able to: (1) incorporate both multiple cross-docks location and vehicle routing scheduling with multiple CDCs simultaneously by a new two-phase MILP model, resulted in an effective relation between two decision levels through the long-term and short-term planning; (2) present a new location model-based optimization in which operating costs at CDCs, multiple time periods to increase the accuracy of the model and capacity restriction on total costs for opening CDCs are considered for the first time in the literature. Also, operational, transportation and inventory costs are taken into account in different time periods; (3) present a new routing scheduling model-based optimization, in which multiple CDCs, multiple products, volume capacity restriction of each vehicle (route) in pickup and delivery processes, working time restriction of each vehicle in pickup and delivery processes, operational cost for each vehicle, due-date to deliver products to retailers (customers) and penalty costs for both early and tardy deliveries to retailers are taken into consideration for the first time in the literature; (4) consider various uncertainties in input data in the cross-docking location and routing problems for the first time in the literature, related to situations where randomness and fuzziness occur in a mathematical modeling framework; and (5) propose an efficient hybrid solution approach to address uncertainties and complexities in parameters that enhances the robustness of the optimization process by presenting the parameters as PDFs and/or fuzzy membership functions.

The rest of the paper is organized as follows. In the next section, the concerned problem is defined, and then the proposed two-phase MILP model is presented for the cross-docking distribution networks in Section 2. The proposed FPSP solution approach is developed in Section 3. Numerical analysis is provided in Section 4. Finally, Section 5 ended up with some concluding remarks.

2. Modeling cross-docking centers location and vehicle routing scheduling problem

This section outlines the cross-docking planning problem. A location and routing problem is considered in cross-docking distribution networks in this paper that consists of suppliers (pickup nodes), CDCs and retailers (delivery nodes). The structure of the multi-echelon distribution network is depicted in Fig. 1.

2.1. Problem description

The first phase of the proposed model focuses on making a decision in a strategic viewpoint where CDCs need to be located and performed in different time periods. It is a vital decision for logistics companies in long-term planning due to salient influences on the distribution networks. The objective in the first phase is to find three types of costs. First, fixed and operating costs at CDCs. Second, transportation costs to transfer units of multiple products from suppliers (pickup nodes) to CDCs and from CDCs to the retailers (delivery nodes). Third, holding costs are associated with inventories at CDCs. Assumptions of the first phase are provided below. Pickup and delivery are fulfilled within their specified time windows. All demands will be covered by the sufficient inventory of each product. Further, there are restrictions on the capacity of CDCs in each period.

The second phase of the proposed model focuses on making a decision in an operational viewpoint where the optimal route and the arrival time of each vehicle with multiple CDCs need to be obtained. It is an important decision for the logistics companies in short-term planning due to salient influences on the distribution networks. The objective in the second phase is to find minimum transportation costs related to transferring multiple products in the pickup and delivery processes separately, operational costs of vehicles and penalty costs for earliness and tardiness deliveries to retailers. Limitations of the second phase are provided below. Each supplier or retailer can only be picked up or delivered once. The total quantity of pickup must equal the quantity to be delivered. The load on the pickup route and on the delivery route for each vehicle cannot exceed the capacity of the vehicle. Also, maximum working time of each vehicle is regarded in pickup and delivery processes. In this phase, it is assumed that vehicles are located in the multiple CDCs, and pickup and split deliveries are not

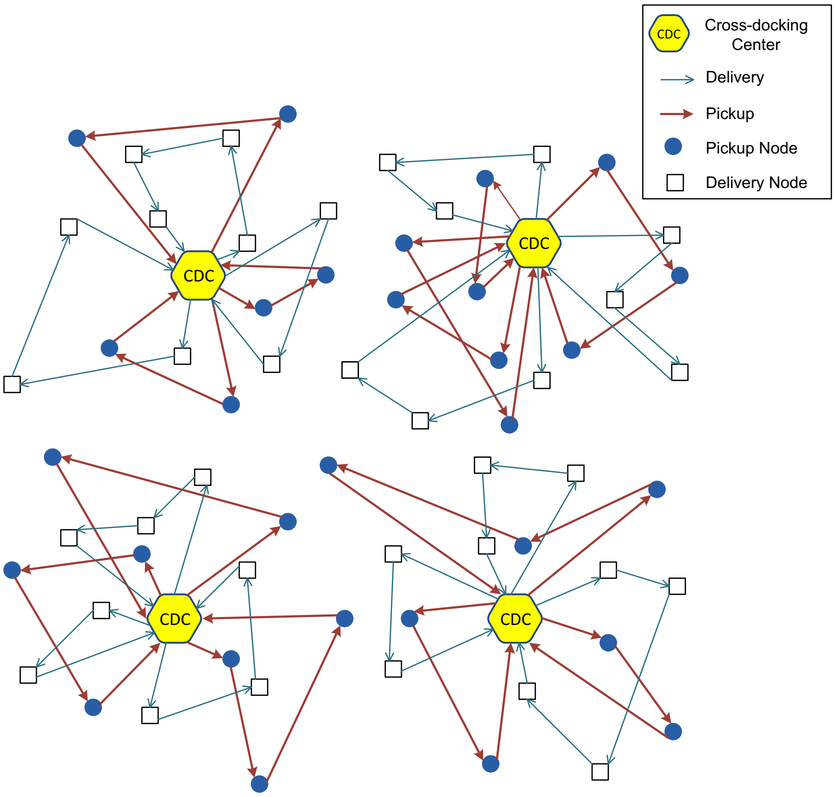

Fig. 1. A proposed network for multiple CDCs in the distribution system.

allowed. The vehicles for the delivery move to the retailers and then return to the same CDC after completing their tours. Moreover, since the cross-docking is basically a just-in-time logistics system for the distribution planning, this paper takes earliness/tardiness penalty into account for early/tardy outbound trucks violating the retailers' due-date in the delivery process. This issue mandates the planning, routing and scheduling in the CDCs to be considered due-dates of various retailers in the delivery process. Just-in-time distribution through the cross-docking is regarded as a modern logistical service allowing logistics managers to operate with low stocks while maintaining optimal service standards, and it brings about numerous advantages for all partners of the supply chain. It is worth to mention that to make a best decision and effective connection between two phases in long-term and short-term planning, critical parameters of the second phase (vehicle routing scheduling) are taken by the values obtained in the first phase (location of multiple CDCs). The parameters are demands, supply quantities, and number of pickup nodes, CDCs and delivery nodes.

2.2. Proposed FPSP model for cross-docking distribution networks

In this sub-section, notations are presented for the formulation of the proposed fuzzy possibilistic–stochastic MILP model in the location problem of multiple CDCs for the first phase and vehicle routing scheduling problem with multiple CDCs for the second phase, respectively.

Sets and parameters
P: Set of pickup nodes ($i = 1,2,\ldots,n$)
D: Set of delivery nodes ($i' = 1,2,\ldots,m$)
K: Set of CDC ($k = 1,2,\ldots,c$)
p: Set of products ($p = 1,2,\ldots,q$)
t: Set of time ($t = t_{\min},\ldots,t_{\max}$)
t': Set of time periods ($t' = 1,2,\ldots,T$)
V: Set of vehicle in pickup process ($v = 1,2,\ldots,V$)
V': Set of vehicle in delivery process ($v' = 1,2,\ldots,V'$)
$Q_{i,p,t'}$: is 1 if product type p is picked up in node i in period t', and 0 otherwise
$Q_{i',p,t'}$: is 1 if product type p is delivered by node i' in period t', and 0 otherwise
$Am_{i,p,t'}$: Amount of product type p in pickup node i in period t'

$Am_{i',p,t'}$: Amount of product type p in delivery node i' in period t'
$\widetilde{Dist}_{i,k}$: Distance of pickup node i from CDC k
$\widetilde{Dist}_{i',k}$: Distance of delivery node i' from CDC k
$\widetilde{C}_{i,k}$: Transportation cost per distance unit from pickup node i from CDC k
$\widetilde{C}_{i',k}$: Transportation cost per distance unit from of delivery node i' from CDC k
TS_i: Lower bound of time window for pickup node i
$TS_{i'}$: Lower bound of time window for delivery node i'
TE_i: Upper bound of time window for pickup node i
$TE_{i'}$: Upper bound of time window for delivery node i'
$cap_{k,t'}$: Capacity of CDC k in period t'
$H\widetilde{C}_{k,p,t'}$: Holding cost per unit product type p in a unit of time at CDC k in period t'
$T_{\min}$, $T_{\max}$: Minimum and maximum of time horizon
$\widetilde{F}_k$: Fixed opening cost to open CDC k
$O\widetilde{C}_{kt'}$: Operating cost at CDC k in period t'
$\widetilde{c}_v$: Operational cost of the vehicle v
$\widetilde{c}_{v'}$: Operational cost of the vehicle v'
$T\widetilde{C}^S$: Maximum total cost which could pay for opening CDCs
p_{ip}: Loaded amount of product type p in node i in pickup process
$d_{i'p}$: Unloaded amount of product type p in node i' in delivery process
$\widetilde{c}_{ij}$: Transportation cost from node i to node j in pickup process
$\widetilde{c}_{i'j'}$: Transportation cost from node i' to node j' in delivery process
$C\widetilde{A}_v$: Volume capacity of vehicle (route) v in pickup process
$C\widetilde{A}_{v'}$: Volume capacity of vehicle (route) v' in delivery process
$\widetilde{f}_p$: Unit volume of product type p
$\widetilde{t}_{ij}^{v}$: Time for the vehicle v to move from node i to node j in pickup process
$\widetilde{t}_{i'j'}^{v'}$: Time for the vehicle v' to move from node i' to node j' in delivery process
$\widetilde{T}_v^S$: Maximum working time of vehicle v in pickup process
$\widetilde{T}_{v'}^S$: Maximum working time of vehicle v' in delivery process
$\alpha_{i'}$: Penalty unit of early delivery from retailer in node i' in delivery process
$\beta_{i'}$: Penalty unit of tardy delivery from retailer in node i' in delivery process
$d_{i'}$: Due-date demanded by retailer in node i' in delivery process.

Decision variables

$X_{p,k,t,t'}^{i}$: 1 if product type p in pickup i goes to CDC k in period t' at time t, and 0 otherwise
$X_{p,k,t,t'}^{i'}$: 1 if product type p in delivery i' is bound for CDC k in period t' at time t, and 0 otherwise
$S_{p,k,t,t'}$: Amount of product type p at CDC k in period t' at time t
x_k: 1 if CDC k is open, and 0 otherwise
x_{ijv}: 1 if point i immediately precedes point j by vehicle v in pickup process, and 0 otherwise $(i,j \in P \cup K)$
$x_{i'j'v'}$: 1 if point i' immediately precedes point j' by vehicle v' in delivery process, and 0 otherwise $(i',j' \in D \cup K)$
Z_{kj}: 1 if pickup node j is allocated to CDC k, and 0 otherwise
$Z_{kj'}$: 1 if delivery node j' is allocated to CDC k, and 0 otherwise
u_{iv}: Auxiliary variable for sub-tour elimination constraints in route v in pickup process
$u_{i'v'}$: Auxiliary variable for sub-tour elimination constraints in route v' in delivery process
w_{iv}: Volume of products to be collected by the vehicle of type v upon arriving at i in pickup process
$w_{i'v'}$: Volume of products remaining to be delivered by the vehicle of type v' upon arriving at i' in delivery process
at_i^v: Arrival time of vehicle v at node i in pickup process
$at_{i'}^{v'}$: Arrival time of vehicle v' at node i' in delivery process
$E_{i'}$: Total earliness delivery penalty demanded by retailer in node i' in delivery process
$L_{i'}$: Total tardiness delivery penalty demanded by retailer in node i' in delivery process.

2.2.1. Multiple CDCs location model (phase one)

The CDCs location problem for the first phase of the proposed fuzzy possibilistic–stochastic model can be formulated in terms of the above notations from a strategic viewpoint as follows:

$$\begin{aligned} \text{Minimize } Z_1 = {} & \sum_{t'=1}^{T}\sum_{k=1}^{c}(\widetilde{F}_k + O\widetilde{C}_{kt'})x_k + \sum_{t'=1}^{T}\sum_{k=1}^{c}\sum_{p=1}^{q}\sum_{t=T_{\min}}^{T_{\max}} H\widetilde{C}_{k,p,t'}S_{p,k,t,t'} + \sum_{t'=1}^{T}\sum_{p=1}^{q}\sum_{i'=1}^{m}\sum_{k=1}^{c}\sum_{t=T_{\min}}^{T_{\max}} X_{p,k,t,t'}^{i'}\widetilde{Dist}_{i',k}\widetilde{C}_{i',k} \\ & + \sum_{t'=1}^{T}\sum_{p=1}^{q}\sum_{i=1}^{n}\sum_{k=1}^{c}\sum_{t=T_{\min}}^{T_{\max}} X_{p,k,t,t'}^{i}\widetilde{Dist}_{i,k}\widetilde{C}_{i,k}. \end{aligned} \tag{1}$$

Subject to:

$$\sum_{k=1}^{c}\sum_{t=T_{\min}}^{Ts_d-1} X^{i'}_{p,k,t,t'} = 0, \quad \forall i', p, t', \tag{2}$$

$$\sum_{k=1}^{c}\sum_{t=TS_d}^{TE_d} X^{i'}_{p,k,t,t'} \leqslant 1, \quad \forall i', p, t', \tag{3}$$

$$\sum_{k=1}^{c}\sum_{t=TE_d+1}^{T_{\max}} X^{i'}_{p,k,t,t'} = 0, \quad \forall i', p, t', \tag{4}$$

$$\sum_{k=1}^{c}\sum_{t=T_{\min}}^{Ts_p-1} X^{i}_{p,k,t,t'} = 0, \quad \forall i, p, t', \tag{5}$$

$$\sum_{k=1}^{c}\sum_{t=TS_p}^{TE_p} X^{i}_{p,k,t,t'} = 1, \quad \forall i, p, t' \text{ and } Q_{i,p,t'} = 1, \tag{6}$$

$$\sum_{k=1}^{c}\sum_{t=TE_p+1}^{T_{\max}} X^{i}_{p,k,t,t'} = 0, \quad \forall i, p, t', \tag{7}$$

$$\sum_{p=1}^{q} S_{p,k,t,t'} \leqslant cap_{k,t'}, \quad \forall k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{8}$$

$$s_{p,k,t',T_{\min-1}} = 0, \quad \forall p, k, t', \tag{9}$$

$$s_{p,k,t,t'} = s_{p,k,t-1,t'} - \sum_{i'=1}^{m} X^{i'}_{p,k,t,t'} Q_{i',p,t'} Am_{i',p,t'} + \sum_{i=1}^{n} X^{i}_{p,k,t,t'} Q_{i,p,t'} Am_{i,p,t'} \ \forall p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{10}$$

$$\sum_{k=1}^{C}\sum_{i=1}^{n} X^{i}_{p,k,t,t'} Q_{i,p,t'} Am_{i,p,t'} - \sum_{k=1}^{C}\sum_{i'=1}^{m} X^{i'}_{p,k,t,t'} Q_{i',p,t'} Am_{i',p,t'} \geqslant 0, \ \forall p, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{11}$$

$$\sum_{k=1}^{C} \widetilde{F}_k x_k \leqslant T\widetilde{C}^S, \tag{12}$$

$$X^{i}_{p,k,t,t'} \leqslant x_k, \quad \forall i, p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{13}$$

$$X^{i'}_{p,k,t,t'} \leqslant x_k, \quad \forall i', p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{14}$$

$$S_{p,k,t,t'} \geqslant 0, \quad \forall p, k, t' \text{ and } T_{min} \leqslant t \leqslant T_{\max}, \tag{15}$$

$$X^{i}_{p,k,t,t'}, \quad X^{i'}_{p,k,t,t'}, \quad x_k \in \{0, 1\}. \tag{16}$$

Eq. (1) indicates the total costs to be minimized in the objective function including costs of holding inventories at the CDC and costs of transportation (i.e., costs of transportation from suppliers to CDCs and then from CDCs to customers), fixed costs and operating costs at the CDC. Constraints (2)-(4) guarantee that each delivery, if necessary, is fulfilled within its specified time window and beyond that range it takes the value of zero in each time period. According to Eqs. (5)–(7), the time window restrictions are taken into consideration for pickups. Constraint (8) considers the potential capacity of CDCs. Eq. (9) corresponds to a zero initial inventory for each product at each CDC. Eq. (10) controls any changes in the inventory level of each CDC at each time. Constraint (11) guarantees the sufficient inventory of each product to cover all demands in each time period. Constraint (12) considers capacity restriction on total costs that can be paid for opening CDCs. Constraints (13) and (14) establish that moving product p from suppliers to CDC and from CDC to retailers in the pickup and delivery processes can be performed only when the corresponding CDC is open in each time period. Finally, constraints (15) and (16) guarantee the non-negativity and binary of the corresponding decision variables.

2.2.2. Vehicle routing scheduling model with multiple CDCs (phase two)

The vehicle routing scheduling problem with multiple CDCs for the second phase of the proposed model can be formulated in terms of the above notations from an operational viewpoint as follows:

$$\begin{aligned}\textit{Minimize}\ Z_2 = &\sum_{i\in P\cup K}\sum_{j\in P\cup K}\sum_{v\in V}\widetilde{c}_{ij}x_{ijv} + \sum_{i'\in D\cup K}\sum_{j'\in D\cup K}\sum_{v'\in V'}\widetilde{c}_{i'j'}x_{i'j'v'} + \sum_{i'\in D}\sum_{j'\in D}\sum_{v'\in V'}\sum_{p}(E_{i'}+L_{i'})x_{i'j'v'} + \sum_{i\in K}\sum_{j\in P}\sum_{v\in V}\widetilde{c}_{v}x_{ijv}\\ &+\sum_{i'\in K}\sum_{j'\in D}\sum_{v'\in V'}\widetilde{c}_{v'}x_{i'j'v'}.\end{aligned} \tag{17}$$

Subject to:

$$\sum_{i\in P\cup K}\sum_{v\in V}x_{ijv} = 1, \quad \forall j\in P, \tag{18}$$

$$\sum_{i'\in D\cup K}\sum_{v'\in V'}x_{i'j'v'} = 1, \quad \forall j'\in D, \tag{19}$$

$$u_{iv} - u_{jv} + nx_{ijv} \leqslant n-1, \quad \forall i,j\in P \text{ and } \forall v\in V, \tag{20}$$

$$u_{i'v'} - u_{j'v'} + mx_{i'j'v'} \leqslant m-1, \quad \forall i',j'\in D \text{ and } \forall v'\in V', \tag{21}$$

$$\sum_{i\in P\cup K}x_{ijv} = \sum_{j\in P\cup K}x_{jiv}, \quad \forall i\in P\cup K \text{ and } \forall v\in V, \tag{22}$$

$$\sum_{i'\in D\cup K}x_{i'j'v'} = \sum_{j'\in D\cup K}x_{i'j'v'}, \quad \forall i'\in D\cup K \text{ and } \ \forall v'\in V', \tag{23}$$

$$\sum_{i\in K}\sum_{j\in P}x_{ijv} \leqslant 1, \quad \forall v\in V, \tag{24}$$

$$\sum_{i'\in K}\sum_{j'\in D}x_{i'j'v'} \leqslant 1, \quad \forall v'\in V', \tag{25}$$

$$-Z_{kj} + \sum_{l\in P\cup K}(x_{ilv} + x_{ljv}) \leqslant 1, \quad \forall i\in K, \quad \forall j\in P \text{ and } \forall v\in V, \tag{26}$$

$$-Z_{kj'} + \sum_{l'\in D\cup K}(x_{i'l'v'} + x_{l'j'v'}) \leqslant 1, \quad \forall i'\in K, \quad \forall j'\in D \text{ and } \forall v'\in V', \tag{27}$$

$$w_{iv} + \sum_{p=1}^{q}\widetilde{f}_{p}p_{ip} - w_{jv} \leqslant (1-x_{ijv})C\widetilde{A}_{v}, \quad \forall i\in P\cup K, \quad \forall j\in P \text{ and } \forall v\in V, \tag{28}$$

$$w_{i'v'} - \sum_{p=1}^{q}\widetilde{f}_{p}d_{i'p} - w_{j'v'} \geqslant (1-x_{i'j'v'})C\widetilde{A}_{v'}, \quad \forall i'\in D\cup K, \quad \forall j'\in D \text{ and } \forall v'\in V', \tag{29}$$

$$at_i^{v} \geqslant \widetilde{t}_{ij}^{v}, \quad \forall v\in V, \quad \forall i\in K \text{ and } \forall j\in P, \tag{30}$$

$$at_{i'}^{v'} \geqslant \widetilde{t}_{i'j'}^{v'}, \quad \forall v'\in V', \quad \forall i'\in K \text{ and } \forall j'\in D, \tag{31}$$

$$at_i^{v} + \widetilde{t}_{ij}^{v} - at_j^{v} \leqslant (1-x_{ijv})\widetilde{T}_{v}^{s}, \quad \forall i,j\in P \text{ and } \forall v\in V, \tag{32}$$

$$at_{i'}^{v'} + \widetilde{t}_{i'j'}^{v'} - at_{j'}^{v'} \leqslant (1-x_{i'j'v'})\widetilde{T}_{v'}^{s}, \quad \forall i',j'\in D \text{ and } \forall v'\in V', \tag{33}$$

$$E_{i'} = \max\left(0; d_{i'} - at_{i'}^{v'}\right)\alpha_{i'}, \quad \forall i'\in D \text{ and } \forall v'\in V', \tag{34}$$

$$L_{i'} = \max\left(0; at_{i'}^{v'} - d_{i'}\right)\beta_{i'}, \quad \forall i'\in D \text{ and } \forall v'\in V', \tag{35}$$

$$x_{ijv}, x_{i'j'v'}, \quad Z_{kj}, Z_{kj'} \in \{0,1\}, \tag{36}$$

$$u_{iv}, u_{i'v'}, w_{iv}, w_{i'v'}, at_i^{v}, at_{i'}^{v'} \geqslant 0. \tag{37}$$

Eq. (17) indicates the total costs to be minimized in the objective function including transportation costs related to moving multiple products in the pickup and delivery processes, penalty costs for total earliness and tardiness delivery to retailers in the delivery process, and operational costs of vehicles in the processes. Eqs. (18) and (19) consider that one vehicle has to arrive at and leave one node in the pickup process, CDC and delivery process. Further, these equations establish that every supplier or retailer belongs to one and only one route. Constraints (20) and (21) make sub-tours impossible. Eqs. (22) and (23) take the consecutive movement of vehicles into consideration. Constraints (24) and (25) show whether or not a vehicle arrives at and leaves a CDC. According to constraints (26) and (27), a retailer or supplier can be assigned to a CDC only if there is a route from that CDC going through that retailer or supplier. Constraints (28) and (29) indicate that the quantity of loaded and unloaded products in a vehicle cannot exceed the maximum capacity of the vehicle in the pickup and delivery processes. Constraints (30) to (33) enforce that all time windows are respected. Eqs. (34) and (35) show the calculations for earliness and tardiness penalty in the delivery process. Finally, constraints (36) and (37) guarantee the binary and non-negativity of the corresponding decision variables.

3. Proposed solution approach

The proposed two-phase model is a fuzzy possibilistic–stochastic MILP that incorporates various uncertainties-concepts into the cross-docking distribution networks. To solve the model, a hybrid solution approach is developed based on fuzzy possibilistic programming and stochastic programming. For this purpose, the original model under uncertainty is transformed into an equivalent auxiliary crisp model by applying an efficient FPSP solution approach resulted from the hybridization of the recent new effective approaches presented by Liu et al. [20], Jimenez et al. [21] and Pishvaee and Torabi [22]: (a) fuzzy possibilistic programming and (b) chance-constraint programming. The proposed hybrid FPSP solution approach is employed to find the final preferred compromise solution under uncertainty.

3.1. Hybrid FPSP solution approach

Parameters in the objective function, technological coefficients and right hand sides of the MILP model are addressed as uncertain values in nature. For this purpose, the chance-constrained programming is integrated within the fuzzy possibilistic framework for taking account of distribution information of the MILP model's right-hand sides. Also, appropriate possibility distributions of the parameters in the objective function are determined according to the definition of the expected interval (*EI*) and expected value (*EV*) of fuzzy numbers. Finally, it results in a hybrid FPSP model as follows:

$$Minimize\ f = \widetilde{C}X. \tag{38}$$

Subject to:

$$\widetilde{A}X \,\widetilde{\geqslant}\, \widetilde{B}, \tag{39}$$

with

$$\widetilde{B} = \left(b_1, b_2, \ldots, b_{m'_1}, b^{(p_1)}_{m'_1+1}, b^{(p_2)}_{m'_1+2}, \ldots, b_{m'}^{\left(p_{m'-m'_1}\right)}\right), \quad x_j \geqslant 0, \quad x_j \in X, j = 1, 2, \ldots, n,$$

where that $\widetilde{C}$ is a triangular fuzzy number, Eq. (40) can be expressed as the membership function of $\widetilde{c}$:

$$\mu_{\widetilde{c}}(x) = \begin{cases} f_c(x) = \frac{x - c^p}{c^m - c^p} & \text{if} \quad c^p \leqslant x \leqslant c^m \\ 1 & \text{if} \quad x = c^m \\ g_c(x) = \frac{c^o - x}{c^o - c^m} & \text{if} \quad c^m \leqslant x \leqslant c^o \\ 0 & \text{if} \quad x \leqslant c^p \text{or} x \geqslant c^o \end{cases} \tag{40}$$

The *EI* and *EV* of triangular fuzzy number $\widetilde{C}$ can be obtained as follows [21,22]:

$$EI(\widetilde{c}) = [E_1^c, E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx\right] = \left[\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^o)\right]$$

and

$$EV(\widetilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4}.$$

Constraints in (39) have fuzzy left hand-side and right hand-side coefficients. In addition, some of the right-hand sides in (39), (i.e., $b^{(p_1)}_{m'_1+1}, b^{(p_2)}_{m'_1+2}, \ldots, b_{m'}^{(p_{m'-m'_1})}$) are presented as probability distributions. Hence, if below conditions hold in terms of level sets,

$$\left\{\mu_{A_{ij}}(a_{ij})|a_{ij} \in [0,1]\right\} = \{\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{iR}\},$$

$$0 \leqslant \alpha_{i1} \leqslant \alpha_{i2} \leqslant \ldots \leqslant \alpha_{iR} \leqslant 1, \quad i = 1,2,\ldots,m'. \tag{41}$$

Then, fuzzy constraints in (39) can be replaced by the following $2R$ precise inequalities, in which R indicates R levels of α-cut [23,24].

$$\overline{A}^l X \leqslant \overline{B}^l, \quad l = 1,2,\ldots,R, \tag{42}$$

$$\underline{A}^l X \leqslant \underline{B}^l, \quad l = 1,2,\ldots,R, \tag{43}$$

where

$$\overline{A}^l = \sup(A^l),$$

$$\overline{B}^l = \sup(B^l),$$

$$\underline{A}^l = \inf(A^l),$$

$$\underline{B}^l = \inf(B^l).$$

Model (38) can be transformed into a conventional linear programming problem based on [20–22] as follows:

$$\mathit{Minimize}\ f = EV(\widetilde{C})X. \tag{44}$$

Subject to:

$$\sum_{j=1}^{n}(\overline{a}_{ij}^{s}X) \leqslant \overline{B}_i^s, \tag{45}$$

$$\text{with} \quad \overline{B}_i^s = \begin{cases} \overline{b}_i^s & \text{when} \quad i = 1,2,\ldots,m_1'; \quad s = 1,2,\ldots,R_1 \\ \overline{b}_i^{s(p_i)} & \text{when} \quad i = m_1'+1, m_1'+2,\ldots,m_1'; \quad s = R_1+1, R_1+2,\ldots,R \end{cases}$$

$$\sum_{j=1}^{n}(\underline{a}_{ij}^{s}X) \geqslant \underline{B}_i^s, \tag{46}$$

$$\text{with} \quad \underline{B}_i^s = \begin{cases} \underline{b}_i^s & \text{when} \quad i = 1,2,\ldots,m_1'; \quad s = 1,2,\ldots,R_1 \\ \underline{b}_i^{s(p_i)} & \text{when} \quad i = m_1'+1, m_1'+2,\ldots,m_1'; \quad s = R_1+1, R_1+2,\ldots,R \end{cases}$$

$$x_j \geqslant 0, \quad j = 1,2,\ldots,n. \tag{47}$$

For the right-hand side of constraint, boundaries of its fuzzy intervals under any α-cut levels have random characteristics. They can be presented as normal distributions as follows:

$$p[\underline{b_2}(s)] = \frac{1}{\sqrt{2\pi}\underline{\sigma}} \exp\left\{-\frac{[\underline{b_2}(s) - \underline{\mu}]^2}{2\underline{\sigma}^2}\right\} \tag{48}$$

and

$$p[\overline{b_2}(s)] = \frac{1}{\sqrt{2\pi}\overline{\sigma}} \exp\left\{-\frac{[\overline{b_2}(s) - \overline{\mu}]^2}{2\overline{\sigma}^2}\right\}. \tag{49}$$

where $\underline{\mu}$ and $\overline{\mu}$ are expected values of $\underline{b_2}(s)$ and $\overline{b_2}(s)$, respectively; Also, $\underline{\sigma}^2$ and $\overline{\sigma}^2$ are the relevant variances [20].

Consequently, the delimited decision space as defined in (45) to (47) will be more robust in dealing with uncertainties for the cross-docking distribution networks. According to [20] as the problem dimension can be expanded to elaborate the uncertain distribution networks, the robustness of the mathematical programming is remarkably enhanced.

3.2. FPSP model of multiple CDCs location (phase one)

According to the above descriptions, the FPSP model for multiple CDCs location problem in the first phase of the proposed MILP model can be provided as follows:

$$\begin{aligned}
MinimizeZ_1 = {} & \sum_{t'=1}^{T}\sum_{k=1}^{c}\left(\frac{F_k^p + 2F_k^m + F_k^o + OC_{k,t'}^p + 2OC_{k,t'}^m + OC_{k,t'}^o}{4}\right)x_k \\
& + \sum_{t'=1}^{T}\sum_{k=1}^{c}\sum_{p=1}^{q}\sum_{t=T_{\min}}^{T_{\max}}\left(\frac{HC_{k,p,t'}^p + 2HC_{k,p,t'}^m + HC_{k,p,t'}^o}{4}\right)S_{p,k,t,t'} \\
& + \sum_{t'=1}^{T}\sum_{p=1}^{q}\sum_{i'=1}^{m}\sum_{k=1}^{c}\sum_{t=T_{\min}}^{T_{\max}}X_{p,k,t,t'}^{i'}\left(\frac{Dist_{i',k}^p + 2Dist_{i',k}^m + Dist_{i',k}^o}{4}\right)\left(\frac{C_{i',k}^p + 2C_{i',k}^m + C_{i',k}^o}{4}\right) \\
& + \sum_{t'=1}^{T}\sum_{p=1}^{q}\sum_{i=1}^{n}\sum_{k=1}^{c}\sum_{t=T_{\min}}^{T_{\max}}X_{p,k,t,t'}^{i}\left(\frac{Dist_{i,k}^p + 2Dist_{i,k}^m + Dist_{i,k}^o}{4}\right)\left(\frac{C_{i,k}^p + 2C_{i,k}^m + C_{i,k}^o}{4}\right).
\end{aligned} \tag{50}$$

Subject to:

$$\sum_{k=1}^{c}\sum_{t=T\min}^{TS_d-1}X_{p,k,t,t'}^{i'} = 0, \quad \forall i', p, t', \tag{51}$$

$$\sum_{k=1}^{c}\sum_{t=TS_d}^{TE_d}X_{p,k,t,t'}^{i'} \leqslant 1, \quad \forall i', p, t', \tag{52}$$

$$\sum_{k=1}^{c}\sum_{t=DE_d+1}^{T_{\max}}X_{p,k,t,t'}^{i'} = 0, \quad \forall i', p, t', \tag{53}$$

$$\sum_{k=1}^{c}\sum_{t=T_{\min}}^{TS_p-1}X_{p,k,t,t'}^{i} = 0, \quad \forall i, p, t', \tag{54}$$

$$\sum_{k=1}^{c}\sum_{t=TS_p}^{TE_p}X_{p,k,t,t'}^{i} = 1, \quad \forall p, i, t' \text{ and } Q_{i,p,t'} = 1, \tag{55}$$

$$\sum_{k=1}^{c}\sum_{t=TE_p+1}^{T_{\max}}X_{p,k,t,t'}^{i} = 0, \quad \forall p, i, t', \tag{56}$$

$$\sum_{p=1}^{q}S_{p,k,t,t'} \leqslant cap_{k,t'}, \quad \forall k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{57}$$

$$S_{p,k,t',T_{\min-1}} = 0, \quad \forall p, k, t', \tag{58}$$

$$S_{p,k,t,t'} = S_{p,k,t-1,t'} - \sum_{i'=1}^{m}X_{p,k,t,t'}^{i'}Q_{i',p,t'}Am_{i',p,t'} + \sum_{i=1}^{n}X_{p,k,t,t'}^{i}Q_{i,p,t'}Am_{i,p,t'}, \quad \forall p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{59}$$

$$\sum_{k=1}^{c}\sum_{i=1}^{n}X_{p,k,t,t'}^{i}Q_{i,p,t'}Am_{i,p,t'} - \sum_{k=1}^{c}\sum_{i'=1}^{m}X_{p,k,t,t'}^{i'}Q_{i',p,t'}Am_{i',p,t'} \geqslant 0, \quad \forall p, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{60}$$

$$\sum_{k=1}^{c}\left(\sup\left[\left(F_k^o - \alpha\left(F_k^o - F_k^m\right)\right), \quad \left(F_k^p + \alpha\left(F_k^m - F_k^p\right)\right)\right]\right)x_k \leqslant \sup\left[\left(TC^o - \alpha\left(TC^o - TC^m\right)\right), \left(TC^p + \alpha\left(TC^m - TC^p\right)\right)\right]^{pi}, \tag{61}$$

$$\sum_{k=1}^{c}\left(\inf\left[(F_k^o - \alpha(F_k^o - F_k^m)), (F_k^p + \alpha(F_k^m - F_k^p))\right]\right)x_k \geqslant \inf\left[(TC^o - \alpha(TC^o - TC^m)), \quad (TC^p + \alpha(TC^m - TC^p))\right]^{pi}, \tag{62}$$

$$X_{p,k,t,t'}^{i'} \leqslant x_k, \quad \forall i', p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{63}$$

$$X_{p,k,t,t'}^{i} \leqslant x_k, \quad \forall i, p, k, t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{64}$$

$$S_{p,k,t,t'} \geqslant 0, \quad \forall p,k,t' \text{ and } T_{\min} \leqslant t \leqslant T_{\max}, \tag{65}$$

$$X^{i}_{p,k,t,t'}, \quad X^{i'}_{p,k,t,t'}, \quad x_k \in \{0,1\}. \tag{66}$$

3.3. FPSP model of vehicle routing scheduling (phase two)

According to the above descriptions, the FPSP model for vehicle routing scheduling with multiple CDCs in the second phase of the proposed model can be provided as follows:

$$\begin{aligned} \textit{Minimize } Z_2 = {} & \sum_{i\in P\cup K}\sum_{j\in P\cup K}\sum_{v\in V}\left(\frac{c^p_{i,j}+2c^m_{i,j}+c^o_{i,j}}{4}\right)x_{ijv} + \sum_{i'\in D\cup K}\sum_{j'\in D\cup K}\sum_{v'\in V'}\left(\frac{c^p_{i',j'}+2c^m_{i',j'}+c^o_{i',j'}}{4}\right)x_{i'j'v'} \\ & + \sum_{i'\in D}\sum_{j'\in D}\sum_{v'\in V'}\sum_{p}(E_{i'}+L_{i'})x_{i'j'v'} + \sum_{i\in K}\sum_{j\in P}\sum_{v\in V}\left(\frac{c^p_{v}+2c^m_{v}+c^o_{v}}{4}\right)x_{ijv} + \sum_{i'\in K}\sum_{j'\in D}\sum_{v'\in V'}\left(\frac{c^p_{v'}+2c^m_{v'}+c^o_{v'}}{4}\right)x_{i'j'v'}. \end{aligned} \tag{67}$$

Subject to:

$$\sum_{i\in P\cup K}\sum_{v\in V}x_{ijv} = 1, \quad \forall j \in P, \tag{68}$$

$$\sum_{i'\in D\cup K}\sum_{v'\in V'}x_{i'j'v'} = 1, \quad \forall j' \in D, \tag{69}$$

$$u_{iv} - u_{jv} + n x_{ijv} \leqslant n-1, \quad \forall i,j \in P \text{ and } \forall v \in V, \tag{70}$$

$$u_{i'v'} - u_{j'v'} + m x_{i'j'v'} \leqslant m-1, \quad \forall i',j' \in D \text{ and } \forall v' \in V', \tag{71}$$

$$\sum_{i\in P\cup K}x_{ijv} = \sum_{j\in P\cup K}x_{jiv}, \quad \forall i \in P\cup K \text{ and } \forall v \in V, \tag{72}$$

$$\sum_{i'\in D\cup K}x_{i'j'v'} = \sum_{j'\in D\cup K}x_{i'j'v'}, \quad \forall i' \in D\cup K \text{ and } \forall v' \in V', \tag{73}$$

$$\sum_{i\in K}\sum_{j\in P}x_{ijv} \leqslant 1, \quad \forall v \in V, \tag{74}$$

$$\sum_{i'\in K}\sum_{j'\in D}x_{i'j'v'} \leqslant 1, \quad \forall v' \in V', \tag{75}$$

$$-Z_{kj} + \sum_{l\in P\cup K}(x_{ilv} + x_{ljv}) \leqslant 1, \quad \forall i \in K, \quad \forall j \in P \text{ and } \forall v \in V, \tag{76}$$

$$-Z_{kj'} + \sum_{l'\in D\cup K}(x_{i'l'v'} + x_{l'j'v'}) \leqslant 1, \quad \forall i' \in K, \quad \forall j' \in D \text{ and } \forall v' \in V', \tag{77}$$

$$\begin{aligned} & w_{iv} + \sum_{p=1}^{q}\Big(\sup\Big[\Big(f^o_p - \alpha\Big(f^o_p - f^m_p\Big)\Big), \quad \Big(f^p_p + \alpha\Big(f^m_p - f^p_p\Big)\Big)\Big]\Big)p_{ip} - w_{jv} \\ & \quad \leqslant (1-x_{ijv})\big(\sup\big[(CA^o_v - \alpha(CA^o_v - CA^m_v)), \quad (CA^p_v + \alpha(CA^m_v - CA^p_v))\big]\big), \quad \forall i \in P\cup K, \quad \forall j \in P \text{ and } \forall v \in V, \end{aligned} \tag{78}$$

$$\begin{aligned} & w_{iv} + \sum_{p=1}^{q}\Big(\inf\Big[(f^o_p - \alpha(f^o_p - f^m_p)), \quad \Big(f^p_p + \alpha(f^m_p - f^p_p)\Big)\Big]\Big)p_{ip} - w_{jv} \\ & \quad \geqslant (1-x_{ijv})\big(\inf\big[(CA^o_v - \alpha(CA^o_v - CA^m_v)), \quad (CA^p_v + \alpha(CA^m_v - CA^p_v))\big]\big), \quad \forall i \in P\cup K, \quad \forall j \in P \text{ and } \forall v \in V, \end{aligned} \tag{79}$$

$$\begin{aligned} & w_{i'v'} - \sum_{p=1}^{q}\Big(\inf\Big[(f^o_p - \alpha(f^o_p - f^m_p)), \quad \Big(f^p_p + \alpha(f^m_p - f^p_p)\Big)\Big]\Big)d_{i'p} - w_{j'v'} \\ & \quad \geqslant (1-x_{i'j'v'})\big(\inf\big[(CA^o_{v'} - \alpha\big(CA^o_{v'} - CA^m_{v'}\big)), \quad (CA^p_{v'} + \alpha\big(CA^m_{v'} - CA^p_{v'}\big))\big]\big), \quad \forall i' \in D\cup K, \quad \forall j' \in D \text{ and } \forall v' \in V', \end{aligned} \tag{80}$$

$$w_{i'v'} - \sum_{p=1}^{q}\left(\sup\left[\left(f_p^o - \alpha\left(f_p^o - f_p^m\right)\right), \left(f_p^p + \alpha\left(f_p^m - f_p^p\right)\right)\right]\right)d_{i'p} - w_{j'v'}$$
$$\leqslant (1 - x_{i'j'v'})(\sup[(CA_{v'}^o - \alpha(CA_{v'}^o - CA_{v'}^m)), (CA_{v'}^p + \alpha(CA_{v'}^m - CA_{v'}^p))]), \ \forall i' \in D \cup K, \quad \forall j' \in D \text{ and } \forall v' \in V', \tag{81}$$

$$at_i^v \geqslant \alpha\left(\frac{t_{ij}^{vm} + t_{ij}^{vo}}{2}\right) + (1-\alpha)\left(\frac{t_{ij}^{vp} + t_{ij}^{vm}}{2}\right), \quad \forall v \in V, \quad \forall i \in K \text{ and } \forall j \in P, \tag{82}$$

$$at_{i'}^{v'} \geqslant \alpha\left(\frac{t_{i'j'}^{v'm} + t_{i'j'}^{v'o}}{2}\right) + (1-\alpha)\left(\frac{t_{i'j'}^{v'p} + t_{i'j'}^{v'm}}{2}\right), \quad \forall v' \in V', \quad \forall i' \in K \text{ and } \forall j' \in D, \tag{83}$$

$$at_i^v + \left(\sup\left[\left(t_{ij}^{vo} - \alpha(t_{ij}^{vo} - t_{ij}^{vm})\right), \quad \left(t_{ij}^{vp} + \alpha(t_{ij}^{vo} - t_{ij}^{vp})\right)\right]\right) - at_j^v$$
$$\leqslant (1 - x_{ijv})\left(\sup[(T_v^o - \alpha(T_v^o - T_v^m)) \quad , (T_v^p + \alpha(T_v^m - T_v^p))]^{pi}\right), \quad \forall i,j \in P \text{ and } \forall v \in V, \tag{84}$$

$$at_i^v + \left(\inf\left[\left(t_{ij}^{vo} - \alpha(t_{ij}^{vo} - t_{ij}^{vm})\right), \quad \left(t_{ij}^{vp} + \alpha(t_{ij}^{vo} - t_{ij}^{vp})\right)\right]\right) - at_j^v$$
$$\geqslant (1 - x_{ijv})\left(\inf[(T_v^o - \alpha(T_v^o - T_v^m)) \quad , (T_v^p + \alpha(T_v^m - T_v^p))]^{pi}\right), \quad \forall i,j \in P \text{ and } \forall v \in V, \tag{85}$$

$$at_{i'}^{v'} + \left(\sup\left[\left(t_{i'j'}^{v'o} - \alpha(t_{i'j'}^{v'o} - t_{i'j'}^{v'm})\right), \quad \left(t_{i'j'}^{v'p} + \alpha(t_{i'j'}^{v'o} - t_{i'j'}^{v'p})\right)\right]\right) - at_{j'}^{v'}$$
$$\leqslant (1 - x_{i'j'v'})\left(\sup[(T_{v'}^o - \alpha(T_{v'}^o - T_{v'}^m)), \quad (T_{v'}^p + \alpha(T_{v'}^m - T_{v'}^p))]^{pi}\right), \ \forall i',j' \in D \text{ and } \forall v' \in V, \tag{86}$$

$$at_{i'}^{v'} + \left(\inf\left[\left(t_{i'j'}^{v'o} - \alpha(t_{i'j'}^{v'o} - t_{i'j'}^{v'm})\right), \quad \left(t_{i'j'}^{v'p} + \alpha(t_{i'j'}^{v'o} - t_{i'j'}^{v'p})\right)\right]\right) - at_{j'}^{v'}$$
$$\geqslant (1 - x_{i'j'v'})\left(\inf[(T_{v'}^o - \alpha(T_{v'}^o - T_{v'}^m)), \quad (T_{v'}^p + \alpha(T_{v'}^m - T_{v'}^p))]^{pi}\right), \ \forall i',j' \in D \text{ and } \forall v' \in V, \tag{87}$$

$$E_{i'} = \max(0; d_{i'} - at_{i'}^{v'})\alpha_{i'}, \quad \forall i' \in D \text{ and } \forall v' \in V', \tag{88}$$

$$L_{i'} = \max(0; at_{i'}^{v'} - d_{i'})\beta_{i'}, \quad \forall i' \in D \text{ and } \forall v' \in V', \tag{89}$$

$$x_{ijv}, x_{i'j'v'}, \quad Z_{kj}, Z_{kj'} \in \{0,1\}, \tag{90}$$

$$u_{iv}, u_{i'v'}, w_{iv}, w_{i'v'}, at_i^v, at_{i'}^{v'} \geqslant 0. \tag{91}$$

4. Computational experiments

In this section, to evaluate the performance of the proposed two-phase MILP model and the usefulness of the proposed FPSP solution approach, numerical experiments are provided. The considered cross-docking distribution in two phases aims to determine the minimum number of CDCs among a set of location sites so that each retailer demands must be met. Then, the optimal schedule is obtained for the vehicle routing problem with multiple CDCs.

Five test problems for two phases of the proposed MILP model are taken into consideration. Their sizes are reported in Table 1. The needed data for pickup nodes, delivery nodes and information of the amount of different products in the three test problems are reported in Tables 2–4. The nominal data are randomly generated by uniform distributions as given in Table 5.

According to Tables 1–5, the proposed two-phase fuzzy possibilistic–stochastic MILP model is solved and reported by GAMS optimization software. The numerical experiments for each size are calculated under three α-cut levels ($\alpha = 0.1, 0.4, 0.6$). It is pointed out that the boundaries of fuzzy intervals for the right-hand side of constraint under α-cut levels have random characteristics, and they can be presented as normal distributions. Also, the values of the probability p_i set to 0.1 and 0.3 in the five test problems. Finally, the computational results for the proposed model in the first and second phases are reported in Tables 6 and 7, respectively.

Computational results demonstrate that the applicability and suitability of the proposed fuzzy possibilistic–stochastic MILP model in an uncertain environment. The proposed FPSP solving approach can explicitly deal with uncertainties and complexities by transforming the mathematical programming model into a deterministic model. Hence, in the proposed hybrid solution m' imprecise constraints are converted into $2Rm'$ precise inclusive constraints that agree with $R\alpha$-cut levels, along with the concept of feasibility degree in the objective function based on the EV and the EI of fuzzy numbers.

Table 1
Sizes of five test problems.

Problem No.	No. of suppliers or pickup nodes (n)	No. of potential cross-docking centers (c)	No. of retailers or delivery nodes (m)	No. of products (q)	No. of vehicles in pickup process (V)	No. of vehicles in delivery process (V')
1	3	2	3	2	2	2
2	4	3	5	3	2	3
3	6	4	7	3	3	4
4	7	4	8	3	4	5
5	8	5	9	4	5	6

Table 2
Data for pickup, delivery and amount of multiple products for the first test problem.

Node	Product		Amount		Time	
	1	2	1	2	Start	End
D_1	√	√	40	35	8	17
D_2	√		30	0	7	19
D_3	√	√	25	15	5	13
P_1	√	√	30	35	10	22
P_2		√	0	29	12	20
P_3	√	√	30	38	15	18

Table 3
Data for pickup, delivery and amount of multiple products for the second test problem.

Node	Product			Amount			Time	
	1	2	3	1	2	3	Start	End
D_1	√		√	15	0	25	4	14
D_2	√	√	√	23	5	34	6	16
D_3		√	√	0	30	11	11	25
D_4	√	√	√	20	15	17	2	19
P_1	√	√	√	10	13	25	7	25
P_2	√	√	√	2	19	30	9	3
P_3	√	√		210	30	0	5	14
P_4	√		√	40	0	18	9	22
P_5	√	√	√	8	28	25	10	19

Table 4
Data for pickup, delivery and amount of multiple products for the third test problem.

Node	Product			Amount			Time	
	1	2	3	1	2	3	Start	End
D_1	√	√	√	22	10	15	5	25
D_2		√	√	0	10	20	8	32
D_3			√	0	0	9	6	25
D_4	√		√	15	0	11	4	20
D_5	√	√	√	5	10	15	11	27
D_6	√			16	0	0	15	28
P_1		√		0	13	0	3	14
P_2	√		√	14	0	27	4	21
P_3	√	√		30	8	0	9	35
P_4	√	√	√	16	21	17	8	38
P_5		√	√	0	6	20	2	19
P_7	√	√	√	10	9	25	3	31

It can be observed that the constraints under various α-cut levels can effectively take complexities of the decision space into consideration. The constraints can assist top managers or decision makers to take account of numerous potential combinations of uncertainties represented in fuzzy intervals and reliabilities represented by α-cut levels in the logistics companies. Hence, the compromise solution is located at the boundary of the two sets of constraints with "$\leqslant$" and "$\geqslant$"

Table 5
Sources of random generation of nominal data for five test problems.

Parameters	Test problems				
	Problem No. 1	Problem No. 2	Problem No. 3	Problem No. 4	Problem No. 5
$HC_{k,p,t'}$	~Uniform (80,250)	~Uniform (100,250)	~Uniform (120,300)	~Uniform (100,320)	~Uniform (80,350)
$Dist_{i,k}$, $Dist_{i',k}$	~Uniform (22,50)	~Uniform (15,55)	~Uniform (11,60)	~Uniform (10,65)	~Uniform (10,70)
TC^s	~Uniform (4000,7000)	~Uniform (8500,13000)	~Uniform (14000,20000)	~Uniform (12,000,20,000)	~Uniform (11000,24000)
$OC_{kt'}$	~Uniform (100,400)	~Uniform (150,500)	~Uniform (200,600)	~Uniform (200,700)	~Uniform (200,800)
F_k	~Uniform (1500,6000)	~Uniform (2000,7000)	~Uniform (3000,8000)	~Uniform (2000,8000)	~Uniform (2000,9000)
$C_{i,k}$, $C_{i',k}$	~Uniform (30,300)	~Uniform (40,350)	~Uniform (50,400)	~Uniform (40,450)	~Uniform (40,500)
c_{ij}, $c_{i'j'}$	~Uniform (100,260)	~Uniform (110,350)	~Uniform (120,400)	~Uniform (100,500)	~Uniform (100,600)
CA_v, $CA_{v'}$	~Uniform (15,40)	~Uniform (15,55)	~Uniform (10,60)	~Uniform (10,65)	~Uniform (10,70)
$t_{ij}^{v}, t_{i'j'}^{v'}$	~Uniform (2,10)	~Uniform (3,15)	~Uniform (5,20)	~Uniform (5,25)	~Uniform (4,30)
$T_v^s, T_{v'}^s$	~Uniform (10,65)	~Uniform (20,70)	~Uniform (15,100)	~Uniform (15,120)	~Uniform (10,140)
$Cap_{k,t'}$	~Uniform (150,600)	~Uniform (250,700)	~Uniform (350,800)	~Uniform (300,850)	~Uniform (200,900)

Table 6
Computational results for five test problems in the first phase of the proposed MILP model under different combination of α and p_i values.

Test problems	Probability values	Objective function values α-cut levels		
		0.1	0.4	0.6
Problem No. 1	$p_i = 0.1$	4812.8	4696	4506.3
	$p_i = 0.3$	4542.1	4488.7	4403.2
Problem No. 2	$p_i = 0.1$	8539	8310.9	8285
	$p_i = 0.3$	8371.5	8277	8047
Problem No. 3	$p_i = 0.1$	13312	12645.4	10881.3
	$p_i = 0.3$	12181.7	11490	10121.6
Problem No. 4	$p_i = 0.1$	18600.2	17903.5	15930.1
	$p_i = 0.3$	16905.8	16206	14979.2
Problem No. 5	$p_i = 0.1$	29760.3	26855.2	24502.1
	$p_i = 0.3$	27099.2	25939.4	24202.9

Table 7
Computational results for five test problems for the second phase of proposed MILP model under different combination of α and p_i values.

Test problems	Probability values	Objective function values α-cut levels		
		0.1	0.4	0.6
Problem No. 1	$p_i = 0.1$	3039	2879.2	2623
	$p_i = 0.3$	2680.4	2539	2301.6
Problem No. 2	$p_i = 0.1$	5012	4910	4675
	$p_i = 0.3$	4865.1	4497.8	4213.7
Problem No. 3	$p_i = 0.1$	7560	7401.5	7216.9
	$p_i = 0.3$	7099.3	6843.6	6700.2
Problem No. 4	$p_i = 0.1$	13408.9	13322	12800.4
	$p_i = 0.3$	12978.7	12510.4	12260.3
Problem No. 5	$p_i = 0.1$	17601.4	16917.5	16087.5
	$p_i = 0.3$	16419.3	15010	14271.7

relationships, respectively. Finally, the proposed fuzzy possibilistic–stochastic MILP model tackling different uncertainties can illustrate a preferred compromise between optimality and reliability of the cross-docking distribution networks. It can properly depict a realistic reflection of the complexities in the networks. In fact, the decision makers can justify the decision schemes for the cross-docking planning under uncertainty through the incorporation of their implicit knowledge and experience.

5. Concluding remarks

To address the location and routing problem in the cross-docking distribution networks, this paper first develops a novel two-phase mathematical programming model. For this purpose, two new mixed-integer linear programming (MILP) are formulated to be closer to the needs of real-life applications. The two models are integrated for the location problem of multiple cross-docking centers (CDCs) in the first phase and a vehicle routing scheduling problem with multiple CDCs in the second phase in the distribution networks. Second, the uncertainty in parameters of the incorporated two-phase model that has critical influence from strategic/tactical and operational decision levels is taken into consideration for the first time in the literature. Third, to tackle uncertain parameters a new hybrid solution approach is introduced by combining fuzzy possibilistic programming and chance-constrained programming from the new recent effective methods. The uncertain parameters include distance of pickup and delivery nodes from CDCs, transportation costs, operating cost at CDCs, operational cost of vehicles, volume capacity of vehicles in pickup and delivery processes, time for vehicles to move between nodes, and maximum working time of vehicles. The proposed solution approach can effectively consider features of cross-docking distribution networks with complex and uncertain information. It can explicitly reflect uncertainties and complexities in such a network without unrealistic simplifications. Also, the uncertain parameters in the proposed two-phase mathematical programming model can be represented as probability density functions and/or fuzzy membership functions in the constraints. They can also be specified based on the concept of feasibility degree in the imprecise objective function by taking account of the strong mathematical concepts including expected interval and expected value of fuzzy numbers. It leads to an enhanced robustness of the optimization process and resulting solution. The computational results under combined α-cut levels and the values of the probability p_i are suitable for the logistics managers or decision makers to make the best decisions for the cross-docking planning through the incorporation of their implicit experience and knowledge. To the best knowledge of the authors, this paper is first to consider location and routing scheduling problem with the cross-docking under the fuzzy-stochastic environment, and to simultaneously propose the fuzzy possibilistic and chance-constrained programming to handle the uncertainty in such a cross-docking distribution network. Finally, the proposed fuzzy possibilistic–stochastic MILP model tackling different uncertainties can demonstrate a preferred compromise solution between optimality and reliability of decision schemes in logistics companies. Since the two-phase MILP model presented in this paper is the primary attempt in the cross-docking, different directions can be recommended for future research. For instance, it is suggested that the proposed fuzzy possibilistic–stochastic solution approach can be hybridized with the robust optimization theory associated with highly complex and uncertain conditions. Also, future research directions can be in addressing the location of cross-docking centers and vehicle routing scheduling problem when distributions are unknown. Integrating the vehicle routing scheduling problem with the sequencing problem of inbound and outbound trucks under uncertainty remains as a future research study. Proposing effective heuristic and meta-heuristic algorithms can be another possible topic to solve large-sized cross-docking problems. In addition, the presented solution approach can be employed in numerous practical problems in which probabilistic distributions and fuzzy possibilistic information can be available concurrently.

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